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AN EXAMPLE OF COLUMN DESIGN OPTIMIZATION FOR FAILURE MODE CONTROL OF REINFORCED CONCRETE FRAMES

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Abstract. *In this paper an example of column design optimization for the control of collapse mechanism for RC-frames is presented. The applied theory has already been developed, by the authors, in previous work.*
The procedure is based on the application of the kinematic theorem of the plastic collapse (TPMC: Theory of Plastic Mechanism Control). In particular, the outcome of the theory is the evaluation of the sum of the plastic moments of the columns required, at each storey, to prevent undesired failure modes such as soft-storey mechanism. In the proposed method the second-order effects, due to vertical loads, can play an important role in the seismic design of reinforced concrete frames; they can be taken into account by mean the mechanism equilibrium curve of the considered collapse mechanism.
In this work the authors show how the classical design methodology based on the beam-column hierarchy criterion does not allow to obtain a global mechanism.
Beam-column hierarchy criterion, commonly suggested by seismic codes, appears only as a very rough approximation when compared to TPMC and its theoretical background.
By applying this classic methodology, if a global mechanism is obtained, than the design of the column section and its reinforcement are not optimized. In fact, only with the theory already recalled we can obtain the minimum of section columns able to provide the development of a global mechanism.
In addition, in the presented design approach, also the role played by joists can be accounted for. In fact, if their contribution is neglected the collapse mechanism can be very worse that the desired one.

1 INTRODUCTION

One of the most important requirement in the seismic design consists in avoiding collapse mechanism characterized by the development of the minimum energy dissipation capacity of the structure, such as “soft-storey” mechanism. In fact the development of a collapse mechanism of global type, characterized by the location of plastic hinges at all the beam ends and at the base sections of first storey columns is always preferred.

Relatively to the moment resisting frames, the maximum number of plastic hinges is obtained when two plastic hinges develops in each bay and they are usually located at beam ends. At the aim of avoiding undesired collapse mechanisms, hierarchy criterion reported in all the modern seismic codes, suggests that at any joint, the sum of the flexural strength of the columns is greater than the sum of the flexural strength of the beams converging in the same joint [1, 2]. Notwithstanding, the beam-column hierarchy criterion, being based on simple joint equilibrium, is generally able to prevent “soft-storey” mechanisms, but it does not assure the development of a collapse mechanism of global type; in fact it is a non-rigorous application of capacity design principles. In addition, several research are devote in recent years to the understand of seismic collapse mode of reinforced concrete frames, the induced loss and the retrofitting technics to be adopted in order to obtain a better and more dissipative collapse mechanism both in case of existing structures [3-9] and in case of new structures [10-16]. In order to overcame this problem, a more sophisticated design procedure, based on the kinematic theorem of plastic collapse and on second order plastic analysis (i.e. the concept of mechanism equilibrium curve) has been presented. “Theory of Plastic Mechanism Control” (TPMC) has been obtained as a powerful tool for the seismic design. In particular, it consists on the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. In fact, for any given structural typology, the design conditions to be applied in order to prevent undesired collapse mechanisms can be derived by imposing that the mechanism equilibrium curve, corresponding to the global mechanism, has to be located below those corresponding to all the other undesired mechanisms up to a displacement level compatible with the local ductility supply of dissipative zones. This design approach has been applied to different steel structural typologies such as MRFs with RBS connections, EB-Frames, dissipative truss-moment frames, MRF-CBF dual systems and MRFs equipped with friction dampers [17-39]. Starting from the above background, the TPMC is developed also with reference to the reinforced concrete frames. In the present paper, a worked example and a validation of the proposed design procedure are presented in [40].

2 THEORY OF PLASTIC MECHANISM CONTROL

In general, three main collapse mechanism typologies that the structure is able to exhibit can be recognized: these mechanisms, depicted in Figure 1, are to be considered undesired because they do not involve all the dissipative zones. In order to apply the TPMC it is of paramount importance the introduction of the concept of linearized mechanism equilibrium curve for each considered mechanism. The mathematical expression of this curve can be written as:

