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PERCOLATION PROPERTIES OF THE FREE VOLUME GENERATED BY TWO ROUGH SURFACES IN CONTACT

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Abstract. *The mechanism of fluid leakage trough the free volume between rough surfaces in contact is relevant in physics and in many engineering applications. In the present study, the normal contact problem between randomly generated fractal rough surfaces is solved using the boundary element method. Then, an algorithm for the evaluation of the network involved in the percolation of fluid is proposed. Numerical results are synthetically collected in diagrams relating the free volume involved in the percolation to the dimensionless statistical parameters of the rough surface.*

1 INTRODUCTION

The mechanism of percolation in seals or simply between two rough surfaces in contact is an interesting topic for physics and engineering [1, 2, 3]. This topic is relevant for many applications such as hydraulic fracturing and sealing of mechanical components in contact. Also, the percolation of fluid in photovoltaic modules can lead to electrical power losses due to the oxidation of the grid line deposited on the solar cell caused by moisture. The free volume and the related network of channels is due to the fact that surfaces are never ideally flat and the actual area in contact is determined by the elastic interactions between asperities. Since the full contact solution could be attained only in case of very high contact pressures, in most of the cases only a small percentage of the nominal contact area is in contact and a significant amount of free volume is present. In this context, a line of research regards the discovery of the relation between the contact mechanics results to the statistical properties of the undeformed parent rough surface with a power-law power spectral density (PSD) function [4]. Yastrebov investigated the role of the lower and upper cut-offs to the PSD function in different contact regimes [5], while Paggi and Ciavarella [6] highlighted the effect of the bandwidth parameter α. Similarly, the features of the leakage domain by considering the statistical proprieties of the surface height distribution were investigated in the literature. As highlighted by Dapp [7], the role of the finite size length and the lower cut-off frequency of the rough surface mainly affect the topology of the free volume and, consequently, leakage characteristics [1]. All of those studies pinpoint the fact that, below the full contact limit, morphological proprieties of the surface affect percolation of a fluid between two rough surfaces. However, during contact, the initial statistics of the deformed surface change as well as consequence of the elastic interactions.

The present study aims at investigating the dependency of the leakage characteristics of free volume on the variation of statistical parameters describing the evolution of the contact domain between rough surfaces. To this purpose, a single rough surface with fractal dimension $D = 2.1$ has been generated and the contact problem has been solved for a series of mean plane separations using the boundary element method (BEM). Afterwards, the free volume has been evaluated and its portion involved in the percolation has been computed, under the assumption of a laminar fluid flow. In Section 2, the numerical framework is briefly summarized. Results are presented in Section 3, in form of diagrams relating the variation of the statistical parameters of the contact domain for decreasing mean plane separations and the relation with the variation of the percolating volume.

2 COMPUTATIONAL METHOD

A single rough surfaces with fractal properties is numerically generated according to the random midpoint displacement (RMD) algorithm detailed in [8]. With this method, it is possible to obtain a detailed square surface by refining an initial square mesh, adding recursively in the mid point an intermediate height. The value of this height is equal to the mean value of the neighboring heights, plus a random number extracted from a Gaussian distribution with zero mean and variance $\sigma_1^2 = \sigma_0^2/2^{(3-D)/2}$, where $\sigma_0^2 = 1/\sqrt{0.09}$ and D is the surface fractal dimension ranging from 2 to 3. The refinement algorithm depends upon the parameter m, which is related to the number of heights per side of the squared generated grid, viz. $2^m + 1$.

Once the surface is generated, the contact problem between an elastically deformable rough surface and a rigid half space is solved using BEM [9] for different values of the far-field closing displacement Δ. For each imposed Δ, the contact problem is solved using the Non-Negative Least Squares (NNLS) algorithm, without warm starting, as detailed in [9].

