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ISO-XFEM FOR TOPOLOGY OPTIMIZATION OF STRUCTURES UNDER MULTIPLE LOAD CASES AND ACCELERATION LOADING

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Abstract. *Iso-XFEM is an evolutionary based topology optimization method for generating high resolution topology optimised solutions using isolines/isosurfaces of a structural performance criterion and eXtended Finite Element Method (XFEM). The conventional approach for topology design of structures includes the use of a density based topology optimization method, such as SIMP, to find the optimal density distribution within the design domain, and thresholding the densities at an arbitrary value to obtain the design boundary. This will then require additional post-processing, such as smoothing and shape optimization, to generate a manufaturable design with smooth and clearly defined boundaries. In the proposed method, the use of XFEM with isoline/isosurface based boundary representation enables generation of topology solutions represented with smooth boundaries which can be straight away translated to a triangular surface file format, for instance STL format, in order to direct to manufacture. From analysis point of view, this allows the use of a coarser mesh during the optimization process to achieve a smooth solution, while this omits/significantly reduces the post processing required before manufacturing. The aim of this paper is to extend the Iso-XFEM method to include multiple loading cases and acceleration loading. Different 2D and 3D test cases are employed to show the effectiveness of the method.*

1 INTRODUCTION

Topology optimization aims to find the best possible layout for a structure within a specified design domain under a set of loads and boundary conditions. There has been a significant interest in topology optimization methods and applications over the last three decades starting from groundbreaking paper of Bendsøe and Kikuchi (1988) introducing homogenization method [1]. Other methods including Solid Isotropic Material with Penalization (SIMP) [2,3], Evolutionary Structural Optimization (ESO) [4,5], level set method [6,7] and genetic algorithms [8] were introduced after that. Although many of the proposed topology optimization algorithms have been developed and been applied on several problems, such as Michelle-type structures and cantilever beams with rectangular domains, there has been less attention on applying these algorithms on 3D real-life structures and real loading scenarios. In some cases the mathematical complexity or the size of the FE design domain doesn't allow the algorithm to be properly implemented. OptiStruct (Altair Engineering Inc.) is an example of software designed to enable the SIMP method of topology optimization to be applied to real components. Other software such as Nastran (MSC Software) and Abaqus FEA (Dassault Systèmes) also have option to apply similar density-based approaches to find the solution to topology optimization problems. Although the topology optimization modules of these software applications are being widely used for research and engineering purposes, a drawback of the density-based approaches (and many other element-based approaches) is that they cannot provide a clear and smooth representation of the design boundaries in converged topologies. This issue brings difficulties in interpreting the solutions, combining them with CAD and manufacturing the topologies. Therefore the solutions usually need post-processing, reanalysing and shape optimization before manufacturing.

Iso-XFEM was developed in a previous study to address the issues related to the boundary representation of the topology [9,10,11]. The idea was to use a simple evolutionary based optimization algorithm (similar to BESO) while improving the boundary representation by implementing isoline/isosurface approach during the optimization. An XFEM integration scheme was also used to increase the accuracy of FE solutions near the design boundary. The method was successfully applied to 2D and 3D structures with complex design domain [11], and the results showed a significant improvement in boundary representation and structural performance of the solutions over the conventional BESO. The aim of this paper is to extend the Iso-XFEM method into problems with multiple load cases and acceleration loading. In the next sections, an overview of the Iso-XFEM method followed by two examples of stiffness design using this method is presented. Then the extension of the method into optimization of structures under multiple load cases and acceleration loading with a few examples of each is presented.

2 AN OVERVIEW OF ISO-XFEM METHOD

The main three elements of the Iso-XFEM method include isoline/isosurface approach to represent the design boundary, XFEM to calculate the elemental sensitivities (a structural performance criterion) near the boundary, and an evolutionary based optimization algorithm. These three elements are explained in this section.

