

Optimum Design, Finite Element Model Updating and Dynamic Analysis of a Full Laminated Glass Panoramic Car Elevator

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Abstract. *A systematic optimum design procedure, including an accurate dynamic analysis of a full glass panoramic car elevator under real dynamic load conditions are presented in this work. The cabin is manufactured entirely of laminated glass (two glass layers and an interlayer of polivinyll butiral -PVB), except the roof and the platform. First, modal identification and structural model updating methods are applied, leading to develop high fidelity finite element model of the glass and its connection subsystems. The identification of modal characteristics of the glass is based on acceleration and stress time histories, which are obtained through an experimental investigation of its dynamic response, in two different states. First, in a support-free state by imposing impulsive loading and second in a fixed-free state by imposing random excitation with the use of an electrodynamic shaker. Single and multi-objective structural identification methods are used for estimating the parameters (material properties) of the FE model, based on minimizing the deviations between the experimental and analytical modal characteristics. Next, a “mixed computational-experimental” analysis method is applied, in order to simulate accurately the dynamic behavior of the complete elevator system, in emergency situations like safety gear engagement. A series of experimental tests were performed under real operating conditions, using an experimental device that was designed exactly for this purpose and aimed at recording the acceleration time histories at the connection points of the frame with the safety gears. These acceleration time histories are subsequently used as base excitation for the FE model of the complete elevator system and the stresses developed under these specific loading conditions are evaluated. On the basis of these numerical results, the critical points of the frame are selected, as corresponding to larger stresses and an optimum design procedure was applied. Finally, in order to test the reliability of the method applied, strain gauges are placed at the critical points of the optimum designed system and a series of measurements are carried out, in order to experimentally verify the developed stresses. Direct comparison of the numerical and experimental data verified the reliability and accuracy of the methodology applied.*

1 INTRODUCTION

Within an elevator system the car is one of the most significant components because it is used to carry goods and most importantly persons. Because of its use the car is subject of strict legislations and standards that differ to each Country or State. In the same time the car is, usually, the only functional component of the elevator that is visible by the final user. This fact alters the car to be also an architectural and decorative element of the building. The latest trends in Architecture is to use fully transparent glass building elements [17]. A frameless glass panoramic elevator could not only fit to this trend but also extend the aesthetic of the building. However, glass is a brittle material and this attribute made the verification of the materials load carrying capacity to be mostly based on experimental studies. Some research has been done on the field of modeling of the behavior of glass components, using finite element methodologies, but these are mainly studying static loads situations aimed at the building construction domain [16-18]. In this work we present a method for modeling glass components in dynamic situations that may occur during an elevator's function e.g. when the safety gear of the elevator is activated during an emergency stop. An efficient modelling method is required for the various elevator parts which are in contact with each other. To achieve these modelling issues, it is important to develop an accurate Finite Element Analysis procedure, in order to simulate the dynamic behaviour of these systems.

Another main issue addressed by the present work is the need for the development and application of new appropriate methodologies for investigating the dynamics of large scale mechanical models in a systematic and efficient way. The equations of motion of mechanical systems with complex geometry are first set up, applying classical finite element techniques. As the order of these models increases, the existing numerical and experimental methodologies for a systematic determination of their dynamic response become inefficient to apply. Moreover, in order to optimize the FE model of a structure, structural model updating methods [7], have been proposed in order to reconcile the numerical (FE) model, with experimental data. Structural model parameter estimation based on measured modal data [1-6] are often formulated as weighted least-squares estimation problems in which metrics, measuring the residuals between measured and model predicted modal characteristics, are build up into a single weighted residuals metric formed as a weighted average of the multiple individual metrics using weighting factors. Standard gradient-based optimization techniques are then used to find the optimal values of the structural parameters that minimize the single weighted residuals metric representing an overall measure of fit between measured and model predicted modal characteristics. Due to model error and measurement noise, the results of the optimization are affected by the values assumed for the weighting factors.

The work presented here is based on previous work where we demonstrated the advantages of applying appropriate numerical and experimental methodologies in order to accurately predict the dynamic response and the identification of the critical points in an elevator system [2]. Also examined the applicability and effectiveness of the updating methods, coupled with robust, accurate and efficient finite element analysis software.

