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# RELIABILITY ANALYSIS IN FRACTURE MECHANICS ACCORDING TO COMBINED FAILURE CRITERIA

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**Abstract.** *The interest in reliability analysis related to Fracture Mechanics field grew during the past decades. This contribution proposes a reliability analysis combining two failure criteria: the crack stability and the plastic collapse by mean of the Failure Assessment Diagram (FAD). FAD and reliability method are coupled to assess the probability of failure. Some issues have been highlighted considering classic gradient-based optimization methods to determine key points for approximation method. A strategy based on surrogate-based evolutionary algorithms is proposed to solve these issues. A discussion on the most influencing parameters and their consequences in terms of reliability based design is also presented. The application of the methodology is demonstrated on a simple test case.*

# 1 INTRODUCTION

A frequent way to treat uncertainties is to consider the worse conditions and run deterministic analysis. Nevertheless, this approach can generate over-sizing and the impact on the conservatism is generally not known. Consequently, reliability analysis tools are developed to assess the probability of failure of a component considering the uncertainties to estimate the level of conservatism.

A typical Fracture Mechanics problem is subject to uncertainties related to the geometry, material properties, loads or crack location and shape. Consequently, Fracture Mechanics is one the main applications for the reliability analysis [1, 2]. To assess the probability of failure in Fracture Mechanics issues [3], two main approaches have been developed, one based on the simulation (*Monte Carlo method*) and another based on the approximation. [4] consider a failure criterion linking the Fracture toughness and Stress Intensity Factors (SIF). Nevertheless, it is necessary to consider another failure criterion to include the plastic collapse to account for the remaining strength of the structure [5]. In [6], a complete reliability study considering these two criteria is presented using *Monte Carlo method* and *First Order Reliability Methods* (*FORM*). They consider the different failure modes associated with each criterion independently and propose a performance function based on the number of cycles to failure. Another approach [7] is to combine both failure criteria using the Failure Assessment Diagram (FAD). A reliability and sensitivity analysis based on this approach is proposed in [8]. They focus the performance function on the time inspection and perform *Monte Carlo* and *FORM* analysis. The present contribution presents reliability and sensibility analysis considering a FAD margin as performance function. Some difficulties are highlighted for *FORM* computation as a classical gradient-based optimisation fails to converge, this contribution proposes to solve the optimization problem of *FORM* using efficient surrogate-based evolutionary algorithm. Our approach allows to evaluate the *FORM* approximation for black box performance function because its gradient is not necessary. In addition to reliability analysis, a complete sensitivity of the FAD margin performance function is performed on a test case.

This paper is structured as follows. The first section introduces the main methods of reliability analysis. The second section reviews the main sensitivity indices and provides recommendations for a good understanding of the reliability problem. The third section presents the FAD margin performance function and the test case. It also includes the enhancement of the *FORM* optimization algorithm and a complete sensibility analysis of the test case. Finally, the last section concludes the and discusses future work.

# 2 APPLICATION TO FRACTURE MECHANICS

## 2.1 Probability of failure

Let us consider $\mathbf{X} = (X_1, \ldots, X_d)$ as a vector of $d$ random variables $X_i$ defined by respective probability density functions $f_{X_i}$. Each random variable $X_i$ is defined by distribution law characterised by a vector of $d_\theta$ parameters $\boldsymbol{\theta}_i = \{\theta_{ik}\},\ k = 1, \ldots, d_\theta$. Realizations of random variable $X$ is denoted by $x$ and a realization vector by $\mathbf{x} = x_1, \ldots, x_d$.