$$\alpha = \alpha_0 - \gamma\delta \tag{1}$$

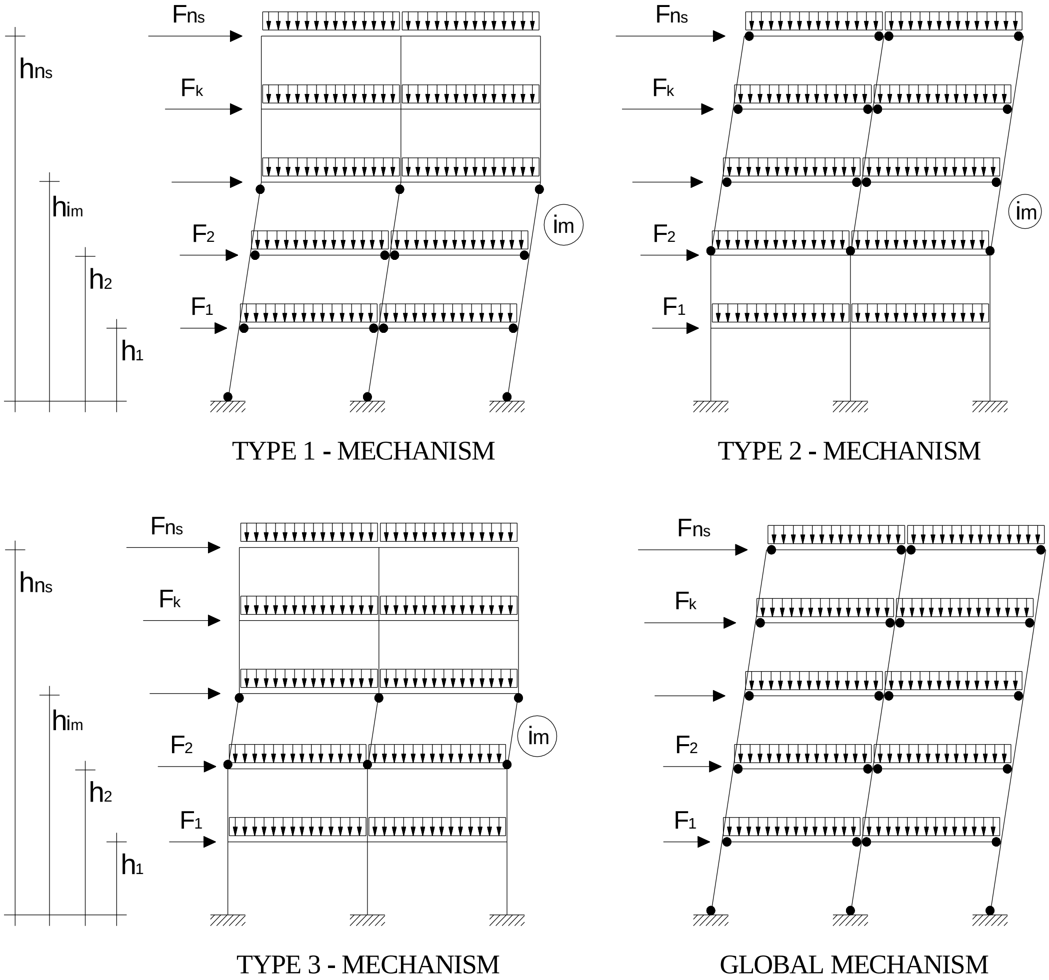

Figure 1: Collapse mechanism typologies

where α_0 is the kinematically admissible multiplier of horizontal forces, γ is the slope of the mechanism equilibrium curve and δ is the top-sway displacement. Both parameters can be derived, according to rigid-plastic theory, using the principle of virtual work. Within the framework of a kinematic approach, for any given collapse mechanism, the mechanism equilibrium curve can be easily derived by equalling the external work to the internal work. In addition, in order to account for second-order effects, the external second-order work due to vertical load is also evaluated. For what concerns the theory herein considered and applied, reference is to be made to [40] where the design algorithm and the presentation of the developed theory is reported. In is useful to remember that in the proposed method the beam section properties are assumed to be known quantities because they are designed to resist vertical loads. As a consequence, the unknowns of the design problem are the column sections. They could be determined by means of design conditions expressing that the kinematically admissible multiplier corresponding to the global mechanism is the minimum among all kinematically admissible multipliers corresponding to all other mechanisms (Figure 1). Obviously, this design condition is able to assure the desired collapse mechanism only in case of rigid-plastic behaviour, while actual structures are characterized by elastic displacements before the development of a plastic mechanism. Due to these elastic displacements, second-order effects of vertical loads cannot be neglected. For the better comprehension of the following, the adopted notation is reported to Table 1.

n_c	number of columns	e	index of beam ends (e=L=left end, e=R=right end)
n_b	number of bays	$M_{c,i,im}$	plastic moment of the i-th column at i_m-th storey
n_s	number of storeys	$M_{c,im} = \sum_{i=1}^{n_c} M_{c,i,im}$	sum of column plastic moments at i_m-th storey
i_m	index of mechanism	$M_V = \sum_{k=1}^{n_s} V_k h_k$	second-order work due to vertical loads in global mechanism
H_o	sum of the interstorey heights of the storeys involved by the generic mechanism	$M_F = \sum_{k=1}^{n_s} F_k h_k$	external work due to horizontal forces in the global mechanism
h_k	height of the k-th storey (with k=1, 2, .., n_s)	$F = \sum_{k=1}^{n_s} F_k$	sum of the horizontal forces
F_k	horizontal force applied to the k-th storey	$M_{b.jk}$	plastic design resistance of beam at j-th bay of the k-th storey
V_k	sum of all the vertical loads acting at k-th storey	$M_{b,Rd,e} = \sum_{k=1}^{n_s}\sum_{j=1}^{n_b} M_{b.jk}$	sum of the plastic design resistances of beam ends (for e end) in the global mechanism

Table 1: Notation

These effects can be taken into account by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all other mechanisms i.e. the upper bound theorem of plastic design is to be satisfied for each value of the displacements δ (Figure 2). However, the fulfilment of this requirement is necessary only up to a selected ultimate displacement δ_u, which has to be compatible with the ductility supply of structural members.

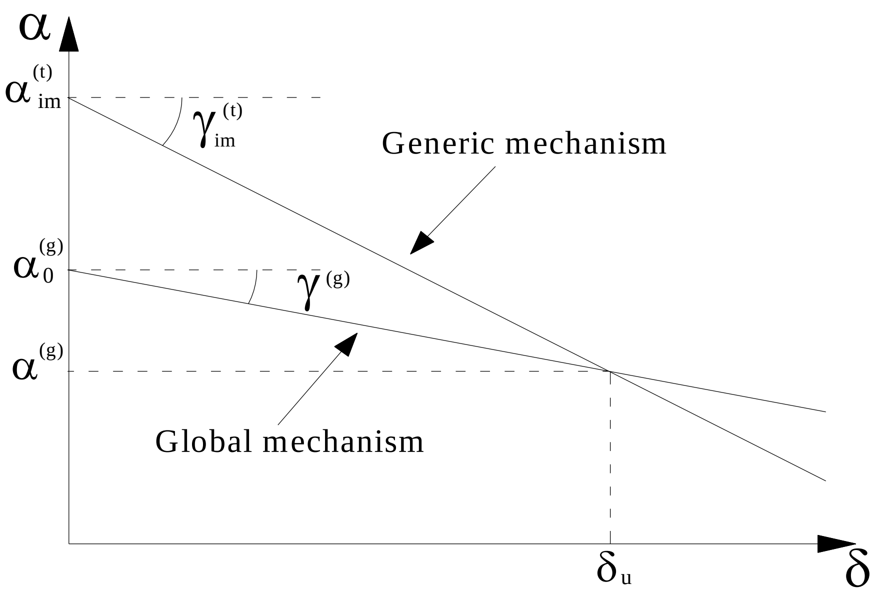

Figure 2: Design condition

This corresponds to impose the following conditions, for LR earthquake:

$$\alpha_{LR}^{(g)} = \alpha_{0,LR}^{(g)} - \gamma^{(g)}\,\delta_u \le \alpha_{0,im,LR}^{(t)} - \gamma_{im}^{(t)}\,\delta_u = \alpha_{im,LR}^{(t)} \tag{2}$$

and for RL earthquake:

$$\alpha_{RL}^{(g)} = \alpha_{0,RL}^{(g)} - \gamma^{(g)}\,\delta_u \le \alpha_{0,im,RL}^{(t)} - \gamma_{im}^{(t)}\,\delta_u = \alpha_{im,RL}^{(t)} \tag{3}$$

for $i_m = 1, 2\ 3, \ldots, n_s$ and $t = 1, 2, 3$.

Therefore, there are $6n_s$ design conditions to be satisfied for a structural scheme having n_s storeys. With reference to i_m-th mechanism of type-1, the kinematically admissible multiplier of seismic horizontal forces is given, for LR earthquake, by:

$$\alpha_{0,i_{m,LR}}^{(1)} = \frac{M_{c,1} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk,L}^{+} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk,R}^{-} + M_{c,im}}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{4}$$

and for RL earthquake by:

$$\alpha_{0,i_{m,RL}}^{(1)} = \frac{M_{c,1} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk,L}^{-} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b} M_{b.jk,R}^{+} + M_{c,im}}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{5}$$

while the slope of the mechanism equilibrium curve, which is the same for both directions, is:

$$\gamma_{i_m}^{(1)} = \frac{1}{h_{i_m}}\frac{\sum_{k=1}^{i_m} V_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} V_k}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{6}$$

With reference to i_m-th mechanism of type-2 the kinematically admissible multiplier of seismic horizontal forces is given, for LR earthquake, by:

$$\alpha_{0,i_{m,LR}}^{(2)} = \frac{M_{c,im} + \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk,L}^{+} + \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk,R}^{-}}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})} \tag{7}$$

and for RL earthquake by:

$$\alpha_{0,i_{m,RL}}^{(2)} = \frac{M_{c,im} + \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk,L}^{-} + \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b} M_{b.jk,R}^{+}}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})} \tag{8}$$

while the slope of the mechanism equilibrium curve is:

$$\gamma_{i_m}^{(2)} = \frac{1}{h_{n_s} - h_{i_m-1}}\,\frac{\sum_{k=i_m}^{n_s} V_k \left(h_k - h_{i_m-1}\right)}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})} \tag{9}$$

Finally, with reference to i_m-th mechanism of type-3, the kinematically admissible multiplier of horizontal forces, is given by:

$$\alpha_{0,i_m}^{(3)} = \frac{2M_{c,im}}{\left(h_{i_m} - h_{i_m-1}\right)\sum_{k=i_m}^{n_s} F_k} \tag{10}$$

In this case the expression is the same for both directions of earthquake because the beams are not involved in this collapse mechanism. In addition, the corresponding slope of the mechanism equilibrium curve is given by:

$$\gamma_{\mathrm{i_m}}^{(3)} = \frac{\sum_{\mathrm{k=i_m}}^{\mathrm{n_s}} \mathrm{V_k}}{\left(\mathrm{h_{i_m}} - \mathrm{h_{i_m-1}}\right)\sum_{\mathrm{k=i_m}}^{\mathrm{n_s}} \mathrm{F_k}} \tag{11}$$

It is important to underline that, for any given geometry of the structural system, the slope of mechanism equilibrium curve attains its minimum value when the global type mechanism is developed. In fact, it is easy to check that $\gamma^{(g)}$, which is equal to $\gamma_1^{(2)}$, is always the minimum value among all the $\gamma_{i_m}^{(t)}$. This issue assumes a paramount importance in TPMC allowing the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve by simply checking relations (2) and (3) for the value $\delta = \delta_u$, as depicted in Figure 2.

3 STUDY CASE

In order to show the practical application of the proposed design procedure, the seismic design of a four-bay six-storey moment resisting frame is presented in this section.

The inelastic behaviour of the designed structure is successively examined by means of a push-over static, confirming the fulfilment of the design goal, i.e. the location of the yielding zones at the beam ends with the only exception of the base section of first-storey columns. The structural scheme of the frame to be designed is shown in Figure 3. The interstorey height is equal to 3m. The characteristic values of the vertical loads acting on the beams are equal to 21.89 kN/m and 8 kN/m for permanent (G_k) and live (Q_k) actions, respectively. The structural materials adopted are concrete C25/30 and reinforcement of steel grade B450C.