2 FINITE ELEMENT MODEL UPDATING METHODS

Let $D = \{\hat{\omega}_r, \hat{\phi}_r \in R^{N_o}, r=1, \dots, m\}$ be the measured modal data from a structure, consisting of modal frequencies $\hat{\omega}_r$ and mode shape components $\hat{\phi}_r$ at N_o measured DOFs, where m is the number of observed modes. Consider a parameterized class of linear structural models used to model the dynamic behavior of the structure and let $\theta \in R^{N_\theta}$ be the set of free structural model parameters to be identified using the measured modal data. The objective in a modal-based

structural identification methodology is to estimate the values of the parameter set $\underline{\theta}$ so that the modal data $\{\omega_r(\underline{\theta}), \underline{\phi}_r(\underline{\theta}) \in R^{N_0}, r=1, \dots, m\}$ predicted by the linear class of models at the corresponding N_0 measured DOFs best matches the experimentally obtained modal data in D . For this, let

$$\varepsilon_{\omega_r}(\underline{\theta}) = \frac{\omega_r^2(\underline{\theta}) - \hat{\omega}_r^2}{\hat{\omega}_r^2} \quad \text{and} \quad \varepsilon_{\underline{\phi}_r}(\underline{\theta}) = \frac{\|\beta_r(\underline{\theta})\underline{\phi}_r(\underline{\theta}) - \hat{\underline{\phi}}_r\|}{\|\hat{\underline{\phi}}_r\|} \quad (1)$$

be the measures of fit or residuals between the measured modal data and the model predicted modal data for the r -th modal frequency and mode shape components, respectively, where $\|\underline{z}\|^2 = \underline{z}^T \underline{z}$ is the usual Euclidean norm, and $\beta_r(\underline{\theta}) = \hat{\underline{\phi}}_r^T \underline{\phi}_r(\underline{\theta}) / \|\underline{\phi}_r(\underline{\theta})\|^2$ is a normalization constant that guaranties that the measured mode shape $\hat{\underline{\phi}}_r$ at the measured DOFs is closest to the model mode shape $\beta_r(\underline{\theta})\underline{\phi}_r(\underline{\theta})$ predicted by the particular value of $\underline{\theta}$.

To proceed with the model updating formulation, the measured modal properties are grouped into two groups. The first group contains the modal frequencies while the second group includes the mode shape components for all modes. For each group, a norm is introduced to measure the residuals of the difference between the measured values of the modal properties involved in the group and the corresponding modal values predicted from the model class for a particular value of the parameter set $\underline{\theta}$. For the first group, the measure of fit $J_1(\underline{\theta})$ is selected to represent the difference between the measured and the model predicted frequencies for all modes. For the second group, the measure of fit $J_2(\underline{\theta})$ is selected to represent the difference between the measured and the model predicted mode shape components for all modes. Specifically, the two measures of fit are given by

$$J_1(\underline{\theta}) = \sum_{r=1}^m \varepsilon_{\omega_r}^2(\underline{\theta}) \quad \text{and} \quad J_2(\underline{\theta}) = \sum_{r=1}^m \varepsilon_{\underline{\phi}_r}^2(\underline{\theta}) \quad (2)$$

The parameter estimation problem is traditionally solved by minimizing the single objective

$$J(\underline{\theta}; \underline{w}) = w_1 J_1(\underline{\theta}) + w_2 J_2(\underline{\theta}) \quad (3)$$

formed by the two objectives $J_i(\underline{\theta})$, using the weighting factors $w_i \geq 0$, $i=1,2$, with $w_1 + w_2 = 1$. The objective function $J(\underline{\theta}; \underline{w})$ represents an overall measure of fit between the measured and the model predicted characteristics. The relative importance of the residual errors in the selection of the optimal model is reflected in the choice of the weights. The results of the identification depend on the weight values used. The optimal solutions for the parameter set $\underline{\theta}$ for given \underline{w} are denoted by $\hat{\underline{\theta}}(\underline{w})$ [8-10].