A performance function $G(\mathbf{X})$ is associated to the failure modes. This function allows splitting the space into two parts: the safe space where $G(\mathbf{X}) > 0$ and the failure space defined as $G(\mathbf{X}) \leq 0$. The border is the so-called limit state function and characterised as $G(\mathbf{X}) = 0$. The aim of reliability analysis is to compute the probability that an event is in the failure space as $P_f = \text{Prob}(G(\mathbf{X}) \leq 0)$. A structural reliability problem is solved using iso-probabilistic transformation that switches the problem from the physics space $\mathbb{X}^d$ to the standard space $\mathbb{U}^d$

defined by uncorrelated standard normal random variables. $P_f$ becomes:

$$P_f = \int_{G(\mathbf{X})\leq 0} f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x} = \int_{H(\mathbf{U})\leq 0} \phi_d(\mathbf{u})\mathrm{d}\mathbf{u}, \tag{1}$$

where $f_{\mathbf{X}}$ is the joint probability density of $X_i, i = 1, \ldots, d$, $\mathbf{U}$ are uncorrelated standard normal random variables, $\phi_d$ is the $d$-variate uncorrelated normal probability density function and $H(\mathbf{U})$ is the limit state function in the standard space.

## 2.2 Combination of failure criteria

The fundamental principle of Fracture Mechanics is that the stress field ahead of a sharp crack in a test specimen or a structural member can be characterized by a single parameter, $K$, which is the stress intensity factor (SIF). One of the main criteria in Fracture Mechanics relies on the SIF, it must not exceed the toughness $K_{1C}$ which is a material property. It is expressed using $K_r$ as:

$$K_r = \frac{K}{K_{1C}} < 1. \tag{2}$$

Another failure mode corresponds to the plastic collapse of the remaining ligament. A failure criterion $L_r$ is developed in order to measure the capacity of the component to sustain the mechanical load:

$$L_r = \frac{\sigma_n}{\sigma_{p0.2}} \leq \frac{\sigma_{p0.2} + \sigma_m}{2\sigma_{p0.2}}, \tag{3}$$

where $\sigma_n$ is the nominal stress in the remaining ligament, $\sigma_{p0.2}$ and $\sigma_m$ are respectively the yield and the ultimate strength.

The combination of both criteria is essential to assess the failure probability in Fracture Mechanics. Both criteria are combined by to the Failure Assessment Diagram (FAD) in $K_r - L_r$ space. In this diagram, the limit between safe and failure domains is defined by R6-rule [9] as a function of parameters by:

$$g(K_r, L_r) = K_r - \begin{cases} \frac{0.3+0.7\exp(-\mu L_r^6)}{\sqrt{1+0.5L_r^2}} & \text{for} \quad L_r \leq 1 \\ \frac{0.3+0.7\exp(-\mu)}{\sqrt{1.5}} L_r^{\frac{n_r-1}{2n_r}} & \text{for} \quad 1 < L_r < L_r^{\max} \\ 0 & \text{for} \quad L_r \geq L_r^{\max} \end{cases}, \quad \text{with:} \tag{4}$$

$$\mu = \min\{10^{-3}\frac{E}{\sigma_{p0.2}}, 0.6\}, \quad n_r = 0.3(1 - \frac{\sigma_{p0.2}}{\sigma_m}), \quad L_r^{\max} = \frac{\sigma_{p0.2} + \sigma_m}{2\sigma_{p0.2}},$$

where $E$ is the Young modulus. A strategy to evaluate the failure of structure in Fracture Mechanics is to determine $K_r$ and $L_r$ to define a point in the FAD diagram and to compute a function of the distance between this point and the R6-rule boundary [7]. This function is the margin, if it is negative it means a failure of the structure, and if it is positive, the structure is safe. As shown in Figure 1, the margin is function of the ratio $\frac{|\mathrm{OB}|}{|\mathrm{OA}|}$. The margin can be defined as the performance function $G(\mathbf{x})$ because it allows to combine the two failure criteria and $G(\mathbf{x})$ can be expressed as:

$$G(\mathbf{x}) = \left(\frac{|\mathrm{OB}|}{|\mathrm{OA}|} - 1\right) \begin{cases} > 0, & \text{safe} \\ \leq 0, & \text{failure} \end{cases}. \tag{5}$$

