

A ROBUST POLYNOMIAL CHAOS KALMAN FILTER FRAMEWORK FOR CORROSION DETECTION IN REINFORCED CONCRETE STRUCTURES

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Abstract. *A main reason behind reinforced concrete structural deterioration is chloride-induced corrosion. Once the critical chloride concentration is exceeded at the rebar level, the structure becomes susceptible to corrosion initiation. Corrosion propagates progressively, degrades the resistance capacity of the structure and decreases the design safety margin. To mitigate this risk, a stochastic sequential data assimilation technique based on chloride concentration measurements and the Polynomial Chaos Kalman Filter (PCKF) is presented. The Power of PCKF lies in its sampling free scheme and polynomial structure to represent uncertainty. In modeling chloride ingress mechanism, different independent sources of uncertainty should be incorporated in the system including, mathematical model simplification errors, parametric errors, boundary condition errors, and time independent sensors errors. In such circumstances, the curse of dimensionality hinders the efficiency and the applicability of PCKF, due to the exponential growth of the required bases to account for the added uncertainties. This study presents a practical framework to maintain an acceptable accuracy of PCKF without scarifying the computational efficiency of the filter. A one dimensional synthetic numerical example is presented to verify the efficiency of the proposed implementation scheme.*

1 INTRODUCTION

Reinforced concrete (RC) structures, like buildings, bridges, dams, tunnels, pipelines, among others, are complex engineering systems, with long life expectancy, that serve society's industrial needs and guarantee economic prosperity. Labor force and financial assets are extensively invested in the design and construction stages of RC projects to achieve two-prime concerns, safety and serviceability through their lifetime. Research efforts and gained experience, influence design codes and governmental regulations to improve current practices in order to minimize the risk of failure and loss of serviceability. However, structures might be exposed to different environmental conditions and severe loading that can lead to structural deterioration. Therefore, continuous monitoring of the changing environmental conditions and accurate prediction of response of structures under real conditions are becoming highly recommended in practice [1]. In technical terms, this process is known as Structural Health Monitoring (SHM). SHM relies on real time measurements, damage detection algorithm and data assimilation technique for early detection of risks and better assessment of the operating conditions. Consequently, early risk alerts are set and maintenance schedules and repair mechanisms are optimized for proactive measures.

Among these damage mechanisms, corrosion in reinforcement bars is recognized as a serious deterioration phenomenon that threatens structural integrity. It is usually referred to as the most common cause for structural deterioration, and it leads to major problems in maintenance and rehabilitation of RC structures. In practice, corrosion of RC can be understood as a two-stage process: the corrosion initiation stage and the propagation stage [2]. During the first stage, the pH of the pore solution is reduced, leading to the de-passivation of the protective film, and exposing the rebar to corrosion initiation. The most common cause of corrosion initiation is typically attributed to the presence of chloride ions; the de-passivation of the protective films occurs when their concentration at rebar level exceeds a certain threshold. The source of chlorides may be admixtures, contaminants, marine environments, industrial brine and deicing salts. Once the corrosion is initiated, the steel starts to corrode generating iron oxides, which marks the starting of the second stage. Corrosion then progressively propagates causing cracking, delamination and spalling of the concrete cover, thus threatening the structure and alerts serviceability failure.

Once the damage occurs, the process becomes irreversible and needs detailed inspections and may require complex repair strategies. This shifts the attention to proactive maintenance measures that include treatment before any considerable damage. The application of this strategy needs adequate knowledge about the state of the structure and close monitoring of the corrosive species. Condition assessment of such structures is typically based on comparing the chloride content at steel level to the critical threshold chloride concentration.

For accurate prediction of corrosion initiation, Polynomial Chaos Kalman filter (PCKF) will be employed, in this study, for parametric identification of the quantities of interest in a probabilistic framework. Based on real time measurements and dynamic model predictions, the PCKF calibrates the chloride profile and model parameters to improve the prediction of the remaining corrosion free service life and to initiate proactive maintenance alert. This study will also suggest a practical implementation of PCKF, when time independent sources of uncertainties are incorporated in the system. The computational cost efficiency and the accuracy of the implementation will be compared to its closest descendent in the Kalman Filter family, the Ensemble Kalman filter (EnKF).