3 EXPERIMENTAL APPLICATION

In this section, the emphasis is placed on applying the methodology proposed to an elevator with a frameless glass car. The ultimate goal was to develop an accurately Finite Element Analysis procedure, in order to simulate the dynamic behaviour of this system. The FE model of the structure examined was created using mainly rectangular and triangular shell finite elements. Additionally, some other elements like solid (hexahedral) and rigid body elements were also used. After development of the overall FE model, the next step was to examine the dynamic response of system when safety gear is activated and the elevator stops. In order to accurately simulate this procedure, the first step is the identification of the braking forces that acting on the system. The next step, is the verification and validation of the

developed FE model. More specifically, the emphasis given in the developing of an accurate FE model of the glass panels and its supports.

3.1 Determination of Acceleration Levels Under Real Dynamic Load Conditions

In order to verify the accuracy of the FE model of the system and also identify the braking forces acting on the system, we select, to examine only the elevator chassis including the platform of the cabin with full load. On this system, triaxial accelerometers are placed at (6) selected positions. These positions along with the measurement directions are presented in Figure 1. The two connection points of the frame with the safety gear (A1, A2) are included along with four other locations (A3-A6) which are used as reference points. A series of experimental trials were performed under real operating conditions (free fall of the elevator frame), using an experimental device designed and constructed exactly for this purpose (by Kleemann Hellas S.A), aimed at recording the acceleration time histories at the selected six points. Also, in Figure 2 presented indicative photos of two measurement locations.

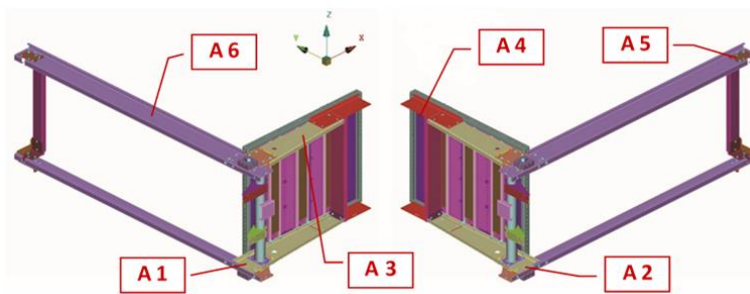


Figure 1: Measurement Locations of Acceleration Time Histories.



Figure 2: Acceleration Measurement Locations at a Connection Point (A1) and at a Reference Point (A4).

Next, the measured acceleration time histories are used in order to determine the braking force that acting on the system. After several experimental trials, the form of the corresponding time-varying braking forces, in each progressive safety gear, were calculated. Then, these force time histories were imported as base excitation in the finite element model of the system. Also, in order to solve this transient response analysis problem in a computationally effective way, a reduction in the dimensions of the original system is performed, so that the results are accurate in the frequency range 0-500 Hz. The total number of degrees of freedom in the reduced model was about 3,500, which is much smaller than the number of degrees of freedom in the original model (1,200,000). The reduced model was solved numerically in order to calculate the maximum stresses developed for the given loading. The identified critical points of the structure include mainly areas of the chassis arms on which the platform is based. Figure 3 shows selected results, in which some indicative

points of the superstructure with maximum stresses are presented.

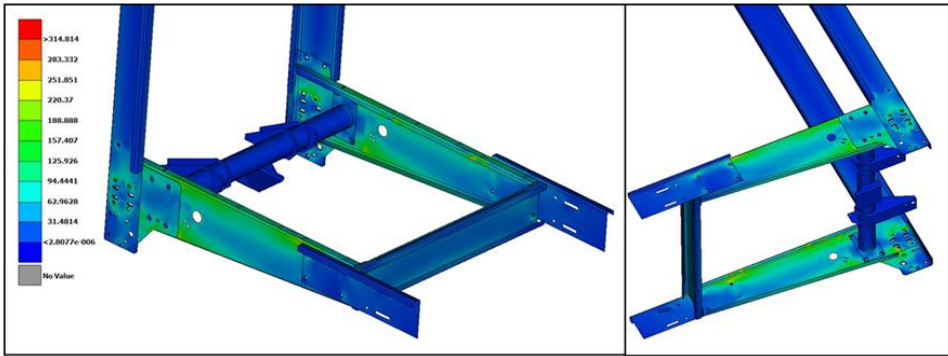


Figure 3: - Locations of the Chassis where Maximum Stresses Appear

3.2 Validation of the Applied Methodology

In order to test the reliability of the method applied, strain gauges were placed at (5) selected critical points of the chassis and a new set of measurements was carried out under similar dynamic loading conditions, to experimentally verify the stress levels developed. These positions are presented in Figure 4 and include locations on the right side of the chassis arms (SG1, SG2 and SG3) and on the left side (SG4 and SG5).