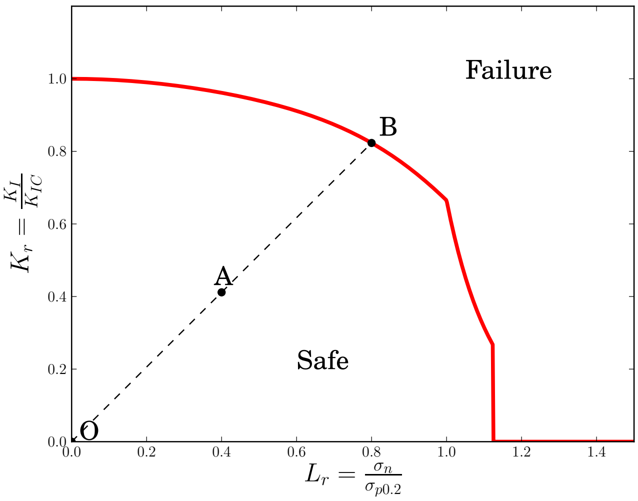


Figure 1: Failure Assessment Diagram: R6-rule and key points to evaluate the performance function.

### 2.3 Model description

A simple test case is defined. It consists in a crack in a beam (Figure 2) subjected to tensile force. The crack propagates according to the Paris law expressed by:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K)^m, \tag{6}$$

where $a$ is the size of the crack, $N$ is the number of cycles, $K$ is the SIF, $C$ and $m$ are crack propagation law parameters.

As shown by Virkler [10], the phenomenon of the crack propagation can be considered as a stochastic process. Indeed, considering sample of the same coupon test, experimental results show that, for a given initial crack, the speed of the crack propagation is different. A common way to integrate these uncertainties is to consider parameters of crack propagation law $C$ and $m$ as random variables. There is a strong correlation ratio between these parameters [11], which can not be independently sampled. Then, it is possible to consider $m$ as random variable and propagate the uncertainty to $C$ according to a linear relationship:

$$\ln C = \xi_1 m + \xi_0, \tag{7}$$

where $\xi_0$ and $\xi_1$ are regression coefficients identified according to experimental test results.

As presented in Table 1, eleven input parameters are set as random variables defined by classical distribution laws according to physical considerations. For the application, values of parameters are invented but representative of the reality.

### 2.4 FORM

The *Fist Order Reliability Method* (*FORM*) [12, 13] is an approximation method based on the *Hasofer Lind reliability index* [14] to compute the probability of failure. This index $\beta$ is a geometric measure of the reliability in relationship with the failure probability as $p_f = \Phi(-\beta)$. It is the shortest distance between the origin and the limit state in the standard space and is

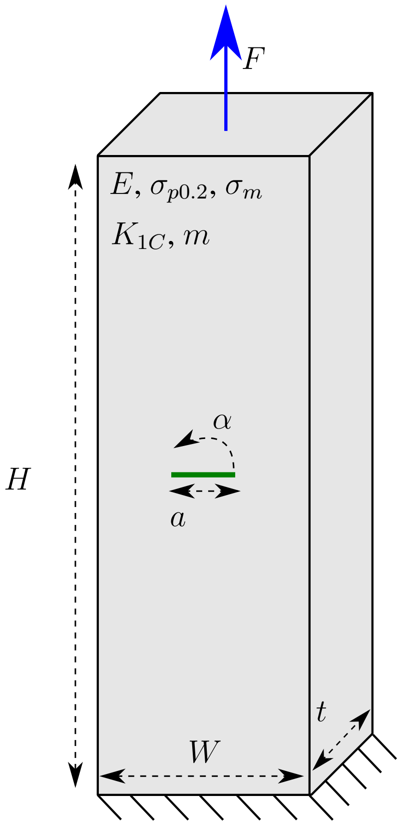


Figure 2: Test case geometry with parameters defined as random variables.

so-called the Most Probable Point (MPP) $P^*$. The coordinates $\mathbf{u}^*$ of $P^*$ are obtained by solving the following constrained optimization problem.

