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CARTESIAN MESHES WITH DYNAMIC LOCAL WAVELET-BASED REFINEMENT FOR FLOW AROUND BODY PROBLEMS

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Abstract. *This document describes our project of making Cartesian multilevel AMR-method for flow around body problems in Eulerian statement. The main idea of Cartesian meshes with local refinement is based on the simplicity of Cartesian algorithms and the necessity of time and memory economy during the calculations. Tree-like data format for mesh representation [1] makes all the algorithms of mesh performance (generation, reconfiguration and traversal) rather simple and fast. We use Free boundary method [2] for geometry treatment on Cartesian meshes, WENO-technology for interlevel solution projecting and B-spline wavelet for data analysis.*

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1 INTRODUCTION

We present modern computational method for fluid dynamics based on both adaptive Cartesian meshes [1] and Free Boundary Method [2] for geometry treatment on Cartesian meshes. Suggested format of data representation based on quadric trees simplify procedures of mesh generation, reconfiguration and traversal. Specific way of cell numeration accelerates neighbor searching. Mesh configuration on each time step depends on the current position of the solids inside the computational domain and the local solution behavior. Free Boundary Method allows us to handle static or moving solids of complex geometry on the meshes with the simple rectangular cells. We use Finite Volume Method with Godunov or Rusanov fluxes as the main calculation scheme. Wavelet decomposition is used for quick parameter-free analysis of numerical solutions. Several different algorithms of multilevel mesh adaption that used binary criteria of function smoothness are compared.

2 MATHEMATICAL MODEL

In this paper, we propose a variant of mesh adaptation to solve the problem of an ideal gas movement, suitable for massive parallelization. The mathematical formulation of the problem is reduced to solving the system of equations

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + div(\rho \mathbf{U}) = 0, \\ \dfrac{\partial (\rho \mathbf{U})}{\partial t} + div(\rho \mathbf{U}\mathbf{U}) + grad(p) = 0, \\ \dfrac{\partial (\rho E)}{\partial t} + div(\rho \mathbf{U} H) = 0, \end{cases} \tag{1}$$

in a region $V(\Gamma) \subset \mathbf{R}^2$ bounded by a given closed curve $\Gamma = \Gamma(x, y)$. Here we introduce the standard notation: $\mathbf{U}$ - velocity vector, ρ - density, p – pressure, $H = E + p / \rho$ - the total specific enthalpy, $E = e + 0.5\mathbf{U}^2$ - the total specific energy, e - specific internal energy.

Equation of the system correspond to the laws of conservation of mass, momentum and energy. To close these systems serve as the equation of state. The gas is assumed ideal, i.e $p = (\gamma - 1)\rho e$.

For the Euler equations put the standard initial boundary value problem in the area.

Also the computational domain can include a solid body bounded be known curve $\Gamma_S \subset \Gamma$.

3 NUMERICAL METHOD

3.1 Basic calculation scheme

This system (1) written in vector form is solved numerically in the time step on a fixed Cartesian grid. This is done using the finite volume method using the approximate solution of the Riemann Problem by Rusanov [3] or its exact solution and counting flows by Godunov [4].

3.2 Free boundary method

This method was designed for numerical solving the Euler equations of compressible fluid dynamics in domains that contain solid, impermeable, and in general case moving entries (objects) with stationary Cartesian grids. The method proposed is based on the approach of immersed boundary, in which calculations are carried out over all cells of a Cartesian grid that covers the whole computational domain including solid entries. The influence of the solid surface on the gas flow is taken into account by introducing effective fluxes of mass, momentum, and energy on the right-hand side of the governing equations. We use Newton iterations for integrating implicit equation with non-zero right-hand side. The Jacobean was derived analytically. The stability of numerical method is guaranteed by satisfying the CFL condition.

4 CARTESIAN MESH REFINEMENT

4.1 Data format

To adapt the originally structured Cartesian grids, where the partition is meant for any preselected the law, it is convenient to use different tree structures. In addition, they are well-known fast recursive algorithms for crawling and rebuild.

We use quadric trees for the 2D adaptive grids and OC3 format for 3D-fields [5]. Each cell can only be broken into four equal division in half in each direction. Take the original rectangular grid (available in uneven) size $M \times N$.

Each cell is described by its level (for the basic mesh size is equal to zero), the virtual position at this level - a couple of parameters: row on level and column on level. In addition, it has a divided flag, indicating whether this cell is the final settlement (the corresponding leaf of the tree) or it has 4 children cells of the next level. For divided cells we save pointers to all its children, for leaf cell – the pointer to corresponding gas dynamic parameters.