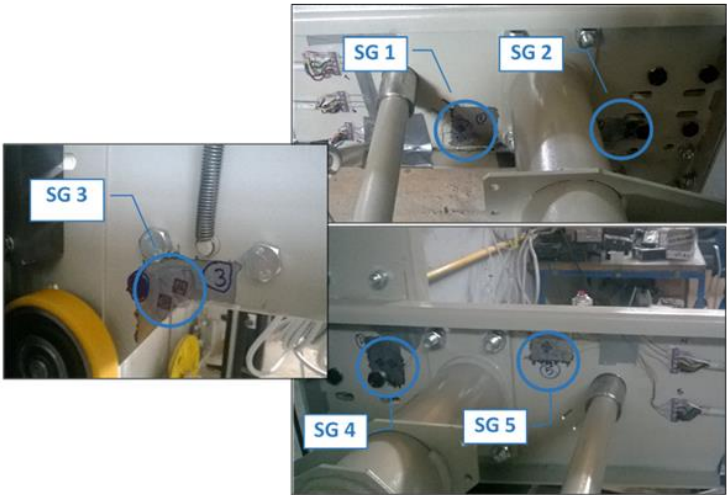


Figure 4: Measurement Locations where Maximum Stress Appears

For a complete monitoring of the stress state, three bridges with a 120o angle rosette were placed at each of the selected points. Some of the experimental and numerical results are summarized in Table 1. This table presents the maximum values of the von Mises stress obtained in four tests (indicated by PG1-PG4) and for all the points where measurements were taken (denoted by SG1-SG5). More specifically, the third from the end column presents the maximum values for all tests, for each measurement location, the penultimate column presents the corresponding maximum values obtained by the finite element model and the last column presents their percentage difference. A comparison of the numerical and experimental data presented in Table 1 verifying that the proposed method is reliable.

	Maximum Value (peak) of Equivalent Stress Von Mises [MPa]				Maximum Value of all Structural Tests	Maximum Value of FEM Solution	Error (%)
Structural Tests	PG1	PG2	PG3	PG4			
Strain Gage Positions							
SG1	90.47	85.48	97.22	92.56	97.22	105.37	7.73
SG2	198.64	193.72	201.14	196.35	201.14	209.82	4.14
SG3	168.61	165.40	170.85	173.21	173.21	170.53	1.57
SG4	160.49	148.25	154.71	152.16	160.49	155.61	3.14
SG5	253.75	246.53	263.22	255.91	263.22	278.24	5.40

Table 1: Maximum Value of Equivalent Von Mises Stress [MPa]

3.3 FE Model Verification of a Glass Panel with the Support System

Next the emphasis given to develop a high fidelity FE model of the glass panels. To achieve this, it is necessary to optimize the numerical FE model of a glass panel including the bottled support. Basic structural model updating methods have been proposed [7], in order to reconcile the numerical (FE) model, with experimental data. Structural model parameter estimation based on measured modal data [1-6] are often formulated as weighted least-squares estimation problems in which metrics, measuring the residuals between measured and model predicted modal characteristics.

3.3.1. FE Model

First, the geometry of the glass panel with the test support device is discretized mainly by solid tetrahedral elements. For the development and solution of the finite element model some appropriate software was used [20, 21]. The detailed FE Model of the experimental device presented in Figure 6. Two typical eignmodes predicted by the nominal finite element model, presented in Figure 5.