2 CHLORIDE INGRESS

Different phenomena govern the ingress of chloride ions into concrete pores depending on the state of the structure and the depths of concrete cover. In unsaturated medium, diffusion is the governing mechanism beyond the convection zone. However, in saturated and fully submerged concrete structures, the diffusion process is the governing mechanism after initial wetting [3, 4]. Since the principle objective is to estimate, the chloride concentration at rebar level, chloride ingress is typically modeled as a diffusion process according to Fick's second law and assuming constant boundary conditions. The general diffusion equation relates the diffusion coefficient and the gradient of the concentration as follows:

$$\frac{\partial C}{\partial t} = \nabla \cdot (D(t) \nabla C) \quad (1)$$

where C (kg/m^3) is the spatial chloride content at a certain location and time t . $D(t)$ is the diffusivity coefficient at t generally represented as a time decaying function:

$$D(t) = D_R \left(\frac{t_R}{t} \right)^m \quad (2)$$

where D_R is the reference diffusion coefficient at time t_R (usually $t_R = 28$ days) and m is a constant depending on concrete mix proportions. $D(t)$ should also account for the effects temperature and relative humidity [5].

An analytical solution, eq. 3, is derived from eq. 1 under several assumptions: 1-D diffusion process, constant surface chloride content, constant diffusion coefficient and infinite medium.

$$C(x, t) = C_s \left[1 - \operatorname{erf} \left(\frac{x}{2\sqrt{Dt}} \right) \right] \quad (3)$$

where C_s is the chloride surface concentration, x is the location away from the surface, and t is the time period.

To maintain generality and robustness, a numerical discrete solution for eq.1 will be implemented based on finite element/finite difference scheme rather than the solution presented in eq. 3 since the latter tends to lose its accuracy for varying environmental and parametric conditions. Moreover, using a numerical discretization scheme, spatial variability in the chloride profile can be updated and incorporated in the state vector for more accurate dynamic model propagation.

3 SOURCES OF UNCERTAINTY

Many sources of uncertainty are identified once developing an accurate mathematical model to simulate the chloride ingress process. First, the model input parameters have a wide margin of variability: the diffusion coefficient depends on the concrete mix and heterogeneous aging of concrete, concrete cover depends on the workmanship skills and on-site quality control, chloride surface concentration depends on type and severity of exposure, chloride concentration threshold values depend on steel bars and cementing material types and admixtures and method of measurement etc. [4]. Additional uncertainties can also be attributed to simplifications associated with the adopted mathematical model, measurement and human errors. To account for the effect of these uncertainties in modeling corrosion, probabilistic models can be used to estimate the probability of corrosion initiation, yet the quantification of

statistical input is usually very difficult and unfavorable in practice due to limited data and wide discrepancy [6].

Therefore, to identify accurately the corrosion initiation time, engineers are resorting to structural health monitoring (SHM). The process of monitoring the corrosion initiation time is theoretically tied to monitoring the free chloride ions in concrete and sometimes to total chloride content [7].

4 SERVICE LIFE PREDICTION

To guarantee safety and integrity of the structure, adequate maintenance is an essence. In the case of corrosion deterioration, the typical forms of repair are removing the concrete cover to clean the corroded steel and restoring the integrity of the formed cracks by certain injections. In some cases, additional structural support is necessary to rehabilitate the loss in designed structural capacity. This reactive maintenance practice is usually unfavorable for two main reasons: First, it pushes the structure to an extreme limit before interfering, which might lead to serviceability failure or even structural failure. Second, in case of corrosive deterioration, the later the maintenance is scheduled the more cost is associated with the repair mechanism. Therefore, pro-active maintenance is highly desirable in practice. The main strategy is to provide protection to reinforced concrete structures to either prevent or halt the problem of corrosion of the reinforcement before any considerable damage.

The probability of corrosion of the structure is computed as the probability of the chloride concentration exceeding the threshold value at the steel bar location. Typically, a reliability tolerance is selected based on the significance of the monitored structure, and accordingly, a maintenance plan is set and recurrently updated to prevent the structure from exceeding the designated probability of corrosion.

