

## MODELLING STRATEGIES OF PRESTRESSING TENDONS AND REINFORCEMENT BARS IN CONCRETE STRUCTURES

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**Abstract.** *This contribution presents an original approach to improve the modeling of steel rebars and prestressing tendons in concrete structures at a reduced cost. Classical 1D meshes and models typically used for civil engineering applications tend to provoke strain localization due to the geometrical singularity and are thus unable to reproduce local mechanical effects. Complete 3D models can be applied in some cases, however their accuracy at the local scale comes at the cost of engineering work on the meshes, especially for complex structures. The 1D-3D model presented in this contribution generates an equivalent volume for the steel bars, based on existing 1D models. Its 3D stiffness and stress state are computed, and then condensed on its interface with the concrete. The condensed degree of freedom are then linked to the surrounding concrete elements by kinematic relations. The presented approach is validated on different representative cases, and is able to predict the 3D effects of the bars and tendons at the local scale. In particular it provides the representativeness and mesh stability of a full 3D model, without the need for a complex mesh.*

## 1 INTRODUCTION

In civil engineering structures, concrete is usually reinforced and/or prestressed using passive and active steel reinforcements. These reinforcements constitute material heterogeneities, and have to be modeled to predict the behavior of the structure. They are usually considered as unidimensional inclusions, considering the ratio of their length to their cross section. Therefore, these reinforcements are typically modeled in FE analyses using 1D elements, with beam or truss kinematics [4]. In some cases, where the three-dimensional effects of the tendon or rebar section have to be taken into account, full 3D models using conform meshes can be applied [8].

In the case of 1D models, the cross section of the steel is neglected, compared to the concrete matrix FE size. The models are superimposed, and the cross section has no influence on stress and stiffness repartition in the concrete. This approach has no specific mesh requirements, and is thus very easy to apply on various (and particularly, large-scale) structures. Therefore, a geometrical singularity is created. Although the global effect of reinforcement and prestressing is well reproduced, this may lead to pathological mesh dependency effects due to stress concentration at the singularity [1]. Also, transversal and cover effects can hardly be modeled with this approach.

In the case of full 3D models, using a geometrical and mechanical representation of the inclusions, the kinematics is much richer. This approach is both representative at the small scale (transversal effects can be predicted) and stable toward the mesh. The counterpart is the additional mesh complexity required to match the geometry of the heterogeneity.

In this contribution, a new approach is proposed, called the 1D-3D model (1D inclusions / 3D finite elements). It aims to combine the advantages of both modeling strategies, without the need for a specific mesh. The proposed model is first described. Then, an application on the numerical model of a nuclear containment building RSV is presented, where the different models are compared [7].

## 2 MODELLING APPROACH

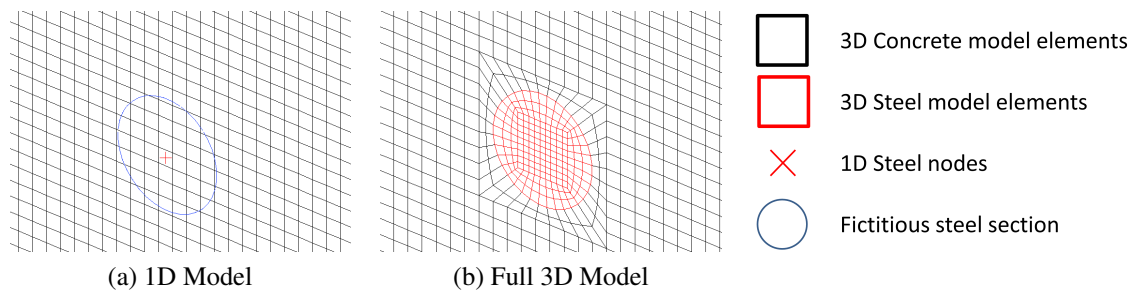


Figure 1: Modelling of a tendon in a concrete volume.

Fig. 1 presents the meshes used to model a reinforcement in a concrete volume, for the 1D and 3D models. The 1D-3D approach aims at combining the upsides of these models. Its main objective is to conserve the ease of use and versatility of the 1D approach, while having the representative local results and robustness of a full 3D approach. It should provide a single model for the reinforcements in concrete, with a validity domain overlapping the 1D and 3D approaches. Moreover, existing FE models can be used with this new model, as it uses 1D meshes. The proposed "1D-3D" approach requires four steps:

- Generation of the 3D equivalent mesh of the bar.