$$\mathbf{u}^* = \arg\min \|\mathbf{u}\| \quad \text{s.t.} \quad H(\mathbf{u}^*) \leq 0. \tag{8}$$

This problem is commonly solved using the Rackwitz-Flessler gradient-based algorithm [15]. *FORM* implies a linearisation of the limit state in the vicinity of MPP. An approximation of the probability of failure is expressed as:

$$P_f \approx P(\underbrace{H(\mathbf{u}^*)}_{=0} + \nabla_{\mathbf{u}} H(\mathbf{u}^*)(\mathbf{u} - \mathbf{u}^*) \leq 0) = \Phi(-\beta_{\text{FORM}}), \quad \text{with: } \beta_{\text{FORM}} = \|\mathbf{u}^*\|. \tag{9}$$

Additional information about relative importance of input variables $u_i$ is provided by cosinus director $\alpha_i$ of the MPP in the $U$-space:

$$\alpha_i = -\frac{\partial \beta}{\partial u_i}|_{\{u^*\}}, \tag{10}$$

so $\alpha_i$ is the derivative of the reliability index according to variables $u$. This information is used for sensitivity analyses developed in the next section.

*FORM* is a fast and efficient method to obtain the failure probability. Moreover, it provides useful sensitivity information. Nevertheless, because it is based on an approximation, the provided value has to be checked using another reliable method such as simulations-based *Monte*

| Class | Variable | Distribution law | Description |
|---|---|---|---|
| Geometry | $L$ | $\mathcal{U}[299.9\text{ mm}; 300.1\text{ mm}]$ | Height of the beam |
| | $W$ | $\mathcal{U}[19.9\text{ mm}; 20.1\text{ mm}]$ | Width of the beam |
| | $t$ | $\mathcal{U}[9.9\text{ mm}; 10.1\text{ mm}]$ | Thickness of the beam |
| Material properties | $E$ | $\mathcal{N}(185000\text{ MPa}; 9250\text{ mm})$ | Young modulus |
| | $\sigma_{p0.2}$ | $\mathcal{N}(600\text{ MPa}; 6\text{ MPa})$ | Yield strength |
| | $\sigma_m$ | $\mathcal{N}(900\text{ MPa}; 9\text{ Mpa})$ | Shear strength |
| | $K_{1C}$ | $\mathcal{N}(200\text{ MPa}\sqrt{\text{mm}}; 2\text{MPa}\sqrt{\text{mm}})$ | Fracture toughness |
| | $m$ | $\mathcal{N}(3; 0.3)$ | Paris law parameter |
| Loads | $F$ | $\mathcal{N}(25\text{ kN}; 2.5\text{ kN})$ | Effort intensity |
| Crack | $a$ | $\mathcal{U}[0\text{ mm}; 0.5\text{ mm}]$ | Initial crack size |
| | $\alpha$ | $\mathcal{N}(0°; 1°)$ | Crack orientation |

Table 1: List of random variables with corresponding distribution laws.

*Carlo method*. The approximation can be enhanced using the Second Order Reliability Method [13] *SORM* which includes the main curvature of the limit state function.

### 2.5 Application of *FORM* with Surrogate-Based Optimization

*FORM* relies on an optimization problem to get the Most Probable Point (MPP) which is commonly solved using Rackwitz-Flessler gradient-based optimization algorithm. It has been applied on our application case but it fails to determine the MPP. Similar issues, such as slow convergence or no convergence, are also observed in [8]. Several starting points have been tested, it is observed that after few iterations, the gradient of performance function becomes close to zero and it breaks the algorithm. Null gradient underlines a saturation phenomenon of the performance function. This phenomenon reveals a structural failure before the end of the life cycle. Indeed, the crack grows and its size can become bigger than the beam size. In this case, the margin in the FAD stays the same, even if input parameters are different. Figure 3 shows the evolution of the performance function in the standard space. In order to plot it in a visual space, only two significant random variables are considered: the length of the crack $a$ and the crack propagation parameter $m$. The phenomenon of saturation can be observed in the failure space. After few iterations, the optimal point is located in the saturation domain and this stops the algorithm.

