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ADVANCES IN GEOMETRICAL PARAMETRIZATION AND REDUCED ORDER MODELS AND METHODS FOR COMPUTATIONAL FLUID DYNAMICS PROBLEMS IN APPLIED SCIENCES AND ENGINEERING: OVERVIEW AND PERSPECTIVES

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Abstract. *Several problems in applied sciences and engineering require reduction techniques in order to allow computational tools to be employed in the daily practice, especially in iterative procedures such as optimization or sensitivity analysis. Reduced order methods need to face increasingly complex problems in computational mechanics, especially into a multiphysics setting. Several issues should be faced: stability of the approximation, efficient treatment of nonlinearities, uniqueness or possible bifurcations of the state solutions, proper coupling between fields, as well as offline-online computing, computational savings and certification of errors as measure of accuracy. Moreover, efficient geometrical parametrization techniques should be devised to efficiently face shape optimization problems, as well as shape reconstruction and shape assimilation problems. A related aspect deals with the management of parametrized interfaces in multiphysics problems, such as fluid-structure interaction problems, and also a domain decomposition based approach for complex parametrized networks. We present some illustrative industrial and biomedical problems as examples of recent advances on methodological developments.*

1 Introduction and motivation: a synopsis

Recent research in Computational Science and Engineering aims at developing and consolidating the capabilities of computational reduction strategies for problems governed by parametrized Partial Differential Equations (PDEs). Parameters might be both physical (material properties, nondimensional coefficients such as Reynolds or Prandtl numbers, boundary conditions, forcing terms) and geometrical (i.e. quantities which characterize the shape of the domain and of the system itself). This research fits into the fields of numerical analysis and scientific computing, with a special interest in computational mechanics, and to applications in the contexts of simulation, optimization and control. In the latter cases iterative minimization procedures entailing several numerical solutions of PDEs (each time with different values of control or design variables, or different physical and geometrical scenarios) are involved, thus requiring high computational efficiency. For this reason, model order reduction techniques, such as reduced basis methods [37, 73], are mandatory to achieve this goal. With the increasing need of real time computing, Reduced Basis (RB) methods have known a remarkable development in the last decade because they make possible a strong reduction of computational times required when solving parametrized PDE problems, owing to a crucial decomposition of the computational procedures. In an offline pre-processing stage, a suitable basis is stored by solving the original problem for a set of parameter values, properly selected in an automatic and optimal way. During an online stage, for each new parameter value the solution is found as a combination of the previously computed basis functions, by means of a Galerkin projection [37, 79]. This problem has a very small size (related with the number of the selected bases, which are typically very few). The resulting procedure is not only rapid and efficient but also accurate and reliable, thanks to residual-based a posteriori error estimators. Research activities in this field have led to a significant development of the reduced order methods for many different problems, and to applications of interest in several real-life scenarios [73]. Moreover, in order to perform efficient numerical simulations in complex and variable geometric configurations, as required for instance in engineering or medical applications, reduced order methods need to be coupled with efficient parametrization techniques for curves and surfaces. Ongoing research aims at deepening the theory and the methodology of reduced order methods for problems in fluid dynamics, characterized by very different physical and temporal scales, but also complex nonlinear problems like bifurcations and instabilities. Another task is devoted in delivering ready tools for applications in naval, nautical, aerospace and mechanical engineering, as well as in medicine (fluid-structure interactions between blood flows and arterial vessels in the human cardiovascular system) [5, 6], biology (motility of cells and micro-organisms) [1, 3], porous media (groundwater flows) [54], and also geophysics (simulation of the Earth's mantle dynamics). The developed methodologies could be properly combined and coupled with novel techniques in data assimilation, uncertainty quantification for the solution of complex inverse problems arising in the multidisciplinary fields previously mentioned. The work is organized as follows: in Section 2 we give an overview about computational challenges to be tackled. In Section 3, we summarize the main idea behind model order reduction with references therein to deepen in the matter. Then, in Section 4, we show some industrial and biomedical applications and challenges in reduced order methods for computational fluid dynamics problems. Finally, in Section 5, some perspectives follow.

Figure 1: Possible examples of CFD applications.