2.1 Isoline/isosurface approach

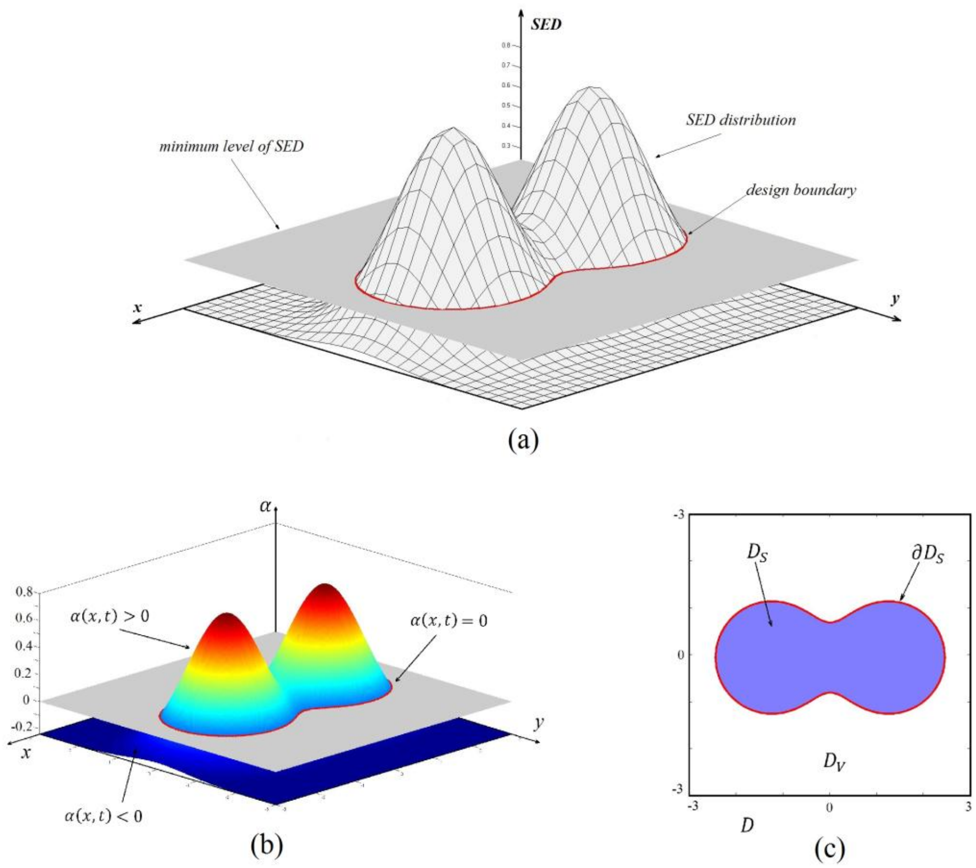
Isolines/isosurfaces are the lines/surfaces that represent the points of a constant value, named the isovalue, in a 2D/3D space. In structural optimization applications [9,11,12,13], the boundaries are defined by the intersection of the structural performance (SP) distribution with a minimum level of performance (MLP), which is typically increasing during the optimization process. Figure 1(a) shows a 2D fixed grid design domain discretized with a 30x30 mesh, where the intersection of strain energy density (SED) distribution as a structural performance criterion with a minimum level of SED gives the design boundary. The relative performance, α, is defined as:

$$\alpha = SP - MLP \tag{1}$$

The design domain can be partitioned into void phase, boundary and solid phase, with respect to the values of relative performance:

$$\alpha(x): \begin{cases} > 0 & solid\ phase\ (D_S) \\ = 0 & boundary\ (\partial D_S) \\ < 0 & void\ phase\ (D_V) \end{cases} \tag{2}$$

Figures 1(b) & 1(c) show how the design space, D, from figure 1(a) is partitioned into D_S, ∂D_S and D_V using the relative performance function $\alpha(x)$, distributed over the design space.



Figur 1: (a) Boundary representation using isolines of a structural performance function (SED in here). The intersection of SP distribution with MLP defines the current state of the boundary. (b) Implicit representation of a 2D design space and the structure's geometry using relative structural performance. (c) Design space decomposed into solid region ($\alpha(x) > 0$), void region ($\alpha(x) < 0$) and boundary ($\alpha(x) = 0$).

2.2 XFEM

By implementing the above isoline/isosurface approach, the design boundary is superimposed on the fixed grid finite elements, making three groups of elements in the FE design space: solid elements, void elements, and boundary elements (the elements which lie on the boundary). The contribution of solid and void elements to the FE framework could simply be considered by assigning solid and void (very weak) material properties to those elements, respectively. In the case of boundary elements, in order to accurately represent the design boundary whilst avoiding expensive remeshing operations, an XFEM approach can be employed. XFEM approximation space for modeling holes and inclusions is given by [14]:

$$u(x) = \sum_i N_i(x)\, H(x) u_i \tag{3}$$

where $N_i(x)$ are the classical shape functions associated to the nodal degrees of freedom, u_i. The value of the Heaviside function $H(x)$ is equal to 1 for the nodes and regions in the solid part of the design and switches to 0 for nodes and regions in the void part of the design domain. This XFEM scheme was realized by dividing the solid domain of the boundary elements into sub-triangles (in 2D problems as shown in figure 2(a)) or sub-tetrahedra (in 3D problems as shown in figure 2(b)), and then performing numerical integration over solid triangles/tetrahedra using Gauss quadrature method.

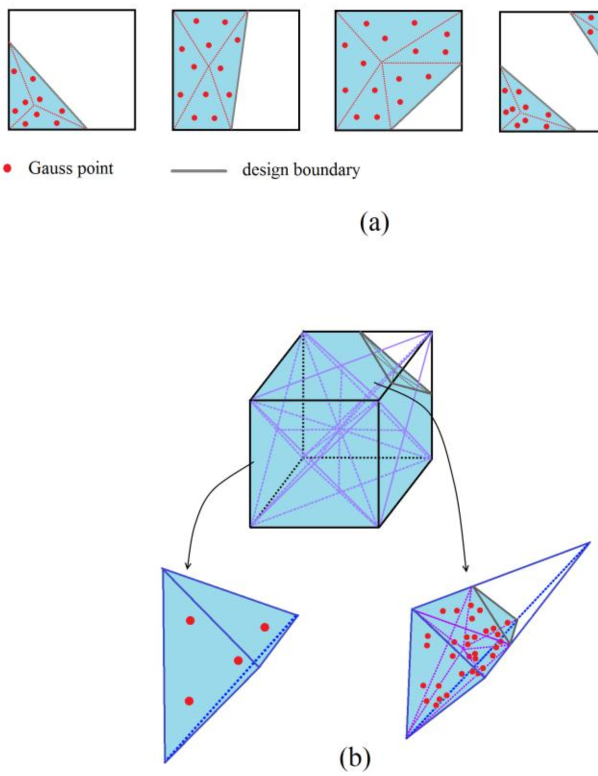


Figure 2: XFEM interation scheme. (a) solid domain of 2D boundary elments are devided into sub-triangles. (b) Solid domain of 3D boundary elements are devided into sub-tetrahedra.