After solving the frictionless contact problem, the grid points in contact are determined and the contact mechanics results and the statistical parameters of the deformed surface are computed for each imposed displacement. The total free volume V is evaluated by summing up the corresponding contributions of the boundary elements not in contact. The complete procedure is described in [10]. Next, the percolating volume V_{pg} is computed as the sum of all the boundary element volumes involved in the leakage phenomena. Those elements are evaluated with an algorithm examining all the available paths for the fluid and retaining in the computation only the free channels connecting one side of the surface to the opposite. Statistical parameters of the deformed surface are evaluated according to Nayak's theory [4]. Nayak investigated the effect of some characteristic moments which are related do the r.m.s. values of roughness ($\sigma = \sqrt{m_0}$), the r.m.s slope ($\sigma_m = \sqrt{m_2}$) and the r.m.s. curvature ($\sigma_k = \sqrt{m_4}$), and the bandwidth parameter $\alpha = \frac{m_0 m_4^2}{m_2}$. Here, this computation is repeated for each imposed contact displacement, using the algorithm published in [11].

3 RESULTS

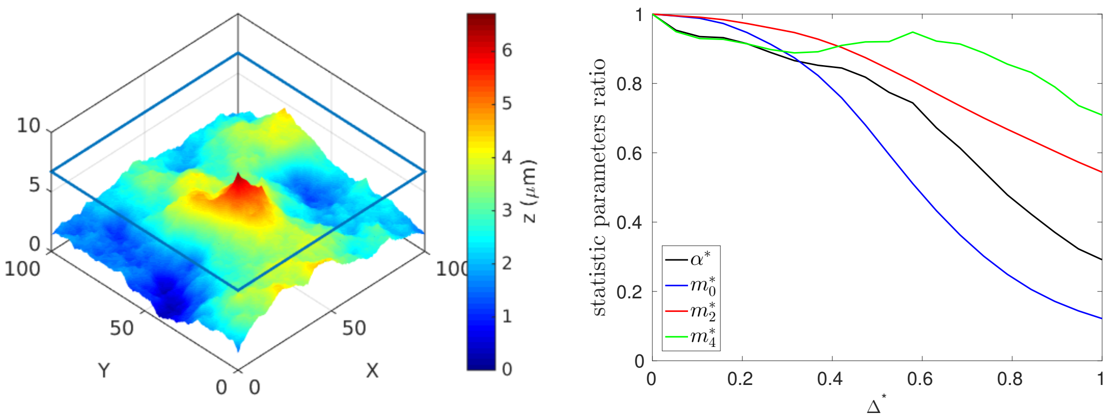
A rough surface with fractal dimension $D = 2.1$ has been generated using the RMD algorithm with a refinement corresponding to $m = 7$, see also [10] for more details. The surface lateral size is $L = 0.1$ mm, and the undeformed surface presents a maximum height equal to

$h_s = 6.72$ μm. Moreover, the mean surface elevation is equal to $\bar{h} = 2.78$ μm and the r.m.s. of heights is $\sigma = 0.95$ μ m^2, the r.m.s. of the profile slopes is $\sigma_m \cong 0.004$, and the r.m.s. of curvatures is $\sigma_k = 0.16$ μm^{-2}. The bandwidth parameter is $\alpha_0 = 921$.

The explored values of the far-field displacement Δ are in the range between $\Delta = 0$ (infinitesimal contact) and the maximum asperity height $\Delta = h_s$ computed in relation to the undeformed surface. This range is subdivided in twenty equal levels. The value of Δ is made dimensionless by dividing it for h_s, i.e, $\Delta^* = \frac{\Delta}{h_s}$. In this way, it is possible to the define the contact level from $\Delta^* = 0$ (infinitesimal contact regime) to $\Delta^* = 1$ (full contact regime). As detailed in [6], the dimensionless free volume $V^* = \frac{V}{L^2\sigma}$ is introduced, where L is the surface lateral size and σ is the r.m.s. value of height distribution. Therefore, the percolating volume V_{pg} is made dimensionless using its value when only one spot is in contact for $\Delta^* = 0$. The spectral moments are also made dimensionless by dividing them for their initial values, i.e., $m_n^* = \frac{m_n}{m_{n0}}$, where m_{n0} is the value of the $n-$th moment corresponding the undeformed surface.