Gradient-based optimization methods are not relevant in our case. Hence, it has been proposed to use genetic algorithm to determine the MPP. Actually, genetic algorithms do not need gradient information to converge to the optimal point. Nevertheless, basic genetic algorithms need a lot of simulations to converge [16]. Then, the interest to use *FORM* in comparison with Monte Carlo can be compromised. In this contribution, a strategy based on a Surrogate-Based Optimization (SBO) to get the MPP using the in-house optimization platform Minamo [17] is built according to the following steps :

1. Firstly, a global objective function is built coupling the objective function (here the distance between the the origin and the Most Probable Point) and the inequality constraint function (here the performance function must be less or equal to zero)

2. Then, a Design Of Experiments is generated using the *Latin Hypercube Sampling* method. It allows to optimally sample the input space on the domain of definition of input parame-

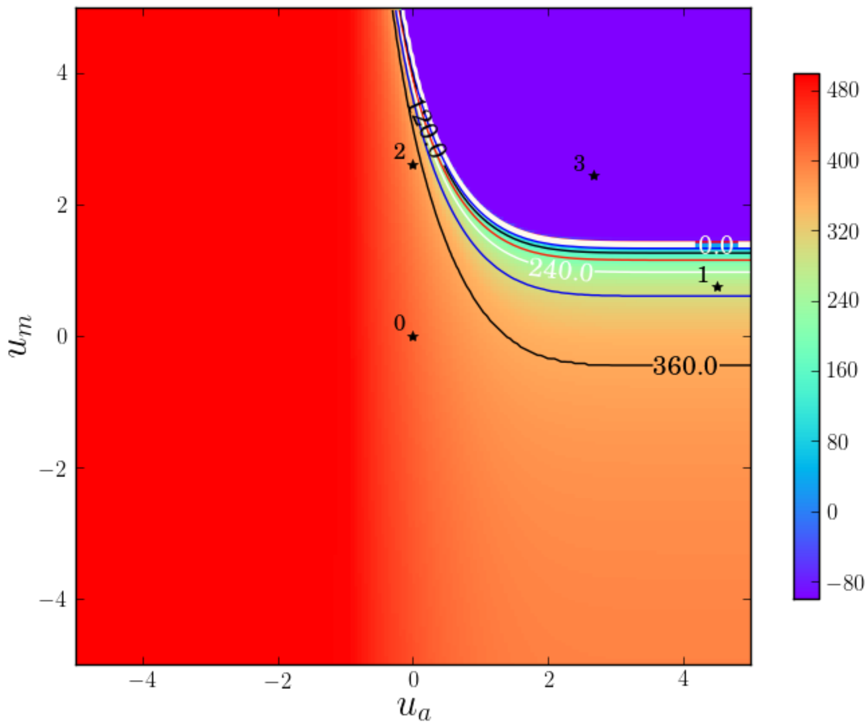


Figure 3: Evolution of the performance function in the standard space $u_a$, $u_m$. The blue part is the failure domain. Gradient in this domain is close to zero. The star points represent the iterations of FORM for the gradient-based optimization. It stops when the point is located in the saturation domain.

ters. Individuals of DOE are so-called training points. In reliability problem, the sample is generated in the standard space and each individual is mapped into the physical space by the inverse of the iso-transformation to compute the performance function at each point of the DOE.

3. A Radial Basis Function (RBF) surrogate of the global objective function in the standard space is built on these training points. The fidelity of the surrogate depends of the size of the DOE.

4. An optimal point is determined based on the surrogate model. The actual objective and constraint function are assessed on this point.

5. Optimization criteria which are typically the maximum number of iterations or the distance between subsequent approximate optimal points, are checked:

   - If criteria are satisfied, it is the end of the optimization process.
   - If they are not. The DOE is enriched by this point and then we return to the third step to build a surrogate with more training points.