2.3 Evolutionary based optimization method

The optimization algorithm used in the Iso-XFEM method is evolutionary based, i.e. based on the simple assumption that the optimized solution can be achieved by gradually removing the inefficient material from the design domain. However, unlike Evolutionary Structural Optimization (ESO) in which the material removal is carried out at an elemental level, in this approach the optimization operates at a global level of structural performance by the use of isoline/isosurface design approach. An appropriate performance criterion is used to characterize the efficiency of material usage in the design domain. Material is then removed from low relative performance regions ($x;\ \alpha(x)<0$) and redistributed to the high relative performance regions ($x;\ \alpha(x)>0$). The target volume of the design for the current iteration needs to be calculated before any region is added to or removed from the structure. The target volume of the design for the current iteration is given by

$$V_{it} = \max(V_{it-1}(1-ER), V^c) \tag{4}$$

where ER is the volume evolution rate and V^c is the specified volume constraint. Once the target volume of the current iteration is found, the minimum level of performance which gives this volume needs to be identified. This could be achieved through an iterative process, for instance by defining upper and lower bands for MLP (which are equal to the maximum and minimum SP in first iteration, respectively), finding the volumes corresponding to the upper and lower bands, averaging and updating the upper and lower bands until the difference between the volumes corresponding to the upper and lower bands is smaller than a minimum value.

3 EXAMPLES OF STIFFNESS DESIGN USING ISO-XFEM

3.1 A comparison study

By employing Iso-XFEM method for topology optimization, we would expect to achieve solutions with smoother boundary than those obtained using conventional element based methods, for instance SIMP and BESO [9,11]. If a density based method, like SIMP, is employed for topology optimization, there is a need to threshold at an arbitrary density to achieve a discrete solution. However this could result in reducing the optimality of the solution. The aim of this test case was to investigate the optimality of Iso-XFEM solution for a benchmark problem and compare it with the solution obtained from a commercial software after thresholding densities.

To address this, Iso-XFEM was used to solve topology optimization problem for the cantilever structure shown in figure 3(a) using 40×20×2 hexahedral elements. Material properties were Young modulus E = 1, and Poisson's ratio ν = 0.3. To minimize compliance, strain energy density was used as structural performance criterion. A volume evolution rate of $ER = 0.02$ was used. The solution converged after 40 evolutionary iterations (figure 3(b)). The same starting mesh was used to solve the cantilever problem using Optistruct, as shown in figure 4. Figure 4(a) is Optistruct solution (optimal density distribution) without implementing minimum member size constraint. It can be seen that the design boundary is not clearly defined as the solution is represented with intermediate relative densities. Figure 4(b) shows the densities thresholded in relative density of 0.5 which results to a solution with approximately the same volume as the prescribed volume constraint. However, it can be seen that thresholding the densities resulted in an unfeasible design as the structure has lost its members' connectivity. It is possible to preserve members' connectivity by thresholding at a lower density. However, this would result in a solution with a higher volume than the volume constraint, which requires further optimization and post-processing.

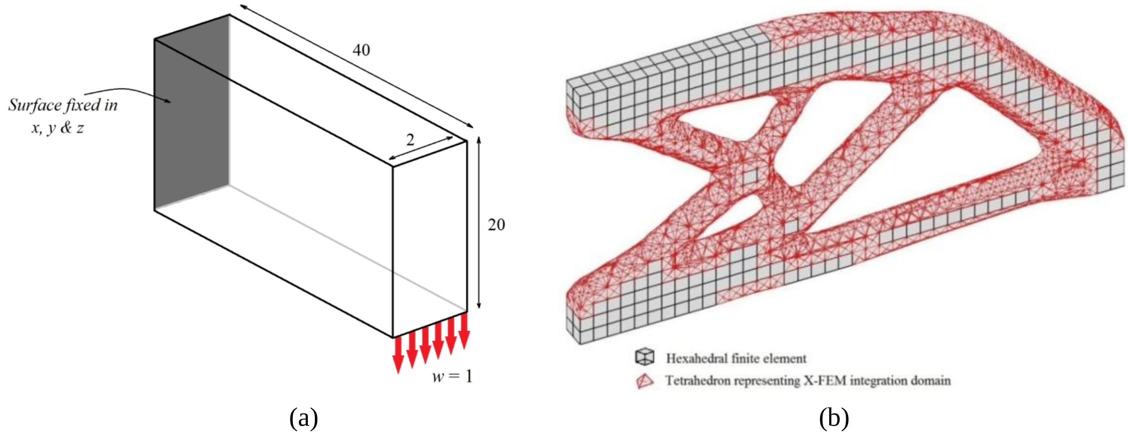


Figure 3: (a) Design domain and boundary conditions (b) Iso-XFEM solution.