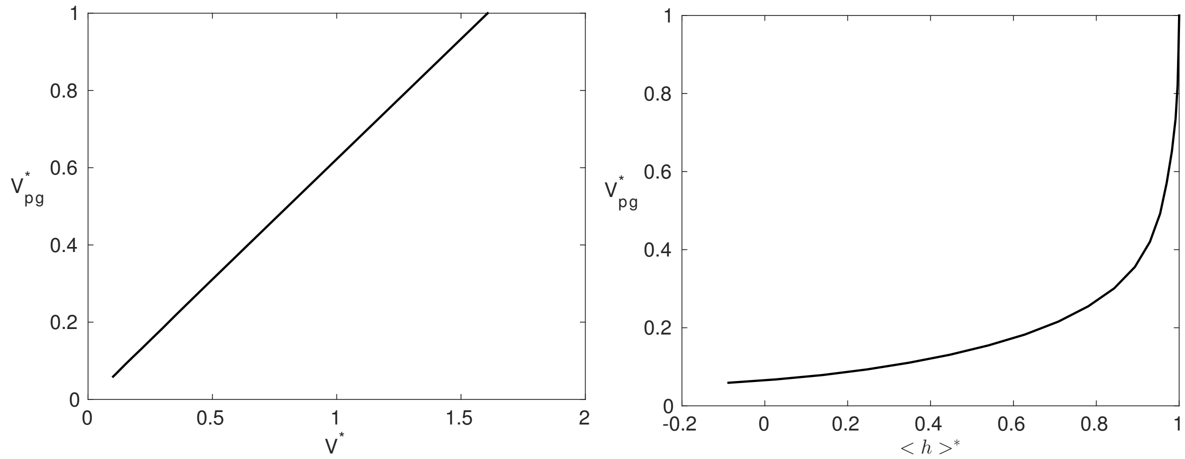
The undeformed surface topography is shown in Fig.1(a), while Fig.1(b) shows the variation of the statistical parameters during contact. By increasing the contact interference Δ^*, statistical parameters diminish in value since the surface becomes flatter and flatter and the related r.m.s. becomes smaller and smaller. More specifically, m_0 presents small variations for infinitesimal contact, before significantly reducing its value for medium and full contact regimes. The same trend is noted for the variance of slopes, m_2. This transitional regime takes place for $\Delta^* \gtrsim 0.4$. The parameter m_4, for low values of Δ^*, tends to be almost constant with variations of about 10% up to $\Delta^* \cong 0.6$, while it significantly reduces afterwards. The variability of α vs. Δ^* is in line with the trend of m_0 and m_2, mitigated by the trend of m_4.

The dependencies of V_{pg} on the free volume V^* and on the dimensionless mean height $\bar{h}^* = \frac{h_s}{\bar{h}}$ are shown in Fig.2. It is important to notice that, for this surface, V_{pg} varies almost linearly with V_{pt}. This might be due by the morphology of the surface, which has the tallest asperity in the center. Moreover, the initial percolating volume V_{pg} is equal to the total free volume V^*. Examining Fig.2(b), the dependency of V_{pg} on the surface mean height presents a highly nonlinear trend, especially in the low contact regime ($\bar{h}^* = 1$). On the other hand, near the full contact limit ($\bar{h}^* = 0$), V_{pg} tends asymptotically to a constant value. Finally, V_{pg} is correlated to m_0^* and m_2^* during contact in Fig.3. In both cases, for an infinitesimal far-field displacement,



(a) Undeformed topography (b) m_n and α vs Δ^*

Figure 1: Topography of the surface and variation of its Nayak's moments for different contact levels.