2 Computational Science and Engineering (CSE) challenges

Simulation-based sciences is a quickly emerging field for mathematics and computational modelling. In every specialization of engineering such as aerospace, biomedical, naval, nautical and more generally mechanical field, as well in applied sciences, such as medicine, it is nowadays fundamental to run simulations to understand how a complex system behaves before actually building or operating on it. Typically, these systems depends upon a certain number of parameters, a priori unknown (e.g. to be optimized) or uncertain (e.g. due to experimental errors), so that several simulations need to be run, each time changing the value of the parameters. Present and future efforts consist in simulating complex problems in order to retain more aspects of the reality, such as multiphysics problems, as well as systems characterized by multiple spatial and temporal scales (see Figure 1). This leads to the growing demand of efficient computational tools for many query and real time computations, parametrized formulations, simulations of increasingly complex systems with uncertain scenarios.

One strategy to solve these challenges is to use the brute force and rely on high performance computing (HPC) platforms. However, for complex applications, such as the industrial ones, the solution of the problem can require a great amount of time even in the HPC framework. This leads to some limitations: in many query context, such as optimization, the computational burden can become prohibitive; for on-spot decision one can not wait for the solution of the problem more than a few minutes.

From the analysis of the physical systems surrounding us we can easily see that the behavior of a system very often changes in a smooth way with respect to the value of the parameters. Moreover, if we are interested only in some output of interest, it often happens that the system itself behaves as a filter, smoothing the output according to changes in the parameter values. This leads to the intuition that, instead of restarting from scratch for every new simulation, we can evaluate the behavior of a system in an easy way exploiting the knowledge of the solution for some (already computed) solutions. This is the rationale that drives all the reduced order models and methods formulations [20].

Thus, the master idea is to create a cooperation and a synergy between high performance computing and reduced order methods (see Figure 2). This leads to the well known offline-online splitting: during the offline stage we rely on HPC platforms to compute some very accurate solution for properly selected values of the parameters, and store them in a database of basis functions; during the online stage, we just need to combine basis functions in the database to evaluate the solution at new values of the parameter.

Although in the last decades several efforts have been spent in this field, a lot of challenges still need to be tackled in order to face and overcome limitations of the state of the art. Reduced order methodologies should be improved for more demanding applications in industrial, medical and, in general, applied sciences settings. In particular, multiphysics and coupled problems

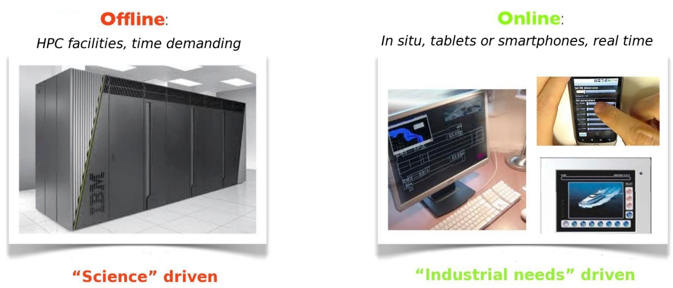


Figure 2: The main idea behind reduced order model: offline-online splitting of computational phases.

shall be considered, as well as a tighter integration with a pre-existing industrial/clinical pipeline (e.g. in data acquisition), in order to export numerical simulations and scientific computing in fields where there is still little exploitation of computational methods. The whole computational science community will need to cooperate to reach these goals, training a new generation of computational scientists. For this reason, SISSA mathLab has released the RBniCS open-source library [10] containing the implementation of several reduced order techniques, based on FEniCS [47].

3 Reduced Order Methods in a nutshell

We present a brief summary about the construction of a reduced order method (ROM) for a general problem at hand, written as:

$$\mathcal{F}(\boldsymbol{u}(\boldsymbol{\mu});\boldsymbol{\mu}) = 0, \tag{1}$$

where the parameters are represented by $\boldsymbol{\mu}$, the state solution is denoted by $\boldsymbol{u}(\boldsymbol{\mu})$ and $\mathcal{F}$ is the operator holding the state equation.

Following the paradigm shown in the previous section, let us denote as $\boldsymbol{u}^{\mathcal{N}}(\boldsymbol{\mu})$ the truth solution, obtained by solving

$$\boldsymbol{F}^{\mathcal{N}}(\boldsymbol{u}^{\mathcal{N}}(\boldsymbol{\mu});\boldsymbol{\mu}) = \boldsymbol{0}, \tag{2}$$

querying an high-fidelity solver characterized by a large number $\mathcal{N}$ of degrees of freedom.