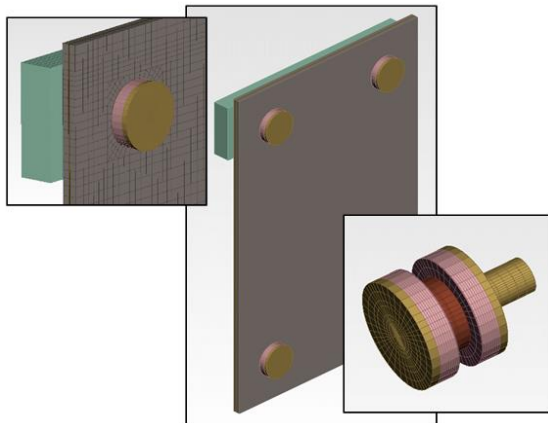


Figure 5: Finite Element Model of the Glass Panel with Support

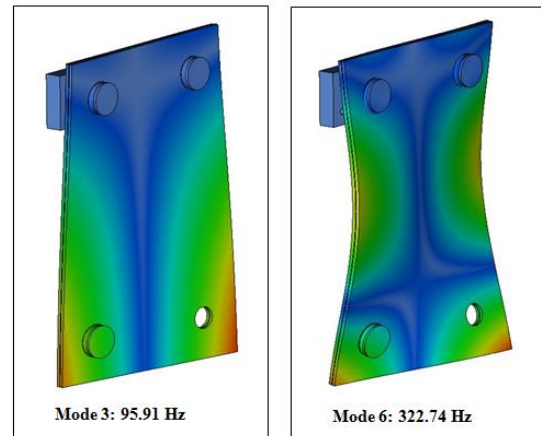


Figure 6: Typical Eigenmodes Predicted by the Nominal Finite Element Model

The FE models of the main frame and the fork parts are updated using the identified modal frequencies and mode shapes shown in Tables 1 and 2. The identified mode shapes include components at all sensor locations. Additionally, we define as design response the total weight of the model, in order to be taken into consideration during the optimization process.

3.3.2. Experimental Modal Analysis

After development of the nominal FE model, an experimental modal analysis of the device was performed to quantify its dynamic characteristics. The system was tested in fixed-free boundary condition. First, all the necessary elements of the FRF matrix required for determining the response of the frame substructure were determined by imposing impulsive loading [1, 2, 8, 9]. The measured frequency range was 0-2048 Hz, which includes the analytical frequency range of interest, 0-400 Hz. An initial investigation indicated that the frame has six natural frequencies in this frequency range. A schematic illustration of the measurement geometry of the experimental device is presented in Figure 7. In this figure, presented the locations of the tri-axial accelerometers, strain gauges and of the electrodynamic shaker.

For instance, Figure 8 shows the magnitude of two typical elements of the FRF matrix before (continuous line) and after (dashed line) application of the Welsh's smoothing method.

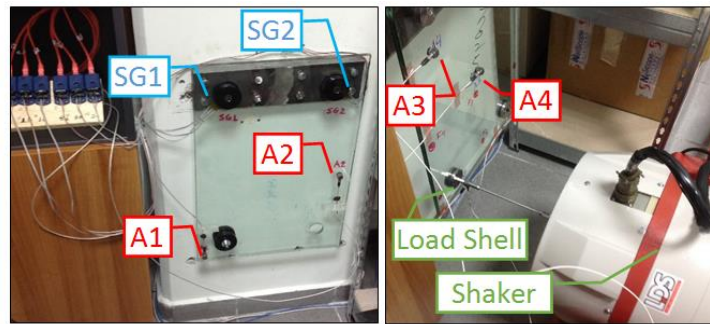


Figure 7: Schematic Illustration of the Experimental Device, Fixed-Free Arrangement with Electrodynamic Shaker, Accelerometers and Strain Gauges Locations