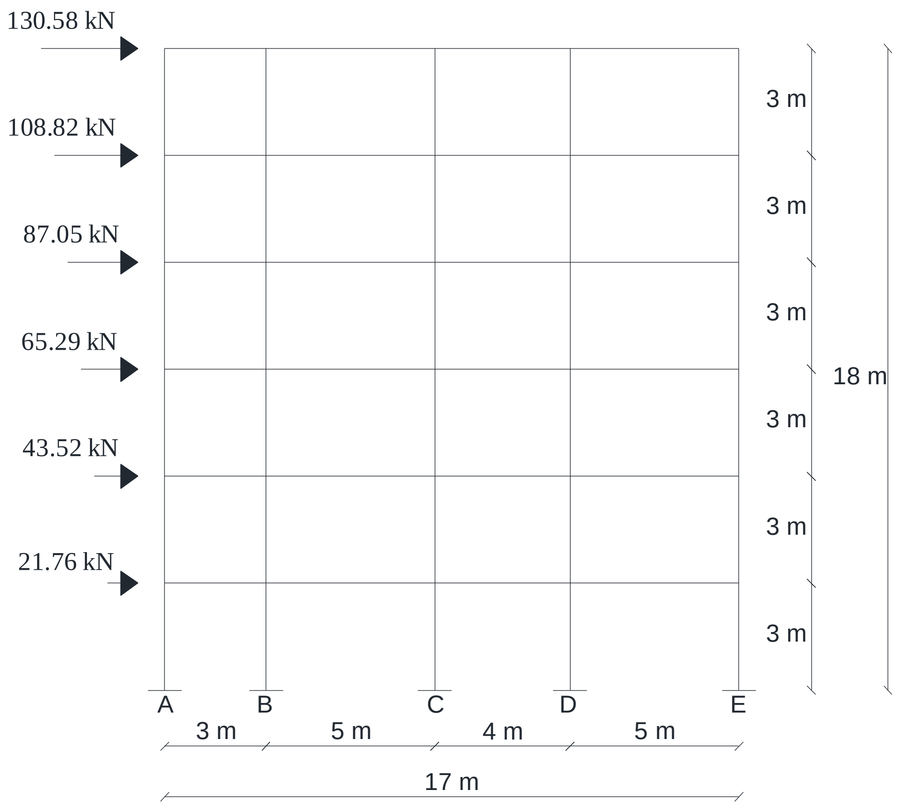

Figure 3: Structural scheme of the designed frame

According to Eurocode 8, the value of the period of vibration to be used for preliminary design is:

$$T = 0.075\, H^{3/4} = 0.075 \cdot 18^{3/4} \approx 0.65\, s \qquad (12)$$

where H is the total height of the frame. With reference to the design spectrum for stiff soil conditions (soil class A of Eurocode 8) and by assuming a behaviour factor q equal to 3.9, the horizontal seismic forces are those depicted in Figure 3. In the following, the numerical development of the design steps for the structural scheme described above is provided.

a) Selection of the design top sway displacement

The selection of the maximum top sway displacement up to which the global mechanism has to be assured is a very important design issue, because the value of this displacement governs the magnitude of second order effects accounted for in the design procedure. A good criterion to choose the design ultimate displacement δ_u is to relate it to the plastic rotation supply of beams or beam-to-column connections by assuming $\delta_u = \theta_u \cdot h_{ns}$ (where θ_u can be assumed equal to 0.01 rad). As a consequence, the design value of the top sway displacement has been assumed equal to:

$$\delta_u = 0.01 \cdot h_{ns} = 0.01 \cdot 18 = 0.18\, m \tag{13}$$

b) Design of beam sections to withstand vertical loads.

The load acting on the frame in the vertical load combination is:

$$Q_{SLU} = 1.3\, G_k + 1.5\, Q_k = 40.46\, kN/m \tag{14}$$

For the design of the beams has been considered a bending moment equal to:

$$M_{max} = \frac{Q_{SLU} \cdot L^2}{8} \tag{15}$$

Therefore, by imposing the base of the section equal to b=30 cm, is possible to calculate the height of the beam through the following design relation:

$$d = r\sqrt{\frac{M_{Sd}}{b}} \tag{16}$$

Assuming $\xi = 0.25$ and $\rho = 0.25$ a value of $r = 0.19$ is obtained. As a consequence the amount of reinforcement is given by:

$$A_s = \frac{M_{Sd}}{0.85 \cdot h \cdot f_{sd}} \tag{17}$$

Obviously the number of steel bars in the beam is such that:

$$M_{Rd} > M_{Sd} \tag{18}$$

where M_{Rd} is the design value of the resistant moment and M_{Sd} is the design value of the applied internal bending moment. The reinforcement at the beam ends are reported in Table 2.

PART OF SECTION	$L_{AB} = 3m$		$L_{BC} = 5m$		$L_{CD} = 4m$		$L_{DE} = 5m$	
	$e = L$	$e = R$	$e = L$	$e = R$	$e = L$	$e = R$	$e = L$	$e = R$
Top	*3F20*	*3F20*	*4F20*	*4F20*	*4F20*	*4F20*	*4F20*	*4F20*
Bottom	*3F20*	*3F20*	*4F20*	*4F20*	*3F20*	*3F20*	*4F20*	*4F20*

Table 2: Reinforcement at the beam ends (L = left and R = right) for the first storey

c) Computation of the slopes of mechanism equilibrium curve $\gamma_{i_m}^{(t)}$.

By means of Eqs. (6), (9) and (11) the slopes of mechanism equilibrium curves are computed. These values are reported in Table 3 and they are the same for both directions of seismic input.

STOREY i_m	$\gamma_{i_m}^{(1)}$	$\gamma_{i_m}^{(2)}$	$\gamma_{i_m}^{(3)}$
1	0.0181	0.0024	0.0181
2	0.0085	0.0027	0.0158
3	0.0054	0.0032	0.0141
4	0.0038	0.0040	0.0127
5	0.0030	0.0056	0.0116
6	0.0024	0.0150	0.0105

Table 3: Slopes of mechanism equilibrium curves (cm^{-1})

In particular it is important to underline that the slope value corresponding to the global mechanism $\gamma^{(g)} = \gamma_1^{(2)}$, is the minimum among all the $\gamma_{i_m}^{(t)}$ values:

$$\gamma^{(g)} = 0.002433\ cm^{-1} \tag{19}$$

d) Computation of the required sum of plastic moments of columns at first storey $M_{c,1}$.

The required sums of plastic moments of columns at first storey for LR and RL earthquake are equal to $M_{c,1,LR} = 1177.99$ kNm and $M_{c,1,RL} = 1177.99$ kNm, respectively.

e) Distribution among the columns proportionally to their number.

According to the global mechanism, axial forces in the columns at collapse state depend both from the distributed loads acting on the beams and from the shear action due to the development of plastic hinges at the beam ends, as depicted in Figure 4 (with reference to the earthquake from Left to Right). So that, the total load transmitted by the beams to the columns is the sum of two contributions. The first one, N_q, is related to the vertical loads acting in the seismic load combination (i.e. the sum of $ql/2$ type contributions).