To represent the probability corrosion initiation, a limit state function relating the chloride threshold to chloride concentration at cover location is presented in equation 4. The limit state, g , is evaluated as the algebraic difference of the chloride critical threshold concentration and the predicted concentration at rebar level. A negative limit state indicates that corrosion has initiated while a positive outcome represents a safe un-corroded structure.

$$g(C_{th}, C_s, D_R, m, x_r) = C_{th} - C(C_s, D_R, m, x_r) \quad (4)$$

To quantify the uncertainties associated with modeling, parametric and boundary condition errors, C_{th} , C_s , D_R , and x_r are modeled as lognormal random variables representing the chloride threshold concentration, chloride surface concentration, diffusion coefficient, and cover location respectively. The age factor m is best modeled as beta distribution between 0 and 1, to abide by the constraining physical behavior. The random process C is the numerical solution of the stochastic diffusion equation approximated using a finite elements/ finite difference scheme.

Thus, the probability of corrosion is theoretically represented as the integral of the joint probability density function, f_x , of the outcome of g , over the negative domain or corrosion domain as shown in equation 5:

$$P_f = \int_{g(C_{th}, C_s, D_R, d, m) < 0} f_x(C_{th}, C_s, D_R, m, x_r) dx \quad (5)$$

The solution of the above integral is estimated using Monte Carlo simulation, which is simple and easy to implement; it consists of simulating the outcome using random samples of

the input parameter and repeating the process sufficiently enough times to represent the pdf of the outcome

5 SEQUENTIAL DATA ASSIMILATION

Sequential data assimilation techniques typically utilize the dynamics of a known process and set of observations of the outcome for improving the knowledge of unknown state model parameters and dynamic variables. A well-known sequential filter, in estimation theory, is Kalman filter, which gives unbiased estimate with a minimum error variance when the system has linear dynamics and Gaussian measurement errors. However, in a non-linear modeling framework and in non-Gaussian framework, the Kalman filter loses many of its pros. Extensions of Kalman filter have been suggested in literature for approximating the posterior measures in complex systems. To address this issue, this study proposes the use of polynomial chaos based Kalman filter for data assimilation purposes. PCKF was suggested as a sampling free alternative [8, 9, 10, 11]. A detailed explanation of its theoretical background and implementation is discussed below.

5.1 Polynomial Chaos Expansion (PCE)

PCE is a non sampling approach to represent uncertainty using deterministic orthogonal polynomials. Let $\xi(\theta)$ be a vector containing random independent variables, where θ is used as an index of the probabilistic character of the components of the vector (ξ_i). Each component of the vector denotes an independent source of uncertainty, where it's considered as a modeling decision to choose the length of the random variables vector or the contributing sources of uncertainty. Non-linear functional of ξ are expanded with respect to a basis in space of square integrable random variables. A suitable set of these bases is utilized by multidimensional polynomials, such that they are orthogonal to the joint probability measure of the random variables vector ξ . Therefore, in accordance with this formulation, if the random variable used is Gaussian, the orthogonal multidimensional basis is Hermite polynomials. Then accordingly, the deterministic coefficients of each basis are determined to parameterize the probabilistic distribution of the expanded function that uniquely represents its distribution and in an L2 sense. To elaborate the PCE more, consider a random process $u(x, \theta)$ that is expanded with respect to the vector ξ , with a finite covariance. Thus, its polynomial expansion is written with respect to multidimensional Hermite polynomials in Gaussian random variables arguments ($\xi_{i1}, \xi_{i2}, \dots, \xi_{in}$) as follows [12]:

$$u(x, \theta) = a_0(x)\Gamma_0 + \sum_{i_1=1}^{\infty} a_{i_1}(x)\Gamma_1(\xi_{i_1}(\theta)) + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} a_{i_1 i_2}(x)\Gamma_2(\xi_{i_1}(\theta)\xi_{i_2}(\theta)) + \dots \quad (6)$$

Where Γ_n is the n th multidimensional Hermite polynomial, and a_{i_n} is its corresponding deterministic coefficient. The above series could be truncated by choosing a finite length independent random variable vector and an upper limit for the degree of the Hermite polynomial. By introducing a one to one mapping to a set of ordered indices, and truncating the series after the P^{th} term, the above expansion is rewritten as follow:

$$u(x, \theta) = \sum_{j=0}^P u_j(x) \psi_j(\theta) \quad (7)$$

The accuracy of the polynomial expansion is usually refined by increasing the M random dimension and/or the maximum order p of the expansion. The total number of terms, $P+1$, can be calculated as following:

$$P + 1 = \frac{(p + M)!}{M! p!} \quad (8)$$

5.2 Polynomial Chaos Kalman Filter (PCKF)

The PCKF consists of two stages: (1) a forecast stage and (2) an analysis stage if a measurement is present. The order and dimension of PC expansion are fixed from the beginning and throughout the processing for simplicity and computational purposes.