This procedure is commonly named *Adaptive sampling*. At each iteration, new points enrich the DOE to refine the surrogate in interesting part of the domain. Therefore, regardless saturation phenomenon, it allows to determine the MPP without performing a lot of simulations. *FORM* approximation can be applied to compute reliability information.

Table 2 presents results of the reliability analysis using *Monte Carlo method* and *FORM with SBO* comparing the probability of failure and the number of simulations. Even if there is a ratio about two between the *FORM* probability and the *Monte Carlo* one, both are in the same order of magnitude. *FORM* approximation is not exact but only about hundred simulations are necessary. A compromise has to be determined between accuracy and speed. In our case, *FORM* seems to be adapted because the scale of the probability is kept with a small number of simulations. Moreover, *FORM* provides interesting sensibility information as explained later.

| Method | $P_f$ | Simulations |
|---|---|---|
| Monte Carlo | $0.02505$ | $100000$ ($\text{cv}_{\hat{P}_f} \approx 2\%$) |
| *FORM with SBO* | $0.03860$ | $115$ (LHS $40$) |

Table 2: Results of reliability methods on the test case.

## 3 Sensitivity analysis

Reliability analysis aims at getting the probability of failure. Nevertheless, it is a fuzzy information because experimental comparison are hard to establish. Therefore, sensibility of the input variables is an interesting point as it defines the significant variables that impact the probability of failure. Then, this contribution presents the main sensitivity indices useful for reliability analysis. Firstly, sensitivity analysis of the performance function is performed. Secondly, reliability sensitivities are provided.

### 3.1 Mechanical sensitivities

Mechanical sensitivities aims at characterising the impact of inputs on the performance function. Even if *screening* and *local sensitivity* are available, mechanical sensitivities are generally associated with global sensitivity providing accurate information about the influence of input variabilities on the whole domain. Based on the *ANalysis Of VAriance* (*ANOVA*) decomposition of model output, the *Sobol method* [18] is a global and model independent sensitivity analysis method that provides indices to measure the impact of the input variabilities on the output variance. First Sobol indices quantify the impact of an input variable without taking into account the interactions with other variable ans the total Sobol indices provide the global impact of a variable. Nevertheless, these indices are only focused on the variance output and can not predict a change of the mean or other statistical moments of the output. Therefore, Borgonovo [19] proposes indices based on the Probability Density Function (PDF) output. They measure the change area of the density output and allow to consider other statistical moments. To completely characterize the global impact of a variable, both indices are complementary.

Global sensitivity analysis of the performance function is achieved. Sobol first order, Sobol total and Borgonovo indices are assessed and shown in Figure 4. On one hand, the variance-based sensibility underlines three significant variables which the length of the crack $a$, the crack propagation parameter $m$ and the load $F$. It sounds in line with the definition of the test. The difference of value between the two Sobol indices can be explained by the fact that there is interaction between these three significant variables. On the other hand, density-based indices show significant indices for ten variables : The three main variables of the variance-based approach have still impact but geometric parameters and material properties have also significant indices. These last parameters impact the output density mean or skewness but they have a very small influence on the output variance. Mechanical sensitivities show that the variable $H$ has no influence on the performance. It is in line with the definition of the test case since it accounts for semi-infinite geometry.

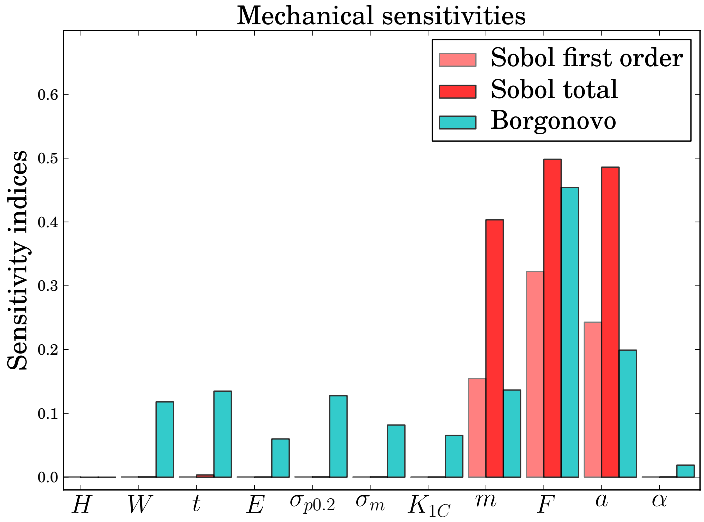


Figure 4: Mechanical sensitivities of the performance function which the margin in the FAD.