- Computation of the equivalent stresses.
- Condensation of the 3D stiffness and stresses.
- Construction of the steel-concrete interface.

## 2.1 Generation of the 3D equivalent mesh

To be able to represent the 3D effects of the steel bars on the concrete, a 3D equivalent mesh must be generated from existing 1D meshes. This allows to keep the adaptability of 1D meshes, as well as applying this new model to existing FE models of structures. The equivalent 1D-3D mesh is generated by projecting a circular section along the axis of the bar. The circular section is meshed with triangles using a concentric circles method. The 3D equivalent volume is therefore meshed with prismatic elements. The 3D concrete domain is not modified.

Fig. 2 presents the 1D-3D equivalent mesh for a reinforcement in a concrete domain, and the nodes at the steel-concrete interface.

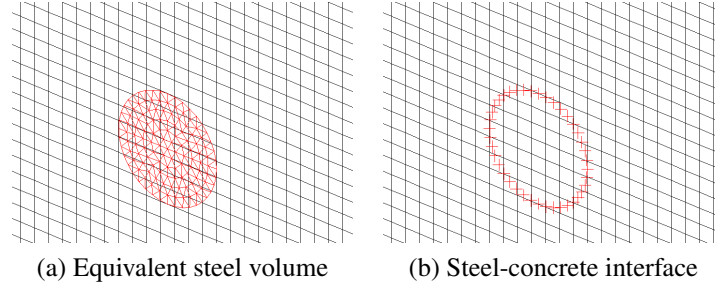


Figure 2: Modelling of a tendon in a concrete volume.

## 2.2 Equivalent stresses

In the case of active reinforcements such as prestressing tendons, or pre-tensioned rods, stresses are applied to the steel member, which transmits it to concrete. The proposed 1D-3D model requires transposing a 1D stress state to the equivalent 1D-3D volume. The 1D stress state of the steel is first converted to nodal forces:

$$F_i = S \int_{L_i} \frac{d\sigma_{1D}}{dl} dl \quad (1)$$

where  $F_i$  is the nodal force vector at node  $i$ ,  $\sigma_{1D}$  the stress value,  $S$  the bar's cross section and  $l$  the abscissa along the bar, integrated on the length  $L_i$  (depending of the finite elements form). These nodal forces are then applied to the 1D-3D model previously generated, creating a stress state such that:

$$F_i = \int_{V_i} \nabla \sigma_{1D3D} dV \quad (2)$$

where  $\sigma_{1D3D}$  is the stress tensor,  $\nabla \sigma_{1D3D}$  its divergence,  $V$  the volume of the bar, integrated on  $V_i$  (depending of the FE geometry and interpolation functions). Thus, the stress state computed using a 1D model can also be applied to the 1D-3D model.

## 2.3 Condensation

The 1D-3D model generates a large number of additional FEs and DOFs compared to classical 1D approaches. This may become an issue in terms of computational cost if large RC structures with high levels of reinforcements are considered. However, an approach is proposed to reduce the number of DOFs, similar to the condensation method presented in [9]. This method is based on Guyan's static condensation [6], and replaces the 1D-3D model by its reduced stiffness matrix and loading, applied on the envelope of the 1D-3D bar. This way, the volume is reduced to a surface.

However, the reduced stiffness matrices are smaller but also fuller than standard stiffnesses, which may impact computational performance when solving the linear system. To avoid this problem, low-rank approximations of the condensed stiffnesses may be used. Depending on the configurations, the condensation may or may not improve computational performance. Since this step is not strictly necessary in the 1D-3D approach, it should be applied only in the situations where it improves performance.

## 2.4 Steel-concrete interface

The steel-concrete bond is ensured by the interface nodes of the bar, related to the adjacent concrete elements by kinematic relations. These relations transmit the stiffness and force field of the 1D-3D model to the adjacent concrete nodes. The displacement  $u_i$  of the steel node  $i$  located at  $x_i$  is such that:

$$u_i = \sum_j u_j N_j(x_i) \quad (3)$$

where  $u_j$  are the displacements of the concrete nodes  $j$  adjacent to  $i$ , and  $N_j$  the interpolation functions of the concrete domain. This model is similar to the perfect bond usually considered for 1D reinforcement bars (in the case of non-corresponding meshes). This relation could be improved to include more representative steel-concrete bond models.