5.2.1. Forecast stage

Let A be the state matrix, which contains the vectors representing the deterministic coefficients of the PCE corresponding to model state and parameters at time t :

$$A = (u_0, u_1 \dots, u_p) \quad u_i \in \mathbb{R}^n, A \in \mathbb{R}^{n \times (P+1)} \quad (9)$$

where u_i corresponds to the i^{th} component of the PCE expansion of the state vector u containing the chloride concentration at nodes, D , m and Cs . In the forecast stage, the uncertainty of the parameters and the dynamic state is propagated forward in time using stochastic Galerkin scheme. A process noise is added after forecast to represent the modeling error.

5.2.2. Analysis stage

This is a learning stage, where parameters and uncertainties are updated when a measurement is available. The measurement probabilistic representation, which is in this case the free chloride concentration at certain locations, is also expanded in a polynomial chaos form,

$$D = \sum_{j=0}^P d_j \psi_j \quad (10)$$

where d_j is the deterministic coefficient of the j th basis. D is stored in a matrix form,

$$B = (d_0, d_1 \dots, d_p) \quad (11)$$

The filtering step relies on a minimum square error estimator of the coefficient of the polynomial chaos expansion. To evaluate the coefficient of the analysis step, the filtering step equation is projected on an approximating space by the basis of polynomial chaos $\{\psi_j\}_{j=0}^P$. this results in:

$$u_j^a = u_j^f + P^f H^T (H P^f H^T + R)^{-1} \cdot (d_j - H u_j^f) \quad (12)$$

where f and a denote the forecast and analysis variables respectively, H acts as a linear measurement operator, to relate the true model state, to the measurements d and P , with f or a indices, and R are the covariance error matrices of the forecast state and the measurements respectively

In a matrix form, it becomes [8],

$$A^a = A^f + P^f H^T (H P^f H^T + R)^{-1} \cdot (B - H A^f) \quad (13)$$

5.3 Curse of Dimensionality and Suggested Implementation

When the process noises and measurement errors are considered in PCKF application as time independent variables, the PCE terms could increase beyond computational applicability. PCKF has gained a noticeable popularity since it was first used in 2007 [10], however, it was rarely applied with time independent process noises and model errors. Moreover, time independent measurements are usually approximated with a finite memory dimension with no clear methodology or error analysis for this representation. The theoretical necessity to add independent dimensions for every independent source of error discouraged researchers from applying polynomial chaos based filters to problems with high independent sources of errors. This issue could be the reason behind limiting the application of polynomial chaos based filters to problems with initial value errors and made the use of sampling filters advantageous for other applications.

Theoretically, each independent source of error will cost an increase in the dimensionality of the problem. However, this study will focus on keeping the computational cost minimal yet without sacrificing the accuracy. The suggested approach to deal with increasing dimensionality of the PCE, due to incorporation of time independent errors, relies on limiting the PCE bases to finite terms, yet keeping an approximate same covariance. This strategy can be achieved by projecting the covariance matrix on a pre-specified number of first order terms when an additional basis is required beyond the fixed expansion length $(P+1)$. This methodology maintains a good approximate propagation of the covariance and the mean of the parameters, and maintains a low-dimensional PCE. Thus, the unbearable cost of curse of dimensionality is shifted to the cost of projection the covariance matrix on first order terms, which is the solution of a system of non-linear equations, with, in most cases, a close initial guess. This approach is influenced by the characteristics of PCKF as being suited as an identification filter rather than inference method, since PCKF provides a good approximation of the posterior mean in Bayesian setting with a random priori estimation error [13]. In addition, it relies on the covariance matrices of the forecasted state vector and the covariance of the measurements, for an optimal identification of the state vector, i.e. maintaining a good estimate of the first two moments is the pillar for a successful implementation of PCKF.