4.2 Wavelet-based analyzer

Suppose on a rectangular grid of discrete specified function $\{g_{ij}\}, 0 \le i < 2N, 0 \le j < 2N$ (e.g., density or pressure of the gas-dynamic flow). Also considered to be specified threshold (the value of which is caused by a step of the computational grid and the order of approximation calculation method). Then we can calculate the wavelet decomposition of a two-dimensional array of data and define a set of flags [6]:

$$flag_{ij} = \left(lh_{\left[\frac{i}{2}\right]\left[\frac{j}{2}\right]} > \delta \right) \vee \left(hl_{\left[\frac{i}{2}\right]\left[\frac{j}{2}\right]} > \delta \right) \vee \left(hh_{\left[\frac{i}{2}\right]\left[\frac{j}{2}\right]} > \delta \right), 0 \le i < 2N, 0 \le j < 2N \qquad (2)$$

Flags define the area where you want to save a fine mesh, and in other areas it is possible to use a coarse grid.

4.3 Multilevel adaptation

Now consider the analyzer as a black box, issuing 1 if the cell is necessary to be divided and 0 if it is not necessary. Black box is applying to some window of cell environment (cross from 5 axis-aligned quasicells (real cells or parent cells without data) equal to chosen by size with the center of chosen cell or a square with a side of 5 cells and center in a given).We projected all the data to current pattern, if one pattern element is part of a larger cell, we give parent's value, if it consists of several subcells – we compute the value as conservative averaged.

Now we need to construct an algorithm that generates a grid configuration that is adequate to real physical field, written on current mesh.

We propose the following procedure:

1. First, hold the mesh coarsening procedure: those cells that can be made larger, according to the wavelet analysis are combined into one without violating the existing tree structure. In the cycle at levels ranging from the penultimate to the zero (base) are looking for a virtual cell, which may be roughened (have four physical descendants) and the criteria coarsening (black box for the parent cell produces zero).

2. After that we make refinement procedure. We pass through the levels from zero to the penultimate, looking at them physical cell, for which the dividing criteria is satisfied (black box provides 1) and divide them into 4 parts.

3. Draw the mesh smoothing process: remove all the collisions, where two neigboring cells differs more than twice by size. We do it by dividing too large cells into subcells until we rich the correct mesh.

5 NUMERICAL RESULTS

5.1 Gas dynamic tests

The advantages of suggested approach were demonstrated by some significant numerical calculations. We have computed single point explosion Sedov test [7] and Liska test 6 [8] on adaptive meshes with different value of maximal adaptation level. Results and statistics of this calculations are presented on figures1 and 2.