As in the classical 1D model, the two materials are geometrically superimposed: the additional stiffness is neglected. This method has been developed to be used without additional adaptations and/or meshing effort on existing 1D meshes. It provides, for a small additional computational cost, more accurate local stress fields and removes the pathological mesh dependency.

# 3 MODELLING OF A PRESTRESSED RSV

## 3.1 Presentation

In this section, a structural application case is considered. It consists in a numerical model of a representative structural volume (RSV) of a prestressed and reinforced concrete containment building, on which a pressure test is performed. This particular structure is strongly dependent on the prestressing tendons, which affect damage localization and failure mode [8]. In particular, previous studies have demonstrated an inclusion effect around the vertical tendon. This effect is not predicted with 1D models, but can be analyzed with full 3D models. The aim is thus to evaluate the capacity of the 1D-3D approach to predict this effect.

Fig. 3 presents the geometry of the structure: it consists in a curved, horizontally prestressed concrete volume on which an internal pressure is applied. A non prestressed vertical tendon (the transverse inclusion), and passive reinforcements are also included. The prestress is first applied through the horizontal tendons, with an associated vertical compressive stress; the internal pressure is then applied.

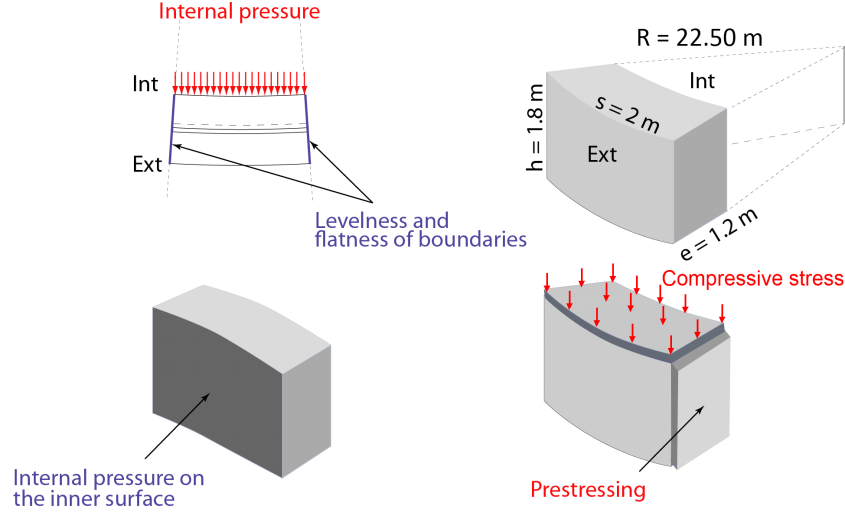


Figure 3: Geometry and loading of the prestressed concrete RSV [8].

The concrete is modelled using Mazars' isotropic damage model [10] (compressive strength  $f_c = 37.6$  MPa, tensile strength  $f_t = 3.57$  MPa). The stress-based nonlocal method is applied [5], with a nonlocal internal length  $l_{C0} = 4$  cm. The passive steel reinforcements are modeled using equivalent shell FEs. All simulations are performed using Cast3M [2], with post-processing using Salome [3]. On this structure, 1D, 1D-3D and full 3D models are compared regarding global and local results.