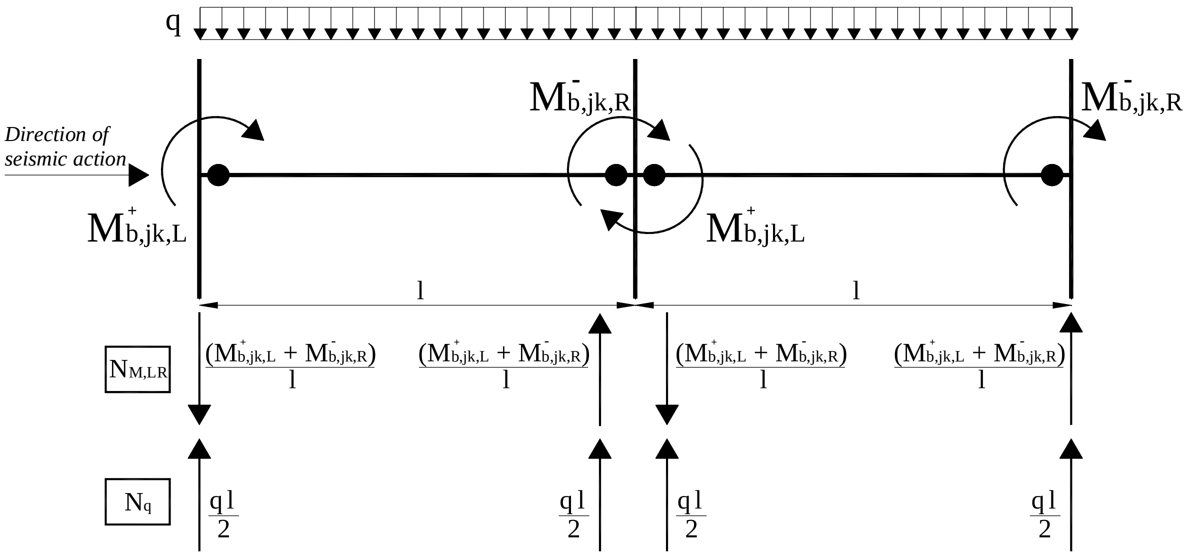

Figure 4: Loads transmitted by the beams to the columns at collapse state for LR earthquake

In Table 4 the axial forces due to vertical loads, for both directions of earthquake, are reported for each storey and for each column.

Storey	Column A	Column B	Column C	Column D	Column E
i_m	N_q (kN)	N_q (kN)	N_q (kN)	N_q (kN)	N_q (kN)
1	218.70	583.20	656.10	656.10	364.50
2	182.25	486.00	546.75	546.75	303.75
3	145.80	388.80	437.40	437.40	243.00
4	109.35	291.60	328.05	328.05	182.25
5	72.90	194.40	218.70	218.70	121.50
6	36.45	97.20	109.35	109.35	60.75

Table 4: Axial forces acting in the columns related to the vertical loads for both directions of earthquake

The second one, $N_{M,LR}$ (or $N_{M,RL}$), is related to the shear actions due to the plastic hinges developed at the beam ends i.e. the sum of $(M^{+}_{b.jk,L} + M^{-}_{b.jk,R})/l$ for earthquake from left to right (or the sum of $(M^{-}_{b.jk,L} + M^{+}_{b.jk,R})/l$ for earthquake from right to left). In Table 5 these contributions are reported.

Storey	Column A		Column B		Column C		Column D		Column E	
i_m	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)	$N_{M,LR}$ (kN)	$N_{M,RL}$ (kN)
1	-508.96	508.96	103.42	-103.42	-38.47	38.47	38.47	-38.47	405.53	-405.53
2	-424.13	424.13	86.18	-86.18	-32.06	32.06	32.06	-32.06	337.94	-337.94
3	-339.30	339.30	68.95	-68.95	-25.65	25.65	25.65	-25.65	270.36	-270.36
4	-254.48	254.48	51.71	-51.71	-19.24	19.24	19.24	-19.24	202.77	-202.77
5	-169.65	169.65	34.47	-34.47	-12.82	12.82	12.82	-12.82	135.18	-135.18
6	-84.83	84.83	17.24	-17.24	-6.41	6.41	6.41	-6.41	67.59	-67.59

Table 5: Axial forces acting in the columns related to the shear actions for both directions of earthquake

Therefore the required bending moment for each column $M_{c,i,1}$, the section, the upper and lower reinforcement and the axial force, for both directions of the earthquake are reported in Table 6.

STOREY	Column	$M_{c,i,1,LR}$ [kNm]	$M_{c,i,1,RL}$ [kNm]	b x h	$A_s = A'_s$	N_{LR} [kN]	N_{RL} [kN]
1°	A	235.59	235.59	30x40	7 Φ 20	-290.26	727.66
	B			30x50	5 Φ 16	686.62	479.78
	C			30x60	5 Φ 16	617.63	694.57
	D			30x60	5 Φ 16	694.57	617.63
	E			30x40	6 Φ 20	770.03	-41.03

Table 6: Final design of the column sections at first storey

The sums of obtained column plastic moments at first storey are: $M_{c,Rd,1,LR} = 1585.19\ kNm$ for LR earthquake and $M_{c,Rd,1,RL} = 1603.24\ kNm$ for RL earthquake which are greater than the required one. As a consequence the value of $\alpha_0^{(g)}$ for LR and RL earthquake are equal to $\alpha_{0,LR}^{(g)} = 1.5179$ and $\alpha_{0,RL}^{(g)} = 1.5227$, respectively.

f) Computation of the required sum of plastic moments of columns $M_{c,im}^{(t)}$ at any storey to avoid undesired mechanism.