### 3.2 Reliability sensitivities

In addition to provide the probability of failure, *FORM* gives some information about the reliability sensibility. Firstly, as random variables are independent, the importance factors $\alpha_i^2$ (Figure 5) describe the weight of each variable on the reliability. As for the variance-based mechanical sensibility, there are three significant variables : $a$, $m$ and $F$. These results allow to determine, by some basic transformations[20], the reliability sensitivities of the inputs distribution parameters of random laws,

$$\frac{\partial \beta_{\text{FORM}}}{\partial \theta_{ik}} = \frac{\partial \beta_{\text{FORM}}}{\partial u_i}\frac{\partial u_i k}{\partial \theta_{ik}} = -\alpha_i \frac{\partial \mathrm{T}_{x\to u}(x_i, \boldsymbol{\theta}_i)}{\partial \theta_{ik}}, \tag{11}$$

where $\mathrm{T}_{x\to u}(x_i, \theta_f) = \Phi^{-1}(F_{X_i}(x_i, \theta_f))$ is the iso-probabilistic transformation. Considering the *FORM* relationship between $P_f$ and $\beta_{\text{FORM}}$, the sensibility of $P_f$ according to the parameters $\theta_{ik}$ is defined as:

$$\frac{\partial P_f}{\partial \theta_{ik}} \approx -\phi(-\beta_{\text{FORM}})\frac{\partial \beta_{\text{FORM}}}{\partial \theta_{ik}}. \tag{12}$$

Reliability sensitivities of the test case (Figure 6) underline two significant variables which are $a$ and $m$. The parameters of the load distribution seem to not impact the reliability. However, reliability sensitivities quantify the impact of the variation of one unity of the input parameter on the reliability. They are not normalised, then, the comparison of the influence of variation is not obvious. Thus, it is also interesting to compare the influence of input variables quantifying

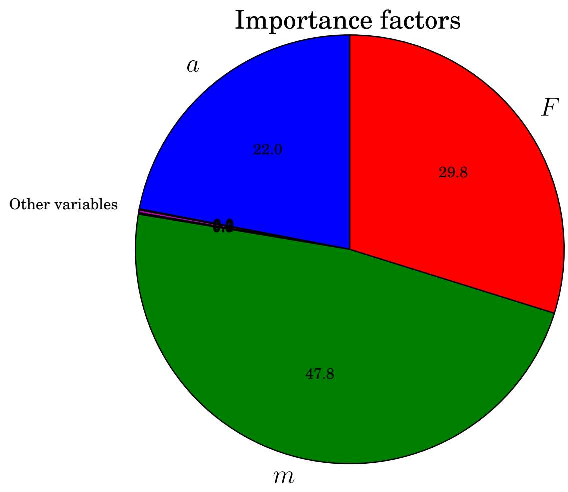


Figure 5: Influence of the parameters on the failure space.

the impact of one percent of variation. This is commonly named *elasticities* and is expressed by:

$$e_{\theta_{ik}} = \frac{\theta_{ik}}{\beta_{\text{FORM}}} \frac{\partial \beta_{\text{FORM}}}{\partial \theta_{ik}}. \tag{13}$$

According to the comparison of elasticities (Figure 7), $F$, $a$ and $m$ are the most significant variables as for the variance-based sensibility and for the importance factors. It can be due to the relatively small influence of other parameters. Indeed, results can be different if other parameters impact more the variance of the performance function. Moreover, $F$, $a$ and $m$ reliability sensitivities are positive, it implies that the reliability decreases when the mean parameters of these variables increase.