The solver can be of very different type, depending on the application and the know-how gained in the past. For applications related reduced order models based on a finite element high-fidelity discretization see [71, 32, 88, 81, 74, 41]), as well as finite volume [34, 35, 28, 52], finite difference method [29, 17, 51], spectral element method [77, 70], extended finite element method [18, 64], boundary element method [53, 83], isogeometric analysis [53, 84] and discontinuous Galerkin methods [43, 2, 67].

Thus, after having solved (2) for some, properly selected values of the parameter, to build a reduced space, we perform a Galerkin projection over the reduced space and obtain the (general) reduced order model:

$$\boldsymbol{F}_N(\boldsymbol{u}_N(\boldsymbol{\mu});\boldsymbol{\mu}) = \boldsymbol{0}, \tag{3}$$

where $\boldsymbol{u}_N(\boldsymbol{\mu})$ is the reduced order solution and $N \ll \mathcal{N}$ is the dimension of the reduced system to be solved. Several numerical recipes are being studied in the community to guarantee the quality of the approximation and enhance the computational speedup of the evaluation, such as efficient assembly of problem operators [13], efficient treatment of nonlinearities [72], stability of the solution [80, 7], error bounds [88, 81, 62], efficient parametrization of extended systems

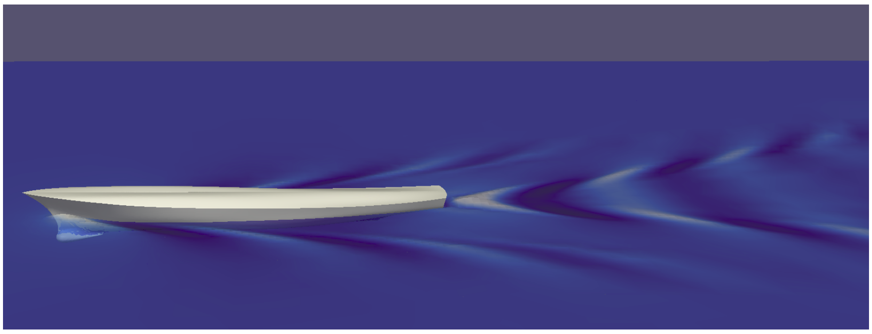


Figure 3: An example of wave pattern simulation around a hull geometry taken from a CAD file.

and complex networks [41, 40], sampling [37] and references therein. After addressing these issues for the particular problem at hand, we obtain an extremely fast system to be solved, which can be queried to obtain real-time input-output evaluation of relevant quantities of interest with an accuracy which is comparable to the high-fidelity solver.

In the following section we will provide a brief overview about some methods and applications we are treating in order to apply ROMs to some real life problems.

4 Some ROM challenges in CFD

In this section we show some, indeed very different, insights about problems arising in the industrial and clinical fields. Moreover, we also provide some ideas on the integration of these frameworks in the context of model order reduction. We first tackle the problem of ship performances prediction using real CAD geometries (Section 4.1), then the problem of the geometrical parametrization (Section 4.2) and the exploitation of isogeometric analysis (Section 4.3) in view of a complete integration of the three aspects in a single pipeline: from geometrical modification of a CAD file to the fast prediction of the wave resistance, thanks to reduced order methods. Moreover, we show new possibilities in parameter studies (Section 4.4) and a multiphysics example regarding fluid-structure interaction (Section 4.5). Finally we provide an insight in biomedical applications (Section 4.6).

4.1 Ship performances: towards the optimization of sea keeping

In the last few years, numerical fluid dynamic simulations have become an increasingly common tool in ship and yacht design. The growing computational resources available have significantly improved the quality of the results obtained. This has naturally generated, among design engineers, the desire to explore new design configurations. Researchers are then faced with the challenge to develop models of higher complexity, which can provide fast and yet accurate prediction of unsteady and nonlinear hydrodynamic loads on the hull, and reliable computations of nonlinear wave-induced ship motions.

SISSA mathLab is currently involved in the development of a model for three dimensional simulations of naval hydrodynamics based on potential flow theory (see Figure 3).