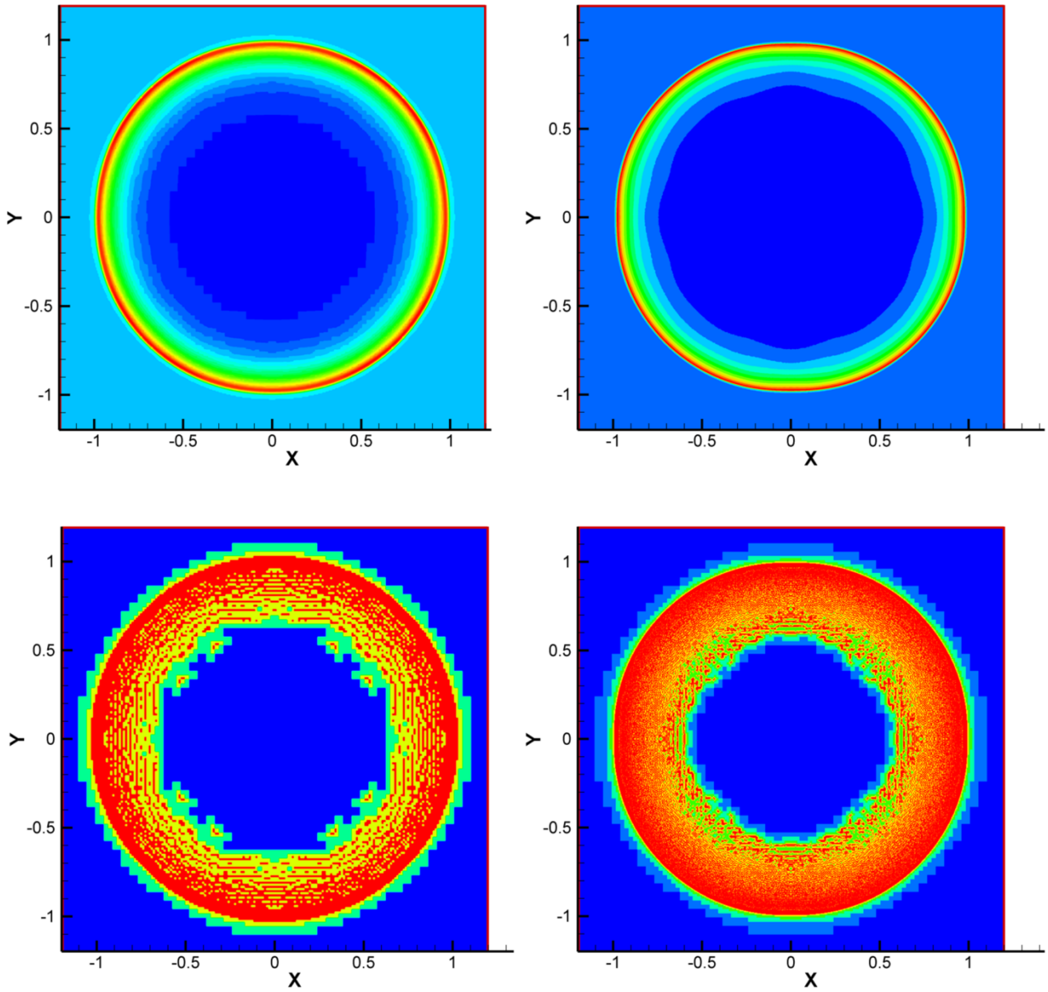

Figure 1: The results of Sedov problem calculations (top row: field density by color from blue to red) in the multi-level adaptive grid and the corresponding configuration of computational grids by the end of the calculation(colors from blue to red shows the level of physical cell). The left column – 4 adaptation levels, right – 6 adaptation levels, the base mesh 50x50.

With an increasing number of adaptation levels significantly increases the accuracy of the calculations (density at the shock wave is approaching to analytical value), decreases the percentage of the smallest. This fact talks about better localization features of algorithm with bigger number of levels. All the calculations were performed with the Rusanov flow to avoid carbuncle effect [9].

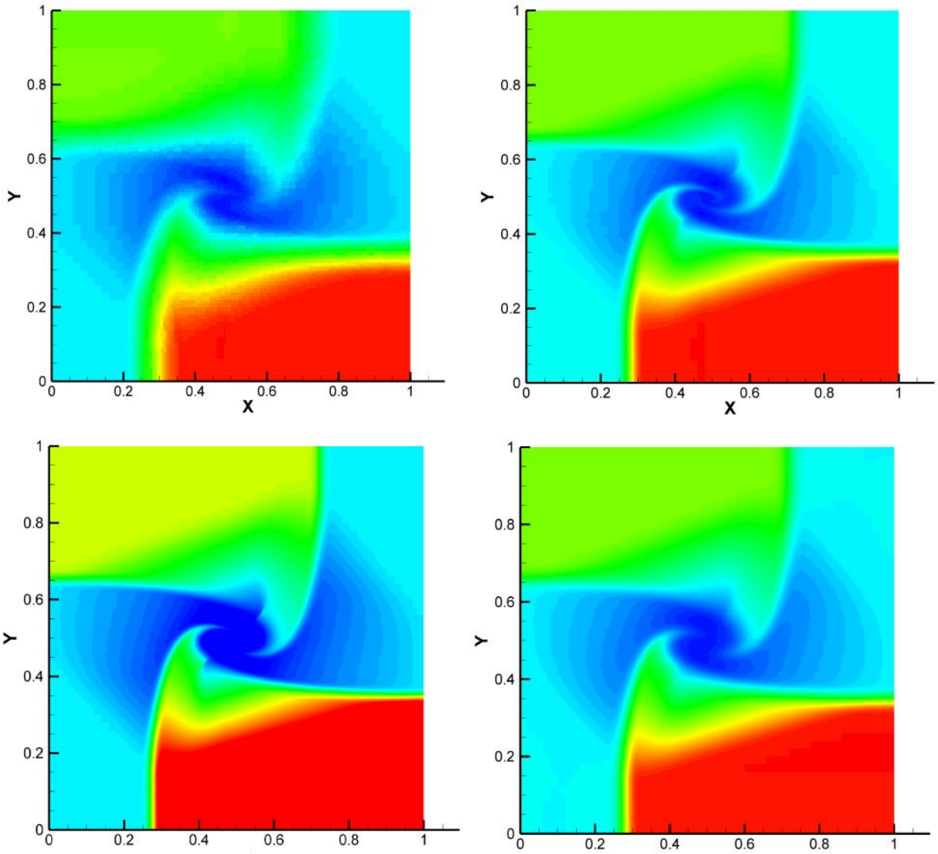

Figure 2: The calculated density fields for Liska test 6 by Godunov method with 3 levels of adaptation (top left), 5 levels of adaptation (top right), 7 levels of adaptation (bottom left) and Rusanov method with 7 levels of adaptation with the base grid of 50x50 cells.