6 NUMERICAL APPLICATION

The efficiency of the presented health-monitoring scheme is illustrated on 1-D diffusion example. The problem represents the chloride ingress in a near shore wall or an interior column exposed to a corrosive environment. A 50x25 cm reinforced concrete column with a 5 cm cover depth is used to demonstrate the efficacy of the presented framework. The sensors

are put in two locations in the zone of interest, near the rebar location, as shown in Table 1, and only one surface is assumed fully saturated.

Sensors	Location (m)
1	0.05 from surface
2	0.03 from surface

Table 1 Sensors' Locations

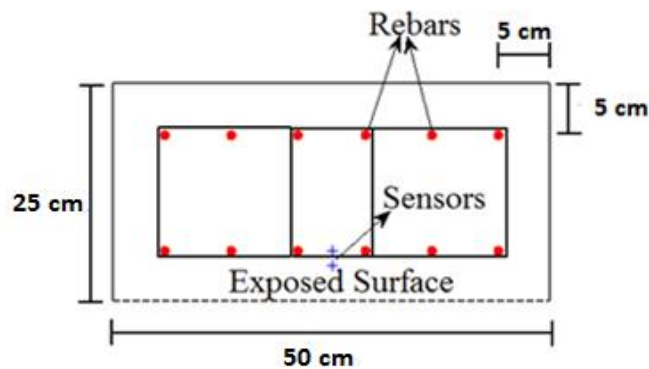


Figure 1 Schematic Drawing

6.1 Input Parameters

Table 2 presents the input parameters assumed for the base model. These parameters are used to simulate the measured chloride concentrations.

Parameter	Value	Comments
Surface Chloride Concentration, C_s	5 kg/m ³	High exposure
Reference Diffusion Rate, D_R	0.00015 m ² /year	At 28 days
Age factor, m	0.07	

Table 2 Base Model Input Parameters

The Crank solution eq. 3, with an average time varying diffusion rate $D_m(t)$ [14], is used to generate the sensors' synthetic data. To represent measurement errors, an additive Gaussian white noise perturbation having a standard deviation equal to 2% of the actual data is added to the simulated chloride concentrations.

Starting with an initial assumption for the state of each parameter, the perturbed synthetic measurements are used to calibrate them as time progress. Table 3 presents the assumed initial probabilistic characteristics for the model parameters. Furthermore, it is assumed that the structures in both problems are chloride free at the beginning of the simulation time.

Parameter	Mean Value	cov	Distribution
Surface Chloride Concentration, C_s	4 kg/m ³	0.07	Lognormal
Reference Diffusion Rate, D_R	0.0001 m ² /year	0.07	Lognormal
Age factor, m	0.05	0.07	Beta

Table 3 Initial Model Parameters

Within the framework of the PCKF, a set of orthogonal polynomials are used to represent the uncertainty within the system. The initial PCE is propagated forward in time using finite element analysis with 0.025 years increment. The model error and parametric errors are represented via an additive Gaussian white noise with a standard deviation equals to 1.5% and 2.5% respectively of the forecasted mean. To represent the spatial variability in concrete, a spatial de-correlation length of 4 cm is assumed; thus a 6 i.d.d random variables are assigned to represent the uncertainty in the concentration profile grid each time model error is incorporated in the system. Whenever measurements are available, the model state vectors are updated using the analysis stage equations. With a real time sensor system, data availability is not a limitation since sensors typically give instantaneous measurements. However, due to the relatively slow nature of the chloride ingress, a one-year measurement interval is adopted.

6.2 Results

6.2.1. PCKF implementation

The benchmark for comparing the results of the suggested implementation will be an EnKF approach using theoretically sufficient ensembles representation, to eliminate sampling errors, that is in this case 5×10^5 ensembles. To assess the effect of number of random variables and maximum order used in the PCE, the methodology will be applied to four independent formulations of PCE with: (1) 37 dimensions and order 1, (2) 37 dimensions and order 2 and (3) 37 dimensions and order 3 and (4) 45 dimensions and order 2. Note that the only constraint for applying the projection on the first order basis is that the number of first order basis (dimension) is at least equal to the length of the state vector, to be able to solve the non-linear system of equations. In addition, two EnKF formulations with 0.25×10^5 and 1×10^5 ensembles will be simulated for comparative purposes. The computational cost will be assessed based on the time (sec) needed to run the simulations for 20 years, or 800 time steps. However to assess the accuracy of the suggested scheme the average deviation, eq. 14, and the maximum deviation, eq. 15, of the first two order moments, from benchmark problem, of chloride concentration at rebar level and the time varying diffusion rate will be calculated for the simulated period, as follows:

Average deviation:

$$\%Ad = \sum_{n=0}^{800} \frac{|(u^n - u_{benchmark}^n)|}{800 \times u_{benchmark}^n} \times 100 \quad (14)$$

$$\%Maxd = Maximum \left(\frac{|(u^n - u_{benchmark}^n)|}{u_{benchmark}^n} \times 100 \right)_0^{800} \quad (15)$$

Where u and $u_{benchmark}$ are first or second order moments of the studied sequential formulation and benchmark problem formulation respectively. The $\%Maxd$ of the studied parameters and the $\%Ad$, for the simulation period are presented in Tables 4 and 5 respectively. Table 6 presents the computational time, in seconds, required for the different sequential filters' formulations, to perform the forecast steps and the analysis steps for the 20 years. These tables reflect the power of the suggested scheme, where the PCKF with dimension 37 and order 2 rendered accurate results, with less than 0.35 $\%Maxd$ and less than 0.094 $\%Ad$ and with better accuracy and computational cost compared to EnKF formulations. Moreover, no significant improvement is shown in the PCE formulation beyond 37 dimensions and second

order. This can be attributed to the emphasize of the lower order terms due to the projection on the first order bases and to conservation of first two moments in all PCE expansions.

	%Ad for mean of C at x_r	%Ad for std of C at x_r	%Ad for mean of D	%Ad for std of D
EnKF: 10^5 Ensembles	0.180	0.571	0.061	0.446
EnKF: 0.25×10^5 Ensembles	0.393	2.100	0.490	1.555
PCKF: Dimensions 37 Order 1	1.074	4.722	1.505	2.930
PCKF: Dimensions 37 Order 2	0.074	0.350	0.072	0.218
PCKF: Dimensions 37 Order 3	0.074	0.350	0.059	0.221
PCKF: Dimensions 45 Order 2	0.074	0.350	0.074	0.219

Table 4 Maximum percentage deviation from the benchmark problem for chloride concentration C at rebar level and diffusion rate D

	%Maxd for mean of C at x_r	%Maxd for std of C at x_r	%Maxd for mean of D	%Maxd for std of D
EnKF: 10^5 Ensembles	0.010	0.159	0.027	0.183
EnKF: 0.25×10^5 Ensembles	0.039	0.346	0.129	0.421
PCKF: Dimensions 37 Order 1	0.106	0.249	0.387	0.506
PCKF: Dimensions 37 Order 2	0.007	0.084	0.024	0.094
PCKF: Dimensions 37 Order 3	0.006	0.082	0.020	0.094
PCKF: Dimensions 45 Order 2	0.007	0.084	0.024	0.094

Table 5 Average percentage deviation from the benchmark problem for chloride concentration C at rebar level and diffusion rate D

	Computational time (Sec)
Benchmark Problem	15168
EnKF: 10^5 Ensembles	3034
EnKF: 0.25×10^5 Ensembles	751
PCKF: Dimensions 37 Order 1	230
PCKF: Dimensions 37 Order 2	372
PCKF: Dimensions 37 Order 3	24480
PCKF: Dimensions 45 Order 2	801

Table 6 Computational time in seconds for 20 years simulation period

6.2.2. Parametric identification

PCKF with 37 dimensions and second order PCE is used for health monitoring of chloride ingress in concrete. Figure 2 shows the mean prediction of chloride concentration at rebar level using PCKF calibration technique compared to using initial value parameters, and Figure 3 shows the mean and 1 std error bars of the calibrated time varying diffusion coefficient as compared to the base case model assumptions. A significant improvement in identifying the chloride concentration at rebar level and the diffusion parameter is reflected in these two figures. Consequently, a more accurate reliability assessment for corrosion initiation can be performed using the updated state vector parameters.