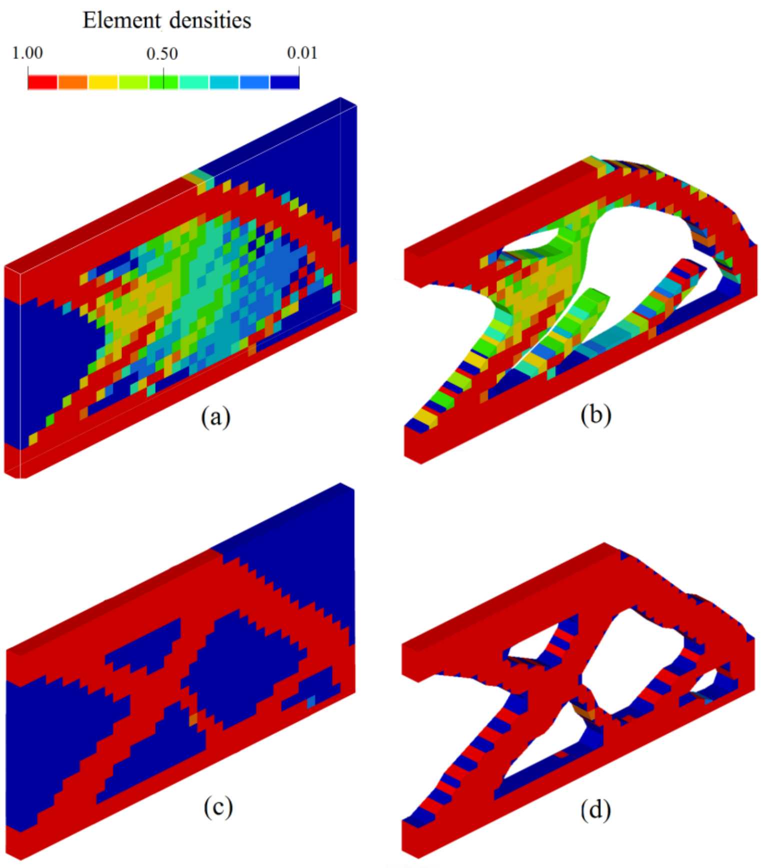


Figure 4: (a) Optistruct solution of cantilever structure without applying minimum member size constraint, and (b) densities of this solution thresholded at isovalue of 0.5. (c) Optistruct solution with minimum member size constraint, and (d) densities of this solution thresholded at isovalue of 0.5.

In order to have a near 0/1 solution and prevent the loss of members' connectivity after thresholding the densities, a minimum member size of 2×element size was applied, resulting in the solution shown in figure 4(c). It can be seen that this is a near 0/1 solution and the members' connectivity is well defined, however, with rough boundaries. Figure 4(d) shows the solution obtained by thresholding the densities of the solution shown in figure 4(c). The densities were thresholded at isovalue of 0.5, resulting in the same volume as the prescribed volume constraint, thus, enabling the comparison of all solutions with the Iso-XFEM solution, shown in 3(b).

To quantify the optimality of the Optistruct and Iso-XFEM solutions, the strain energy of the solutions has been measured by performing FEA on the solutions, as presented in table 1. It can be seen that the design obtained by thresholding densities of optistruct solution has a higher value of objective than Optistruct solutions with/without minimum member size constraint. This is because the design obtained by thresholding the densities of a topology optimised solution, is not an optimised solution anymore, and will require additional shape optimization and further post-processing until it becomes an optimised manufacturable design. However, it can be seen that the strain energy of the Iso-XFEM solution is lower than the other three designs, indicating that the Iso-XFEM solution is a more optimal solution compared to the Optistruct solutions before/after thresholding the densities. The results of this experiment clearly show that although thresholding the densities can improve the boundary representation, it can reduce the optimality of the solutions.

Different solutions	SE (N.m)
Optistruct solution without minimum member size constraint (figure 4a)	7.34
Optistruct solution with minimum member size constraint (figure 4c)	7.17
Design obtained by thresholding densities of the Optistruct solution, with minimum member size constraint (figure 4d)	7.63
Iso-XFEM solution (figure 3b)	6.56

Table 1: Comparison of strain energies of Optistruct and Iso-XFEM solutions

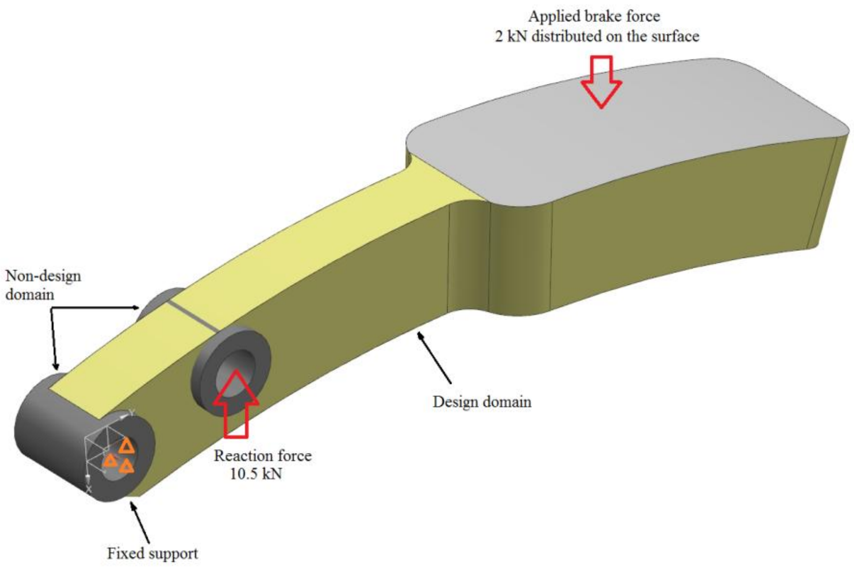
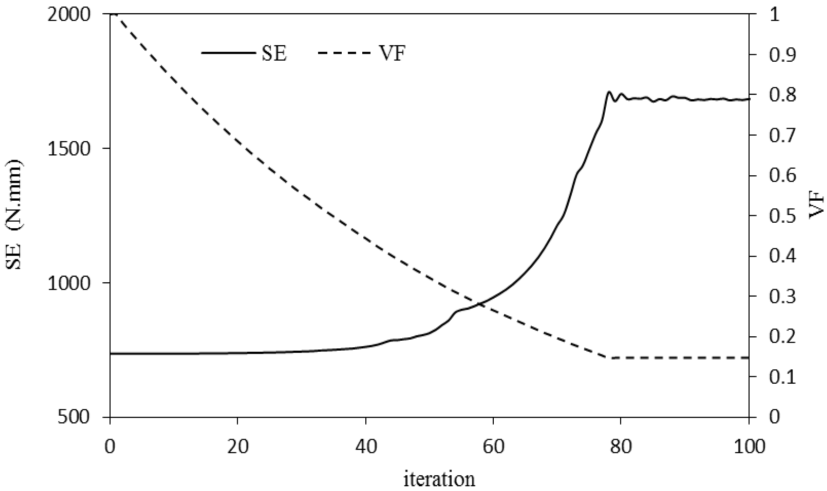