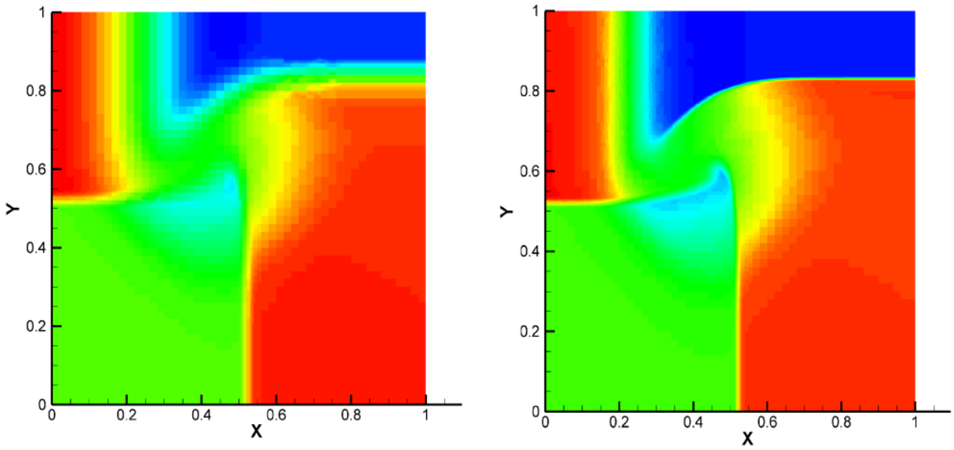

Figure 3: The calculated density fields for Liska test 16 by Godunov method with 3 levels of adaptation (left) and 6 levels of adaptation (right) with the base grid of 50x50 cells.

Increasing the number of adaptation levels when using Godunov method allows to identify the spatial effects of Rayleigh-Taylor instability for the test №6 along the coordinate axes and the rarefication zone for the test №16 (see. figure 3). Calculations using Rusanov doesn’t shov this features even using a lot of adaptation levels.

5.2 Flow around body problems

To show the opportunities of the our approach with geometrical solids we present the result of calculating the flow around aircraft fuselage (see fig. 4).

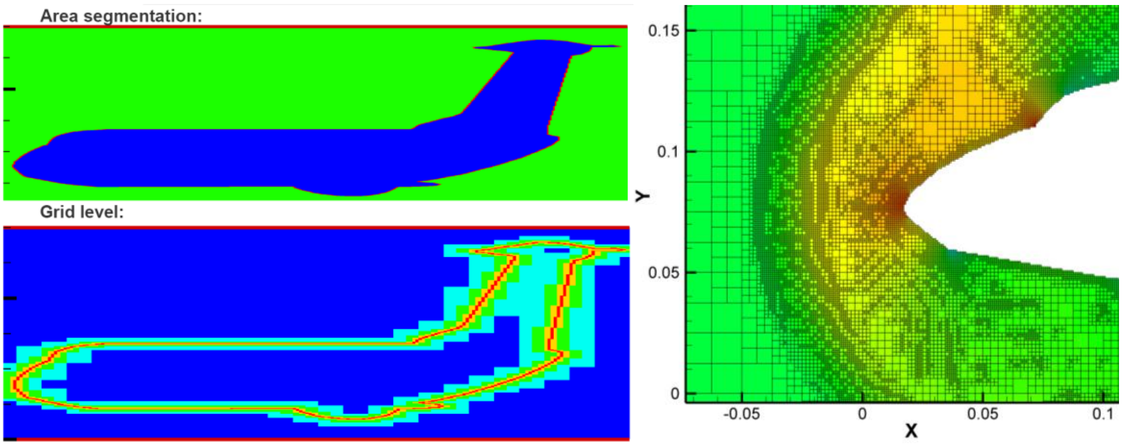

Figure 4: Area segmentation and pressure field for flow around aircraft model

6 CONCLUSIONS

We have designed the program code that allow us to calculate different CFD problems in the computational domains with several solids of complex shape using Cartesian meshes that dynamically rebuilds according to solids positions and solution behavior.

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