The sum of the plastic moments of columns governing the column design at each storey is given in Table 7 and Table 8 by the underlined values. It can be recognized that, in the examined case, the need to avoid type-1 mechanism always governs the design of columns.
So for earthquake from Left to Right:

STOREY i_m	$M_{c,im;LR}^{(1)}$ [kNm]	$M_{c,im,LR}^{(2)}$ [kNm]	$M_{c,im,LR}^{(3)}$ [kNm]
1	<u>1585.19</u>	-	1585.19
2	<u>1509.60</u>	770.57	1140.09
3	<u>1963.60</u>	51.38	1007.49
4	<u>2131.34</u>	-476.97	827.18
5	<u>1917.40</u>	-719.07	599.16
6	<u>1226.37</u>	-579.49	323.43

Table 7: Sum of plastic moments of column required at each storey to avoid undesired mechanism

And for earthquake from Right to Left:

STOREY i_m	$M_{c,im,RL}^{(1)}$ [kNm]	$M_{c,im,RL}^{(2)}$ [kNm]	$M_{c,im,RL}^{(3)}$ [kNm]
1	<u>1603.24</u>	-	1603.24
2	<u>1499.68</u>	784.46	1142.07
3	<u>1957.25</u>	61.29	1009.27
4	<u>2127.97</u>	-470.62	828.67
5	<u>1916.21</u>	-715.70	600.25
6	<u>1226.37</u>	-578.30	324.03

Table 8: Sum of plastic moments of column required at each storey to avoid undesired mechanism

g) Design of column sections at each storey.

The required sum of column plastic moments M_{c,i,i_m}, the section, the upper and lower reinforcement, the axial force for both directions of the earthquake are reported in Table 9.

STOREY	Column	$M_{c,im,LR}$ [kNm]	$M_{c,im,RL}$ [kNm]	b x h	$A_s = A'_s$	N_{LR} [kN]	N_{RL} [kN]
2°	A	301.92	299.93	30x50	7 Φ 20	-241.88	606.38
	B			30x40	6 Φ 20	572.18	399.82
	C			30x50	6 Φ 16	514.69	578.81
	D			30x50	6 Φ 16	578.81	514.69
	E			30x50	6 Φ 20	641.69	-34.19
3°	A	392.72	391.45	30x50	6 Φ 24	-193.50	485.10
	B			30x50	6 Φ 20	457.75	319.85
	C			30x50	6 Φ 20	411.75	463.05
	D			30x50	6 Φ 20	463.05	411.75
	E			30x50	6 Φ 24	513.36	-27.36
4°	A	426.26	425.59	30x50	6 Φ 24	-145.13	363.83
	B			30x50	7 Φ 20	343.31	239.89
	C			30x50	7 Φ 20	308.81	347.29
	D			30x50	7 Φ 20	347.29	308.81
	E			30x50	6 Φ 24	385.02	-20.52
5°	A	383.48	383.24	30x50	6 Φ 24	-96.75	242.55
	B			30x50	7 Φ 20	228.87	159.93
	C			30x50	7 Φ 20	205.88	231.52
	D			30x50	7 Φ 20	231.52	205.88
	E			30x50	5 Φ 24	256.68	-13.68
6°	A	245.27	245.27	30x40	7 Φ 20	-48.38	121.28
	B			30x40	6 Φ 20	114.44	79.96
	C			30x40	6 Φ 20	102.94	115.76
	D			30x40	6 Φ 20	115.76	102.94
	E			30x40	6 Φ 20	128.34	-6.84

Table 9: Design of column sections at each storey

h) Checking of technological condition

By observing Table 6 and Table 9 it can be noted that there are some column sections at the first storey which are smaller than the corresponding ones required at the second storey, therefore, a technological condition at the first storey is not satisfied. As a consequence, the values of $M_{c,Rd,1,LR}$ and $M_{c,Rd,1,RL}$ need to be updated and the procedure needs to be repeated from the step e). In Table 10 and Table 11 the new value of required sum of plastic moments of columns $M_{c,im}^{(t)}$ at any storey are reported for both directions of earthquake.

4 VALIDATION OF THE DESIGN PROCEDURE

In order to validate the design procedure, a static non-linear analysis (push-over) has been carried out to investigate the actual seismic response of the designed frame by means SAP2000 computer program [41]. This analysis has the primary aim to confirm the development of the desired collapse mechanism typology and to evaluate the obtained energy dissipation capacity, testing the accuracy of the proposed design methodology. Regarding the structural modelling, the mechanical non-linearities, have been concentrated at beam and column ends by means of plastic hinge elements.

STOREY i_m	$M_{c,im}^{(1)}$ [kNm]	$M_{c,im}^{(2)}$ [kNm]	$M_{c,im}^{(3)}$ [kNm]
1	1660.80	-	1660.80
2	1468.05	828.74	1148.39
3	1937.01	92.92	1014.96
4	2117.21	-450.38	833.41
5	1912.42	-704.95	603.73
6	1226.37	-574.51	325.93

Table 10: Sum of plastic moments of column required at each storey for LR earthquake

and for the other direction of earthquake:

STOREY i_m	$M_{c,im}^{(1)}$ [kNm]	$M_{c,im}^{(2)}$ [kNm]	$M_{c,im}^{(3)}$ [kNm]
1	1689.21	-	1689.21
2	1452.45	850.59	1151.52
3	1927.02	108.53	1017.77
4	2111.91	-440.39	835.75
5	1910.54	-699.64	605.45
6	1226.37	-572.63	326.86

Table 11: Sum of plastic moments of column required at each storey for RL earthquake

In Table 12 are reported the final value of the columns.