Figure 5: Design domain and boundary conditions of the brake pedal.

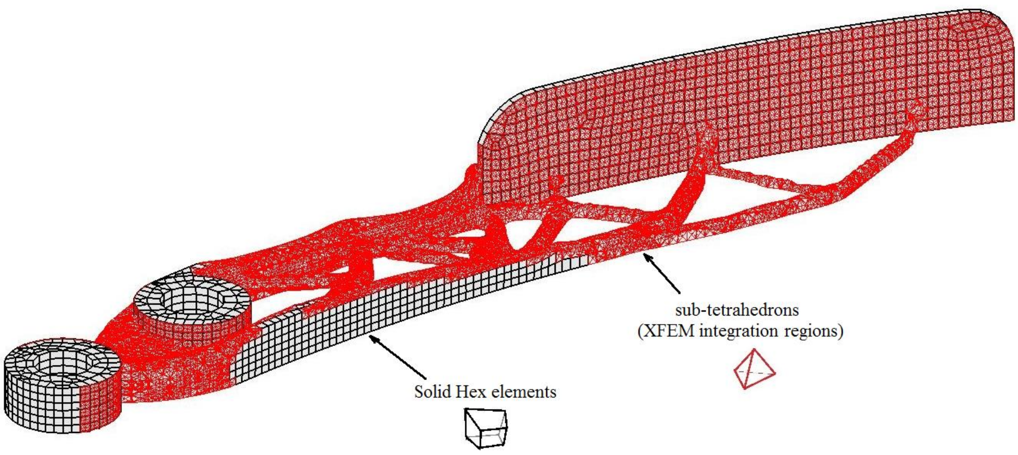
3.2 An industrial test case: a brake pedal

The method was applied to optimize a brake pedal with the loads and boundary conditions shown in figure 5. The objective was to minimize the compliance for the target volume of 15% of the initial design domain. The material used was Ti-6Al-4V. Due to the symmetry of the problem, only half of the structure was considered for the analysis using approximately

16000 hexahedral elements. A volume evolution rate of *ER = 0.02* was used. The solution converged after 90 evolutionary iterations as shown in figure 6(a). Figure 6(b) shows the Iso-XFEM solution for the brake pedal. It can be seen that despite using a coarse starting mesh, a fairly smooth solution has been achieved. Figure 7(a) shows the optimized design after inclusion of all non-design elements fallowed by a few iterations of smoothing. The part was built through Selective Laser Melting (SLM) process using Renishaw AM250 machine, as shown in figure 7(b).

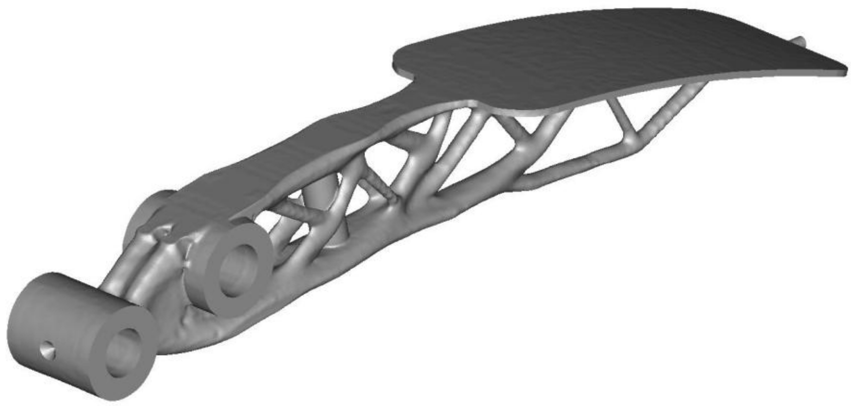


(a)