### 3.2 Results

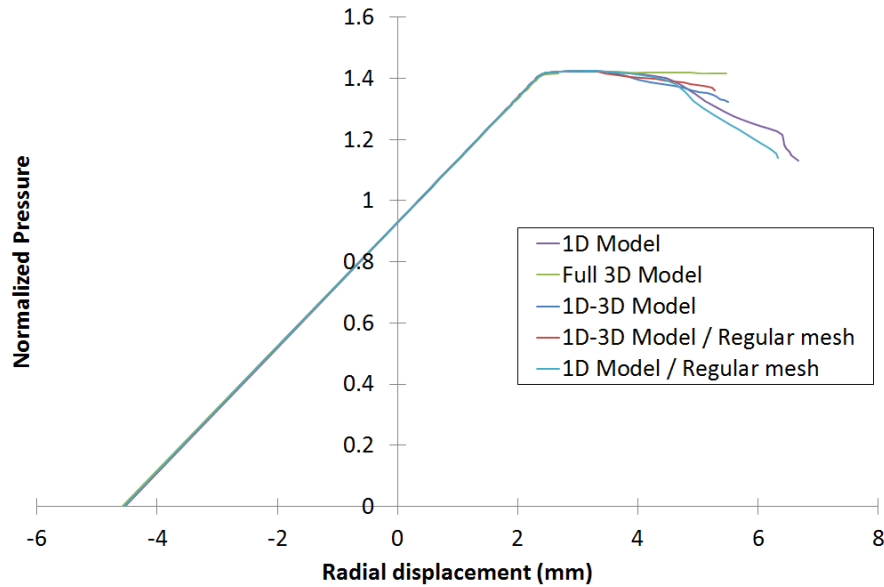


Figure 4: Pressure vs. Radial displacement evolution of the RSV with different tendon models.

To analyze the global nonlinear behavior, Fig. 4 presents the pressure-radial displacement evolutions for the different models. The prestress provokes an initial inward radial displacement. After the elastic part of the behavior, a nonlinear behavior is then observed: the pressure decreases while the (outward) radial displacement increases. All models provide very similar

results in this respect: the choice of the model used for the steel tendons (both horizontal and vertical) has little impact on the global behavior.

To analyze local effects, and in particular the inclusion effect of the vertical tendon, Fig. 5 compares for the different models the damage profiles in the RSV, for a radial displacement of 4.5 mm. The 1D-3D approach in particular, is applied using both the explicit 3D mesh with geometrical representation of the tendons, and a regular mesh, with 1D representation of the tendons. It appears that, excluding the inside of tendons (there is superimposition in the 1D-3D and not in the full 3D model), the distributions are nearly identical between 1D-3D, with both explicit and regular meshes, and the explicit 3D model. The effect of the vertical tendon on damage initiation and propagation is well reproduced. The 1D approach, however, does not reproduce this effect and features a more distributed damage in the whole volume, as well as damage localized around the horizontal singularities. Therefore, the proposed approach clearly improves the description of the mechanical behavior at the local scale. It is able to predict the effect of the vertical tendon without the additional meshing effort required to represent it explicitly.

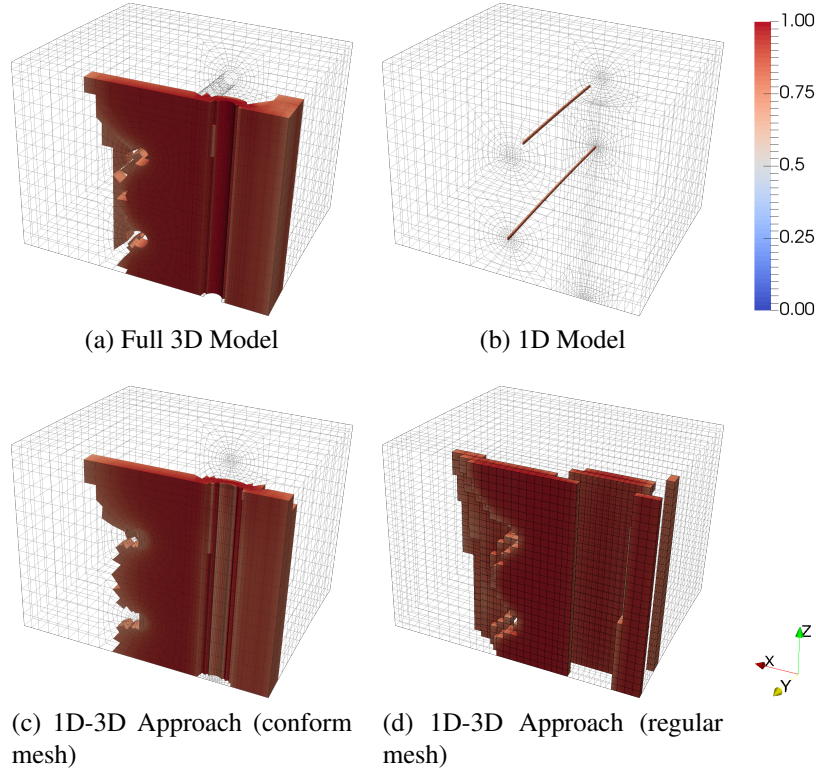


Figure 5: Damage profiles for a radial displacement of 4.5 mm (only the areas with  $D \geq 0.80$  are represented).