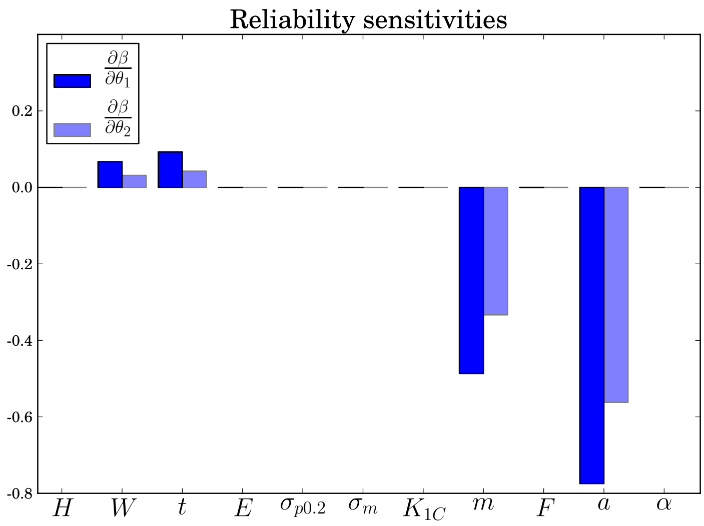


Figure 6: Reliability sensitivities of distribution parameters.

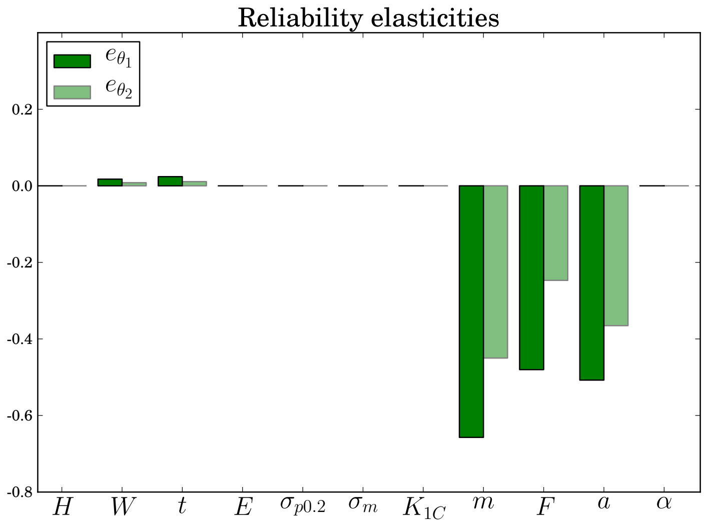


Figure 7: Reliability elasticities of distribution parameters.

To summarise, three parameters have significant influence on the variance performance function and on the reliability:

- the length of the crack $a$,
- the crack propagation parameter $m$,
- the loads $F$.

It can be explained by the fact that these parameters directly drive the crack propagation. In order to run accurate reliability analysis, specific effort has to be made to carefully fit the random laws of these parameters.

## 4 CONCLUSIONS

This contribution presents a reliability and sensibility analysis of a typical Fracture Mechanics problem which combines two failure criteria. Both are combined in the performance function using a margin function in the FAD diagram. The classical gradient-based algorithm used for the *FORM* reliability method failed to determine the Most Probable Point which is necessary to apply this method. It is due to a phenomenon of saturation of the performance function. This contribution fixes the problem using an efficient Surrogate-Based Optimization algorithm to allow to determine key information to apply the *FORM* method with a limited number of simulations. A complete sensibility analysis is performed and it underlines the significant impact other random variables as the length of the crack $a$, the Paris law crack propagation parameter $m$ and the loads $F$. Therefore, a particular consideration must be given to the modelling of these random variables.

The *FORM* method is applied here using a Surrogate-Based Optimization algorithm. Nevertheless, simulation methods can also be used using surrogate model to reduce the number of simulations. Moreover, in the surrogate enhancement, the phenomenon of saturation of the performance function should be taken into account.

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