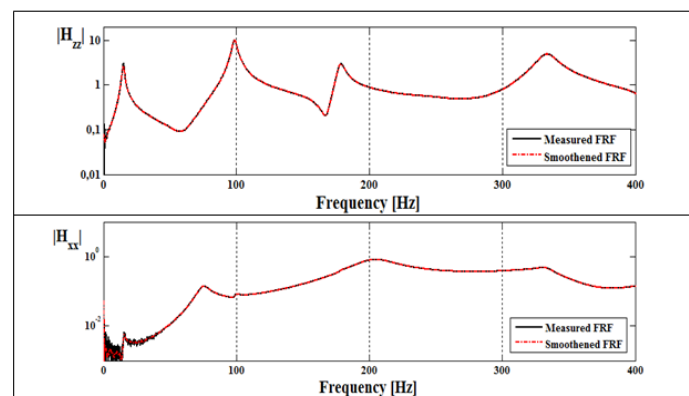


Figure 8: Typical Elements of the FRF Matrix

Based on the measured FR functions, the natural frequencies and the damping ratios of the frame substructure were estimated. As an outcome of the above procedure, the first column of Table 2 presents the values of the lowest 6 natural frequencies (ω_{FE}) of the system examined, while the corresponding damping ratios are included in the fourth column. In the same table,

the second column presents the values of the natural frequencies obtained from the analysis of the nominal finite element model ($\omega_{rN_{FE}}$) and the third column compares these frequencies with the corresponding frequencies obtained by the experimental data. The errors determined between the nominal FE model and the experimental measurements are not insignificant, indicating that the FE model updating process is necessary.

Mode	Identified Modal Frequency	Nominal FE Predicted Modal Frequency	Difference between Identified and FE Predicted Modal Frequencies	Identified Modal Damping Ratio
	$\omega_E [Hz]$	$\omega_{rN_{FE}} [Hz]$	$\frac{\omega_{rN_{FE}} - \omega_E}{\omega_{rN_{FE}}} 100\%$	$\zeta_{rE} (\%)$
1	15.15	17.06	11.20	4.78
2	73.69	87.23	15.52	7.14
3	98.70	95.91	2.91	1.74
4	177.80	170.94	4.01	1.56
5	203.01	227.32	10.69	7.90
6	333.10	322.74	3.21	2.29

Table 1: Modal Frequencies and Modal Damping Ratios

3.3.3. FE model parameterization and Updating Results

The parameterization of the FE model of the experimental device are introduced in order to demonstrate the applicability of the proposed FE model updating method. The parameterized model consisting of six parts which are shown in Figure 9. At each of these parts are used as design variables the Young's modulus and the density. Thus, the final number of the design parameters are twelve (12) variables. In Table 2 presented the initial values that have been set in each parameter, which are identical to the nominal FE model, with the upper and lower limits, which were selected to be used for the optimization process. The last column of the table shows the step of design, which is set at 1% of the respective previous value for all cases. The finite element model is updated using the lowest six identified modal frequencies and mode shapes shown in Table 1. The identified mode shapes include components at all 4 sensor locations.

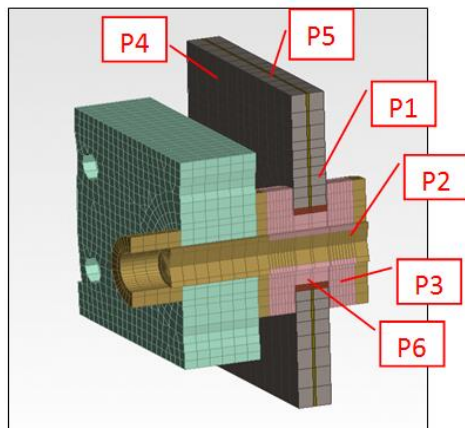


Figure 9: Parts of the Parameterized FE Model

Part	Initial Density [kg/m ³]	Initial Young's Modulus [Gpa]	Move Limit
	LB - UB	LB - UB	
P1	1840 1000 – 2500	0.6 0.3 – 0.9	1%
P2	7850 7200 – 8500	210.0 190 – 220	1%
P3	1100 800 – 2000	0.018 0.006 – 0.05	1%
P4	2500 2000 – 3000	63.0 60 – 70	1%
P5	1840 1000 – 2500	0.6 0.3 – 0.9	1%
P6	2500 1500 – 3000	0.6 0.3 – 0.9	1%

Table 2: Design Variables and Optimization Design Limits

The results from the FE model updating method are shown in Table 3. In this table presented a comparison between identified (ω_{rE}) and ($\omega_{rO_{FE}}$) optimal FE predicted modal frequencies.