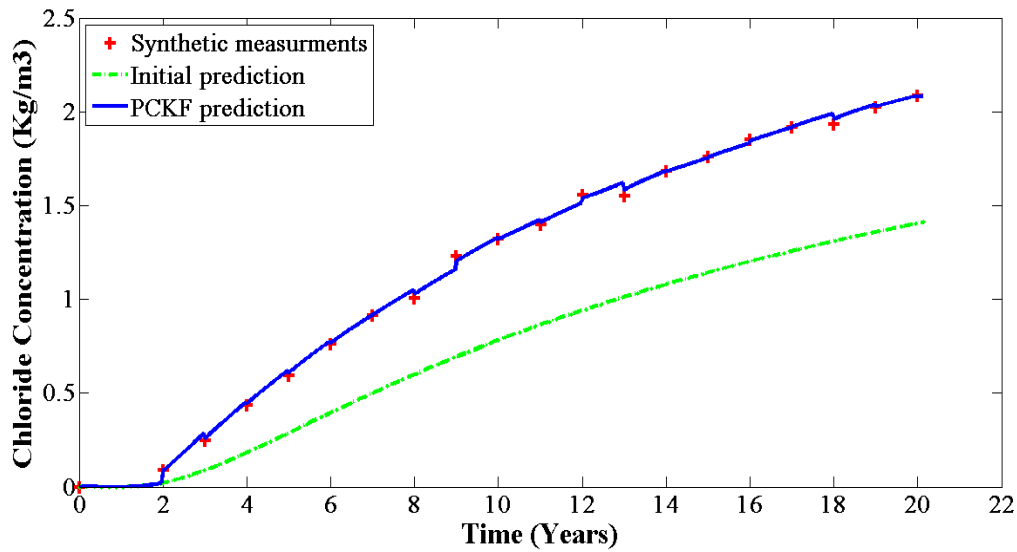


Figure 2 Measured Vs. Predicted chloride concentration chloride concentration using PCKF and initial parameters at rebar level

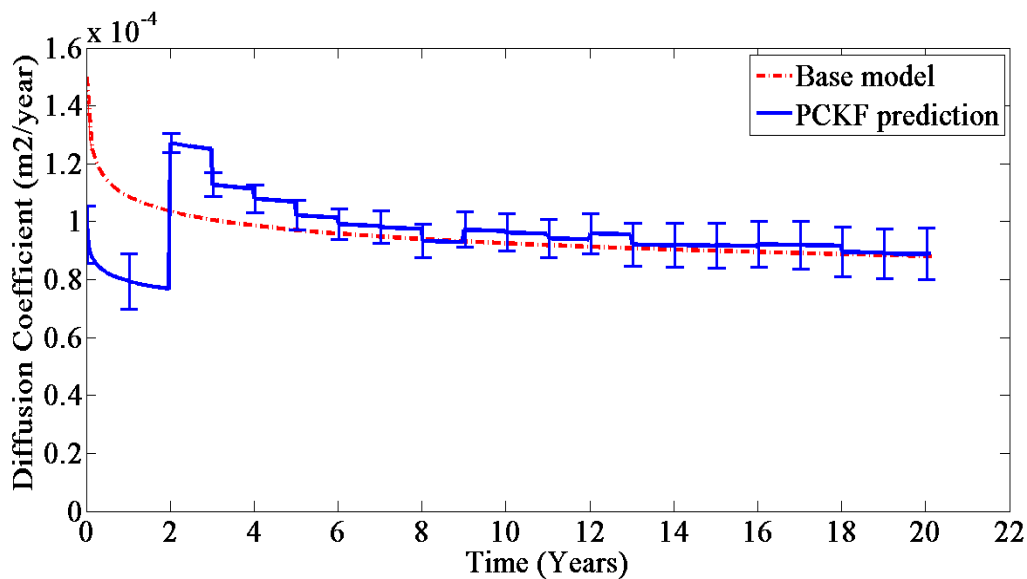


Figure 3 Base Case Vs. Mean calibrated time varying diffusion coefficient with 1 std error bars

7 CONCLUSIONS

In this study, PCKF was employed as a parametric identification tool using a suggested practical framework to mitigate the curse of the dimensionality. The power of the suggested scheme was reflected by comparing its accuracy and computational efficiency to the well-known EnKF. The suggested methodology was used for calibration of dynamic variables and model parameters of 1-D diffusion problem for better assessment of corrosion initiation time.

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