STOREY	Column	$M_{c,im,LR}$ [kNm]	$M_{c,im,RL}$ [kNm]	b x h	$A_s = A'_s$	N_{LR} [kN]	N_{RL} [kN]
1°	A	235.59	235.59	30x50	6 Φ 20	-290.26	727.66
	B			30x50	5 Φ 16	686.62	479.78
	C			30x60	5 Φ 16	617.63	694.57
	D			30x60	5 Φ 16	694.57	617.63
	E			30x50	5 Φ 20	770.03	-41.03
2°	A	293.61	290.49	30x50	7 Φ 20	-241.88	606.38
	B			30x50	7 Φ 16	572.18	399.82
	C			30x50	6 Φ 16	514.69	578.81
	D			30x50	6 Φ 16	578.81	514.69
	E			30x50	6 Φ 20	641.69	-34.19
3°	A	387.40	385.40	30x50	6 Φ 24	-193.50	485.10
	B			30x50	6 Φ 20	457.75	319.85
	C			30x50	6 Φ 20	411.75	463.05
	D			30x50	6 Φ 20	463.05	411.75
	E			30x50	6 Φ 24	513.36	-27.36
4°	A	423.44	422.38	30x50	6 Φ 24	-145.13	363.83
	B			30x50	7 Φ 20	343.31	239.89
	C			30x50	7 Φ 20	308.81	347.29
	D			30x50	7 Φ 20	347.29	308.81
	E			30x50	6 Φ 24	385.02	-20.52
5°	A	382.48	382.10	30x50	6 Φ 24	-96.75	242.55
	B			30x50	7 Φ 20	228.87	159.93
	C			30x50	7 Φ 20	205.88	231.52
	D			30x50	7 Φ 20	231.52	205.88
	E			30x50	5 Φ 24	256.68	-13.68
6°	A	245.27	245.27	30x40	7 Φ 20	-48.38	121.28
	B			30x40	6 Φ 20	114.44	79.96
	C			30x40	6 Φ 20	102.94	115.76
	D			30x40	6 Φ 20	115.76	102.94
	E			30x40	6 Φ 20	128.34	-6.84

Table 12: Design of column sections at each storey for earthquake

The constitutive law of such plastic hinge elements is provided by a rigid plastic moment-rotation curve. The type of hinge depends on the element considered i.e. by its internal action. In fact, for the beams and the columns M3 and P-M3 hinge type have been considered, respectively. In case of P-M3 hinge type, the interaction domain P-M has been evaluated for each column and used in SAP2000 computer program. The results of the push-over analysis are mainly constituted by base shear – top sway displacement curve which is depicted in Figure 5. In the same figure also a straight line is given, i.e. the one corresponding to the linearized mechanism equilibrium curve of global mechanism whose expression, for the designed frame and for earthquake from Left to Right, is:

$$\alpha_{LR}^{(g)} = 1.5179 - 0.002433\,\delta \tag{20}$$

For earthquake to Right to Left, the expression is:

$$\alpha_{RL}^{(g)} = 1.5227 - 0.002433\,\delta \tag{21}$$

As already underlined there is a mechanism equilibrium curve for both direction of earthquake. The two mechanism equilibrium curves are different but only for what concern the α_0 value while the slope is the same as reported in Figure 5.

As it was expected, also the LR push-over curve is different from RL one. This difference can be easily understood if we consider that the axial forces in the columns are different in the two considered push-over and, as a consequence, also the plastic moment is different. Notwithstanding, in this case, the two curves are very close one each other but there is no proof of the fact that this represent a general result. So that both curves should be always considered when a non-symmetric moment-resisting frame is analized. Obviously, the base shear is, in this case, obtained by multiplying the value of α, given by Eq. (20) and Eq. (21), for the design base shear corresponding to $\alpha = 1$. The comparison between the push-over curve and the global mechanism equilibrium curve provides a first confirmation of the accuracy of the proposed design procedure. In fact, the last branches of push-over curves parallel to global mechanism equilibrium curves as showed in Figure 5.