Mode	Identified Modal Frequency	Optimal FE Predicted Modal Frequency	Difference between Identified and FE Predicted Modal Frequencies
	$\omega_{rE} [Hz]$	$\omega_{rO_{FE}} [Hz]$	$\frac{\omega_{rO_{FE}} - \omega_{rE}}{\omega_{rO_{FE}}} 100\%$
1	15.15	15.29	0.92
2	73.69	76.25	3.36
3	98.70	99.30	0.60
4	177.80	177.00	0.45
5	203.01	202.50	0.25
6	333.10	319.10	4.39

Table 3: Comparison Between Identified and Optimal FE Predicted Modal Frequencies

The acceleration time history and the FRF predicted by the optimal FE model (red dashed dot line) of the glass panel are compared in Figure 10 with the acceleration time history and the FRF computed directly from the measured data (blue continuous line) at one indicative measurement locations of the glass panel (A1) in the frequency range [0Hz, 400Hz]. The acceleration time history and the FRF of the initial nominal model (black dashed line) is also shown in these figures to be inadequate to represent the measured acceleration time history and the FRF. Compared to the FRF of the initial nominal model, it is observed that the updated optimal model tend to considerably improve the fit between the model predicted and the experimentally obtained FRF close to the resonance peaks.

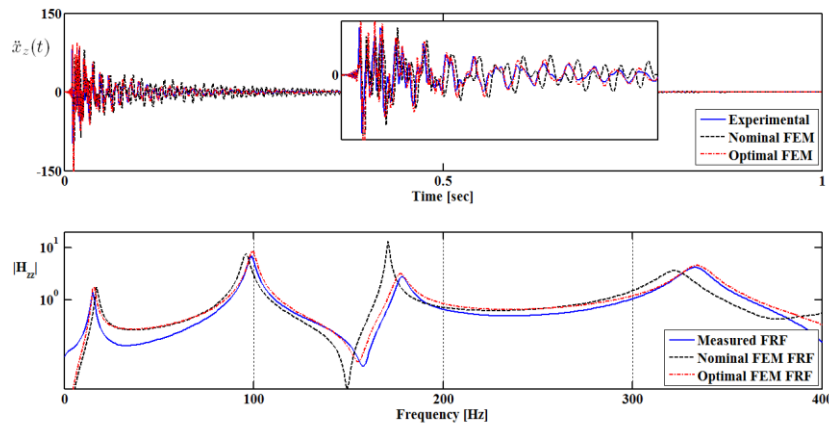


Figure 10: Comparison between measured, nominal and optimal acceleration time histories and FRF in the measured location A1

3.4 Analysis of the FE Model of Full Elevator System - Frameless Full Glass Cabin

Finally, a detailed finite element model of a full elevator system using a frameless full glass car was build. The model was solved numerically in transient response analysis, using the method which was presented in previous sections 3.1-3.2, in order to calculate the maximum stresses developed [19]. Based on the results of this analysis the elevator chassis

and the bottled glass supports was redesigned and optimized, in order to achieving the minimum design stresses at the glazing components during dynamic load conditions caused by e.g. emergency safety gear engagement. The final FE model with indicative numerical results presented in Figure 11.