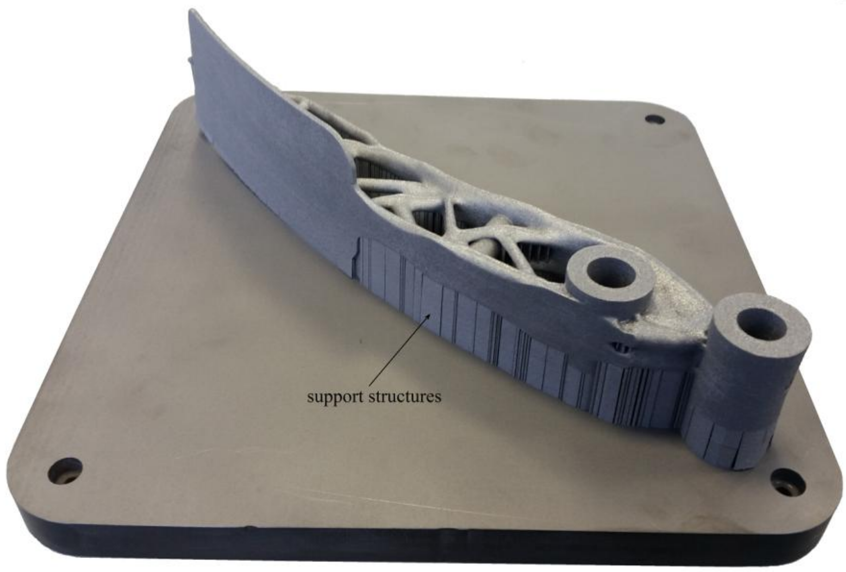


(b)

Figure 6: (a) Evolution histories of objective function (SE) and volume fraction (VF) of the brake pedal (b) Iso-XFEM solution (half of the pedal).



(c)



(d)

Figure 7: (a) Final design of the brake pedal after inclusion of all non-design regions (b) the part built using SLM process. Support structures will be removed after stress relieving process.

4 ISO-XFEM FOR MULTIPLE LOAD CASES

Most real-life structures are subjected to multiple load cases. Moreover, moving loads can be simplified to a finite number of load cases acting sequentially along the load path. In the case of stiffness optimization, in order to account for all load cases, the objective function can be defined as weighted average of mean compliance of all load cases [15]:

$$\begin{gathered} minimize: f(x) = \sum_{i=1}^{m} w_i C_i \\ subject\ to: \sum_{e=1}^{n} v_e^s = V^c \end{gathered} \tag{5}$$

where m is the total number of load cases, n is the total number of elements, C is compliance, w_i is the weighting factor of i^{th} load case, and v_e^S is the volume of the solid part of the element, and V^c is the design volume constraint. In this case, the structural performance associated to element e can be defined as

$$SP_e = \frac{1}{2}\sum_{i=1}^{m} w_i(u_e^T k_e u_e)_i / v_e \tag{6}$$

where u_e and k_e are the element's displacement vector and stiffness matrix, respectively, and v_e is the total volume of the element.

4.1 Example

The rectangular plate with the load and boundary conditions shown in figure 8(a) is considered. The objective was to find the stiffest design for the target volume of 30% of the initial design domain. Two unit loads were applied on the top edge of the rectangular plate. Material properties were Young modulus E= 1, and Poisson's ratio ν = 0.3. Equal weighting factors were assumed for two load cases ($w_1 = w_2 = 0.5$). A mesh of 60×30 was used for the FE model of the plate. A volume evolution rate of ER = 0.02 was used for the optimization.

Figures 8(b) and 8(c) compare the topology obtained when the two loads are applied as a single load case (8b) with the one obtained from two different load cases (loads in different times), (8c). It can be seen that two different topologies have been obtained. However, the solution obtained for the structure under two load cases (figure 8(c)) can be more stable than the other solution, since it has converged to a triangulated frame structure which is more stable than a trapezoidal frame structure. Figure 9 shows the Evolution histories of objective function and volume fraction for the structure under two load cases. It can be seen that a stable material removal was carried out through the Iso-XFEM optimization process.

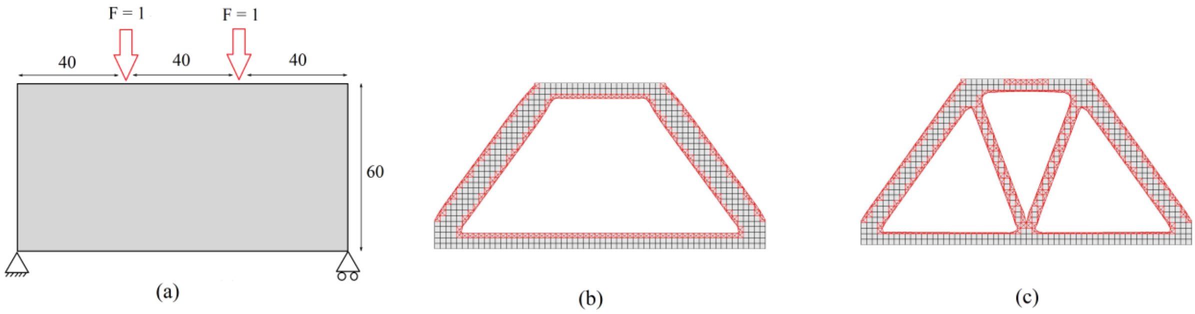


Figure 8: (a) Design domain and boundary conditions of the 2D structure (b) Iso-XFEM solution of the structure under a single load case of two unit force (c) Iso-XFEM solution of the structure under two load cases of a unit force each.