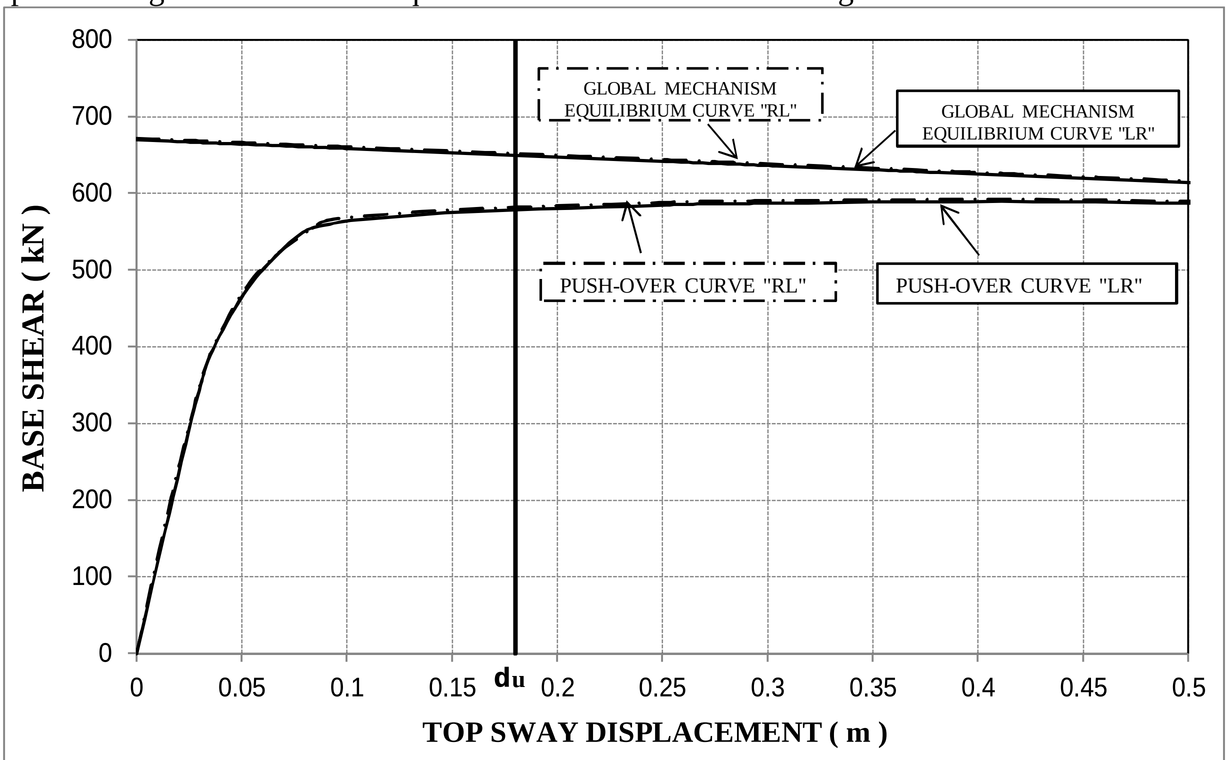

Figure 5: Overlap of the push-over curve with the global mechanism equilibrium curve

A further confirmation, even the most important, of the fulfilment of the design objective is represented by the pattern of yielding developed at the occurrence of the design ultimate displacement. In fact, developed plastic hinges are shown in Figure 6 and their pattern is in perfect agreement with the global mechanism. In order to fulfill the serviceability requirements the interstorey drift have been checked with reference to the limit reported in the Eurocode 8.

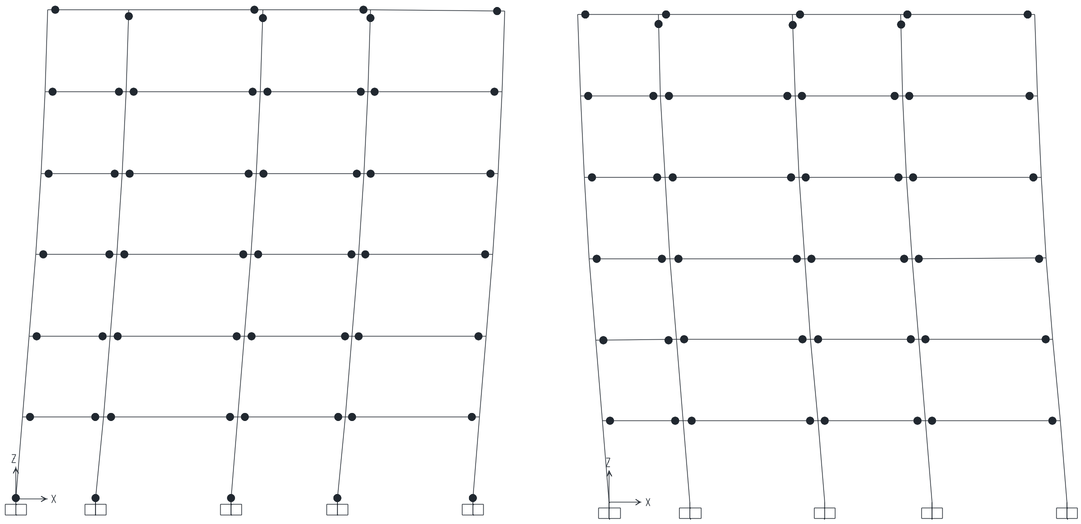

Figure 6: Pattern of yielding of the designed frame at $\delta = \delta_u$ for LR and RL earthquake direction

In particular the considered limit refers to buildings having non structural elements of brittle materials attached to the structure:

$$d_r\, \nu \leq 0.005\,\mathrm{h} \tag{22}$$

where d_r is the the maximum relative displacement between two consecutive storeys and h is the correspondinbg stprey height. If this serviceability requirement is not verified the structural stiffness can be improved by increasing the beam sections or the ultimate design displacement. In fact, in both cases the final results will be a more rigid structure with respect to the one obtained in the worked example herein presented. In Table 13 the final results are reported.

STOREY	d_s [cm]	d_r [cm]	ν	$d_r\, \nu$	0.005 h
6°	1.7687	0.7173	0.5	0.3586	1.5
5°	1.5848	1.0963		0.5481	1.5
4°	1.3037	1.4030		0.7015	1.5
3°	0.9439	1.5538		0.7769	1.5
2°	0.5455	1.4072		0.7036	1.5
1°	0.1847	0.7203		0.3601	1.5

Table 13: Limitation of interstorey drift

5 CONCLUSIONS

In this work a methodology able to control the failure mechanism of reinforced concrete moment resisting frames has been applied. On the base of the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve, the Theory of Plastic Mechanism Control allows to evaluate the sum of plastic moments of the columns required at each storey in order to develope a collapse mechanism of global type.

The closed form solution of the design conditions makes the design procedure very easy to be applied. In fact, in the author opinion it could also be suggested for code purpose by definitely solving the problem of collapse mechanism control whose importance in seismic design is universally recognised. Beam-column hierarchy criterion, commonly suggested by seismic codes, appears only as a very rough approximation when compared to TPMC and its theoretical background. The reliability of the proposed design procedure has been also demonstrated through its application to a four-bays, six-storeys frame, leading to the fulfilment of the design objective, i.e. the development of a collapse mechanism of global type, as it has been confirmed by the results of both push-over analysis.
The proposed methodology can be considered as belonging to the Performance Based Seismic Design philosophy [42]. In fact, in order to satisfy the limit states of "Life Safe" or "Near Collapse" the designer has to promote a dissipative collapse mechanism avoiding the so called "soft storey mechanism". In addition, it is useful to underline that the proposed procedure constitutes a rigorous application of the capacity design principles. In fact, beams are designed in order to bear external loads, while columns are designed according to the maximum internal actions transmitted by the dissipative zones. It is important to underline that the proposed procedure can be applied for MRFs characterized by a non-symmetric scheme and a non-symmetric reinforcement in each beam section. In the future development, the procedure should account also for the joists influence. In fact resistance of the joists is always neglected, but, as shown in [43] it can be really significative.

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