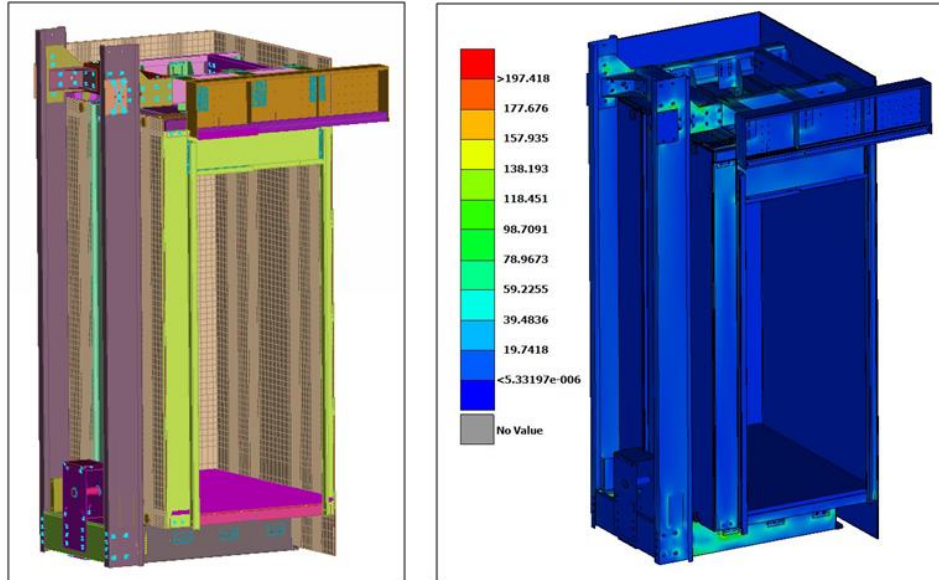


Figure 11: Frameless Full Glass Car FE model and Transient Analysis Results

Based on the results of all the above procedure, a full glass frameless panoramic elevator (Figure 12) was designed and developed. In order to test the reliability of the method applied, strain gauges were placed at (4) selected critical points of the glass panels and a new set of measurements was carried out under similar dynamic loading conditions, to experimentally verify the stress levels developed. A comparison of the numerical and experimental data verifying that the proposed method is quite reliable.

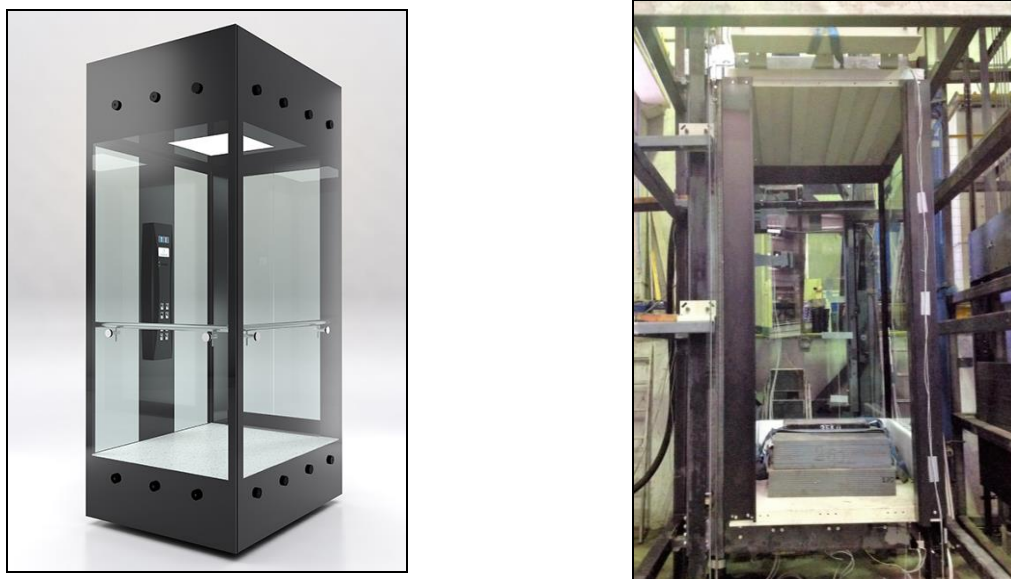


Figure 12: Full Glass Frameless Car with Experimental Verification Test (Patent Pending)

4 SUMMARY

A systematic method was presented for determining the dynamic response and identifying the critical points of a full glass panoramic car elevator system when subjected to dynamic load conditions. The special feature in this work is that the cabin is manufactured entirely of laminated glass, except the roof and the platform. First, modal identification and structural model updating methods are applied, leading to develop high fidelity finite element model of the glass and its connection subsystems. Next, a “mixed computational-experimental” analysis method is applied, in order to simulate accurately the dynamic behavior of the complete elevator system, in emergency situations like safety gear engagement. Based on the results of the method applied the elevator chassis and the bottled glass supports was redesigned, optimized. Finally, a full glass frameless panoramic elevator was designed and developed. Comparison of numerical and experimental results indicated that the methodology applied gives accurate results and provides a useful tool in predicting the critical stress levels developed in the elevator.

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