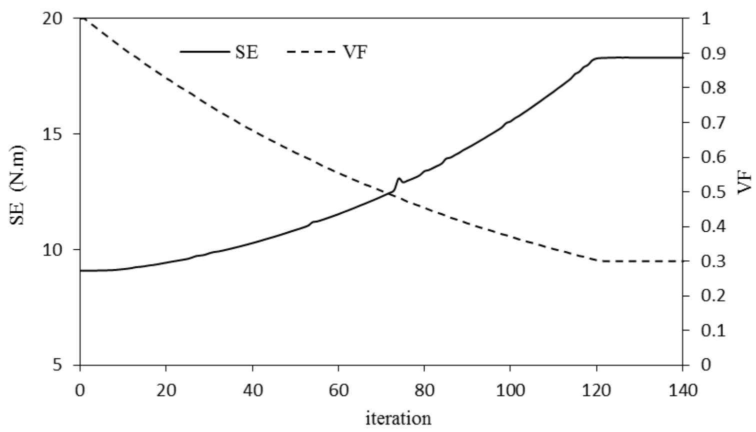


Figure 9: Evolution histories of objective function (SE) and volume fraction (VF) of the rectangular design domain subjected to two load cases.

5 ISO-XFEM FOR GRAVITY AND ACCELERATION LOADING

Gravity load is an important consideration in many engineering design applications such as deigning civil structures. Other forms of acceleration loading may also exist in a mechanical or an aerospace component. Acceleration loading is different from fixed external loading in that the applied force includes design dependent acceleration load, requiring the force vector F in the FE system to be updated in each optimization iteration. Therefore, we use the same objective function as compliance (total strain energy) for stiffness design, and update the nodal forces by

$$F_g^j = \sum_{e=1}^{k} v_e^s \rho g / k \tag{7}$$

where F_g^j is the acceleration force of node j in direction of acceleration g, ρ is the density, and k is the number of elements connected to node j. Note that for simplicity, we assumed that number of elements connected to a node is equal to the number of nodes of an element (for instance for 4-node quadrilateral elements $k = 4$, and for 8-node hexahedral elements, $k = 8$).

5.1 A 2D plate under gravity load

A simply supported 1 m × 0.5 m plate shown in figure 10(a) is considered. The objective was to find minimize the compliance of the plate under gravity load for the target volume fraction of 15% of the initial design domain. Material properties were Young modulus of 200 GPa, Poisson's ratio of 0.3, and density of 78 kg/m^3. A mesh of 60×30 was used for the FE model of the plate. A volume evolution rate of ER = 0.02 was used for the optimization. Figure 10(b) shows the converged solution. The results are in agreement with BESO solution in [15].

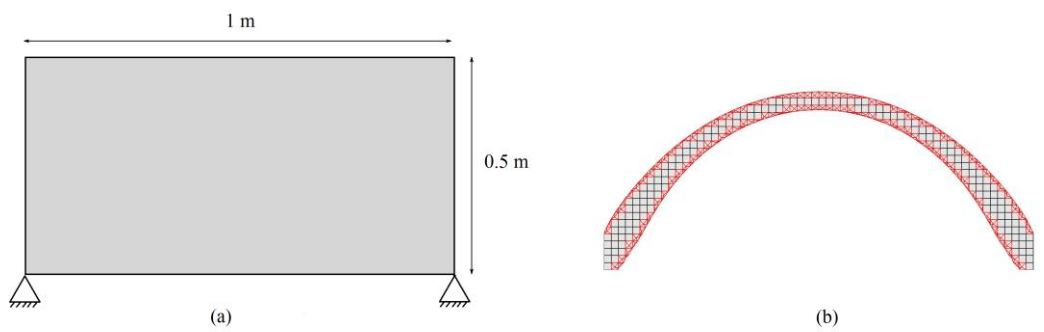


Figure 10: (a) Design domain and boundary conditions of the 2D structure under gravity load (b) Iso-XFEM solution.

5.2 3D structure under gravity load

The cuboid design domain shown in figure 11(a) was used as the 3D test case under gravity load. The structure was simply supported at all four bottom corners. The objective was to minimize the compliance of the structure under self-weight for the target volume fraction of 5% of the initial design domain. Material properties were Young modulus of 200 GPa, Poisson's ratio of 0.3, and density of 78 kg/m^3. A volume evolution rate of ER = 0.04 was used for the optimization. The structure is symmetric in two directions. Therefore only a quarter of the design space was considered for FEA using 32×32×40 elements. The final Iso-XFEM solution is shown in figure 11(b). Figure 11(c) shows the design obtained after mirroring the Iso-

XFEM solution and converting the solution to STL file format. No further post-processing/smoothing was performed to generate the design shown in figure 11(c).

Figure 12 shows the evolution histories of objective function and volume fraction of the 3D structure under self-weight. It can be seen that the Iso-XFEM material removal process for this structure is highly stable. Also, unlike compliance minimization problems for the structures under external loads where by removing material, the compliance increases, here, the material removal resulted in decreasing the compliance. Evidently, this is because the gravity load is a design dependent load and by removing material less load was applied to the structure.