## 4 1D-3D APPROACH FOR UNCERTAIN GEOMETRY

### 4.1 Presentation

As presented earlier, one of the main advantages of the 1D-3D approach in modeling reinforcement bars and prestressing tendons in concrete is its flexibility. As it is based on a simple 1D geometrical description of the reinforcements and can be used without a specific concrete

mesh, it is well suited for applications such as sensitivity analyses or uncertainty quantification (e.g SFE analyses), including geometrical uncertainty.

In this section, an example of application of the 1D-3D approach is presented, that uses the same structural case than the preceding section. In particular, the sensitivity of the damage profile in the prestressed RSV to the position of the vertical tendon will be studied. The 1D-3D approach, using a regular mesh for concrete, will be applied for this study. Three configurations are modeled and analyzed. In the first, the cover at which the vertical tendon ( $\varnothing$  84 mm) is placed in the RSV is increased by 46 mm. In the second, the tendon is located at its nominal position (as in the preceding section), with a concrete cover of 208 mm. In the third, the cover is reduced by 46 mm. The damage profiles obtained with the three configurations are compared, in order to observe the local effects and how it is influenced by the tendon position.

## 4.2 Results

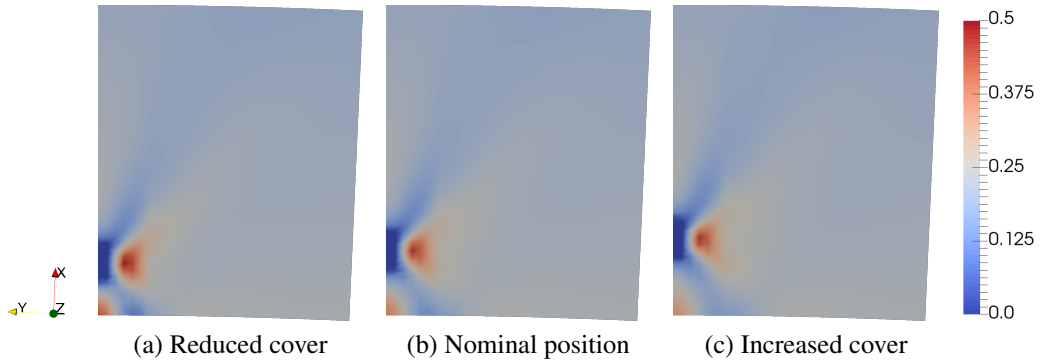


Figure 6: Damage profiles for a radial displacement of 3.0 mm with different positions of the vertical tendon (top view).

Fig. 6 presents and compares the damage profiles obtained with the three tendon positions. Damage profiles of each the increased/reduced configurations are compared locally to the reference configuration of the modeled RSV. In order to observe more easily the difference in localization of damage around the tendon, damage is observed at an earlier stage (radial displacement of 3.0 mm). We observe at first that the inclusion effect, leading to localized damaged areas, is still reproduced with all three positions using the 1D-3D model. In all three cases, damage seems to initiate on the side of the vertical tendon, following the perturbed position. Also, it appears that the concrete elements located inside the 1D-3D tendon are protected in all three simulations. This protected area follows quite correctly the perturbed position of the 1D-3D tendon. A second damaged area appears on the inside face of the RSV, in front of the vertical tendon. This area seems to be more damaged when the concrete cover of the tendon is reduced. The effect of the vertical tendon (and in particular the inclusion effect) is well reproduced in those configurations, and the influence of its position can be observed and quantified. This validates the applicability of the 1D-3D approach (in particular, using a regular mesh), for analyses in a uncertain geometry context.

## 5 CONCLUSIONS

A new approach, called 1D-3D, for the modeling of prestressing tendons and reinforcement bars in concrete structures has been presented. This method creates an equivalent volume from

the 1D mesh, with static condensation of the stresses and stiffness to limit the computational cost: no additional meshing requirements are necessary compared to classical 1D mesh. The reduced stiffness and nodal forces of the steel are applied on the envelope of the equivalent volume, and bonding to the concrete is then performed using kinematical relations. This method is validated on a representative structural volume of prestressed concrete containment buildings, on which geometrical uncertainties are considered. The 1D-3D approach avoids the well-known stress concentration around singularities observed with 1D models. It is also able to reproduce local structural effects, such as transversal effects. Therefore, the proposed 1D-3D approach combines in a single model the versatility and efficiency of classical 1D models at the large scale with the representativeness and the numerical stability of full 3D models.

## ACKNOWLEDGEMENTS

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