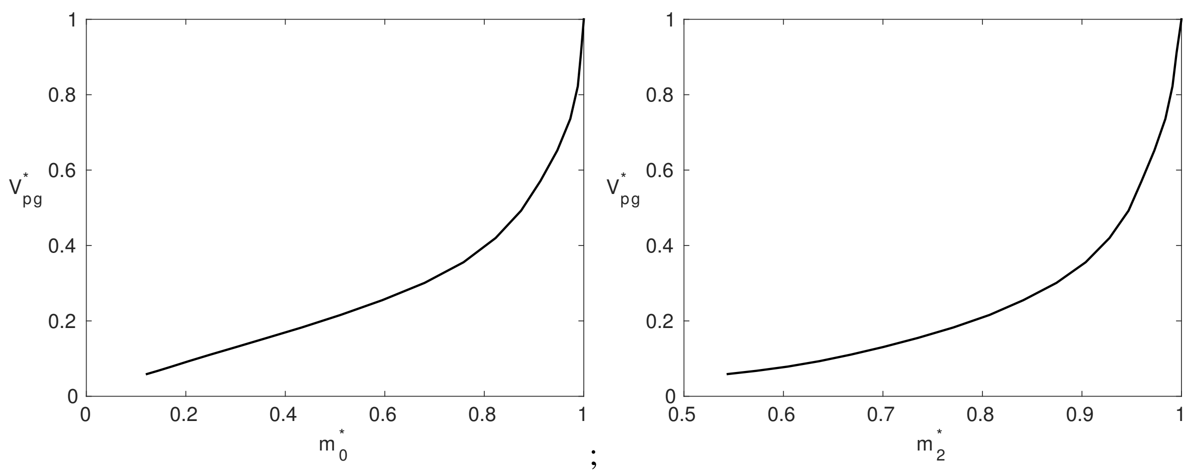
(a) V_p^* vs V^* (b) V_p^* vs $\bar{h}^*$

Figure 2: Variation of the leakage volume during contact. Fig.2(a) shows the dependency of the dimensionless leakage volume V_{pg}^* with respect to the overall free volume V^*. Fig.2(b) shows the dependency of V_{pg}^* on the dimensionless mean height of the deformed rough surface.

a strong dependency of V_{pg} on the moments of the PSD is reported. On the opposite, the percolating volume tends to be almost independent of the statistical proprieties of the surface near full contact. This could be connected directly with Persson theory of leakage [2, 3], which asserts that leakage phenomena do not depend on surface statistic in the full contact regime.

4 CONCLUSIONS

In this work, the free volume generated by two fractal rough surfaces in contact has been computed, determining also its percolating part, which is its portion through which a laminar fluid flow can cross the entire surface from on side to the opposite. A single surface has been generated with the RMD algorithm, and the contact problem has been solved with a BEM method for different far-field displacements, from the infinitesimal to the full contact regimes.



(a) V_p^* vs m_0 (b) V_p^* vs m_2

Figure 3: Variation of the leakage volume during contact. Fig.3(a) shows the dependency of the dimensionless leakage volume V_{pg}^* on m_0^*. Fig.3(b) shows the dependency on m_2^*.

Then, the percolating volume has been evaluated, considering its dependency on the statistical parameters of the surface computed according to Nayak's theory of roughness. Results shows that there is a linear dependency between the free volume V_* and the percolating one V_{pg}. Moreover, examining the height and slope r.m.s. values, it is possible to notice that the percolating volume strongly depends on their variation for small imposed far-field displacements. On the other hand, the percolating volume nearby the full contact limit appears to be almost independent of the statistical parameters variation, in agreement with the Persson's theory of leakage in seals. Future developments of the present study will regard a statistical analysis of the obtained results based on a wider population of numerically generated contact surfaces. Moreover, the computation of the portion of the free volume which is involved in leakage is also an important result, and its dependency on the statistics of the rough surfaces is also a crucial issue to deepen. This knowledge could be helpful to avoid surface configurations implying trapped fluid.

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