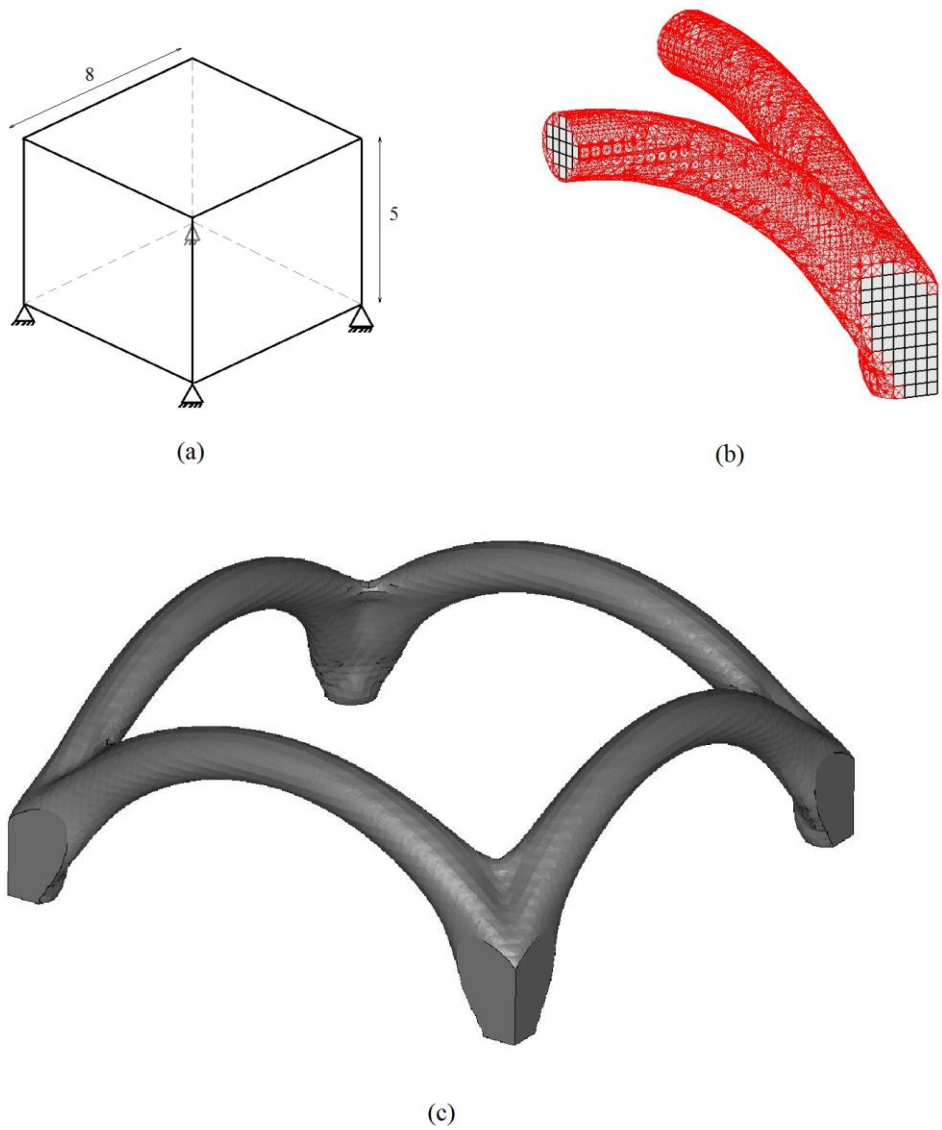


Figure 11: (a) Design domain and boundary conditions of the 3D structure under gravity load (b) Iso-XFEM solution considering only a quarter of the structure (c) Final design.

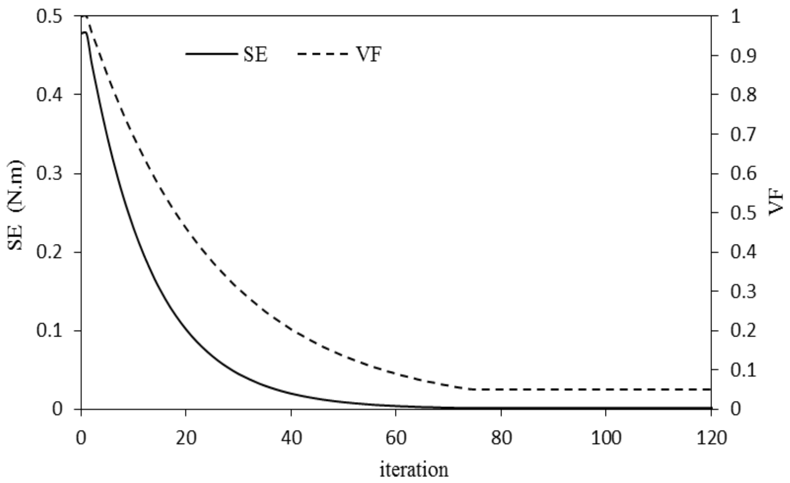


Figure 12: Evolution histories of objective function (SE) and volume fraction (VF) of the cuboid design domain under gravity load.

6 SUMMARY AND CONCLUSION

Iso-XFEM is an evolutionary structural optimization method which can be used for shape and topology optimization of continuum structures. The use of XFEM with isosurface approach has enabled achieving high resolution topology optimized solutions which require no more or only a little post processing before manufacturing. It was shown that the method is capable of optimizing geometrically complex structures, for instance the brake pedal studied in this paper.

In this study, the method was further extended to include topology optimization of structures subjected to multiple load cases and acceleration loading. It was shown that design for multiple load cases can be different from the one for a single load case. The method was also found to be stable for optimization of structures under acceleration loading.

The Iso-XFEM method was found to be a promising method for topology optimization of real-life structures with real loading scenarios. The boundary of the solutions is represented with triangles (rather than element densities in density based topology optimization methods). This makes it straightforward to convert the solutions into STL format for additive manufacturing. The amount of post-processing before manufacturing is highly depended to the mesh size used in the analysis. Considering the fact that even using a coarse mesh, relatively high resolution solutions can be achieved using this method, by taking account the resolution of the 3D printer before creating FE mesh, the post-processing after topology optimization can be avoided and the solutions can be sent directly to additive manufacture.

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