Despite the simplifying assumptions upon which the potential flow theory is based, adopting fully nonlinear free surface boundary conditions on the water free surface makes this reduced model extremely accurate in predicting both water elevation field and hull resistance [58]. In addition, the governing Laplace equation allows for a spatial discretization based on the Boundary Element Method (BEM), which only requires computational grids covering the the flow domain boundaries. Such surface grids in three dimensions are ideal for the front tracking free surface treatment at hand, as they massively reduce the problems related to mesh deformation and generation. The solver described has been implemented in a stand-alone C++ application, which is only based on open source software libraries. In particular, the deal.II library [11, 12] is used for the spatial BEM discretization of the equation, and the IDA package of the Sundials library [38] is used to time advance the resulting Differential Algebraic Equations (DAE) system. Exploiting the relative simplicity of the surface quadrilateral grids required by BEM, an automatic mesh generation module has been developed to complement the flow solver [59]. Making use of the classes of open source C++ library OpenCASCADE [66], such module is able to import the CAD file describing the hull geometry and use it to generate the computational grid [57, 24]. After its most recent development, the model is also accounting for the rigid ship motions. To this end, the three dimensional rigid body equations of motion for the hull, have been strongly coupled to the fluid dynamic solver, so as to obtain a Fluid Structure Interaction (FSI) model able to compute hydrodynamic equilibrium position and orientation of the ship. At every time instant, the CAD model of the hull is displaced in the current position, and is used to compute the correct positioning of the water nodes in contact with the ship surface.

The model described is able to provide accurate estimations of hull resistance, sink and trim [60] at a computational cost which is significantly lower with respect to models based on the solution of Navier–Stokes equations, such as RANS or LES. For this reason, a very stimulating development of the model would be that of including the effect of non-calm sea conditions on the ship resistance and motions. On one hand, this would allow for the estimation of the added resistance component due to waves characterized by different wave lengths. On the other hand, it would open the door towards time domain manoeuvring and seakeeping simulations able to assess the ship stability and the hydrodynamic loads on the hull in presence of waves. Moreover, considering specific historic databases for local wave conditions, it would be possible to design a hull shape in order to minimize wave resistance, wave loads and motions given the specific sea conditions on the route in which the ship will operate. Naturally, this intriguing scenario requires an extremely high number of calls to the FSI solver, to test the effects of variations of both parameters related to the hull geometry and to the wave field encountered. Thus, the application of reduced order methods to the – already reduced – fully nonlinear potential fluid dynamic solver is currently under study. A first example of reduced order methods applied to BEM has been presented in [83, 53].

4.2 Efficient and accurate geometrical parametrization techniques: the free-form deformation setting

Shape optimization is nowadays a major field of interest in the design community. In this framework, we need to change some (possibly a few) geometrical parameters, compute the output of interest (cost functional) and iterate till we obtain the optimal geometry. To perform these operation, we need to rely on efficient and accurate geometrical parametrization techniques. Among many possibilities [82, 8, 5], we employ the free-form deformation (FFD) [44, 86]. This techniques is made up by three steps, summarized in Figure 4. First of all, the physical domain is mapped to a reference one. Then, design variables are varied (by moving few control points) and the deformed configuration is mapped back to the physical domain.

FFD can be applied to a CAD file or to the mesh directly. Both the approaches have some pros and cons. On the one hand, the application to the CAD allows an intuitive and easy use of the tool by engineer and designers; on the other hand, however, it requires the meshing step for each new configuration. This step can be computational expensive for some, very complex,

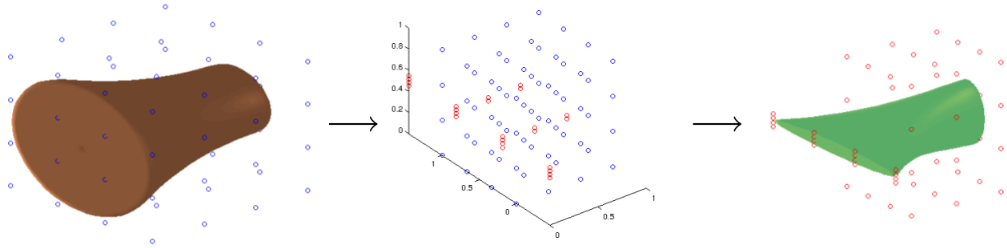


Figure 4: FFD in action: initial configuration, creation and modification of the lattice, final configuration.

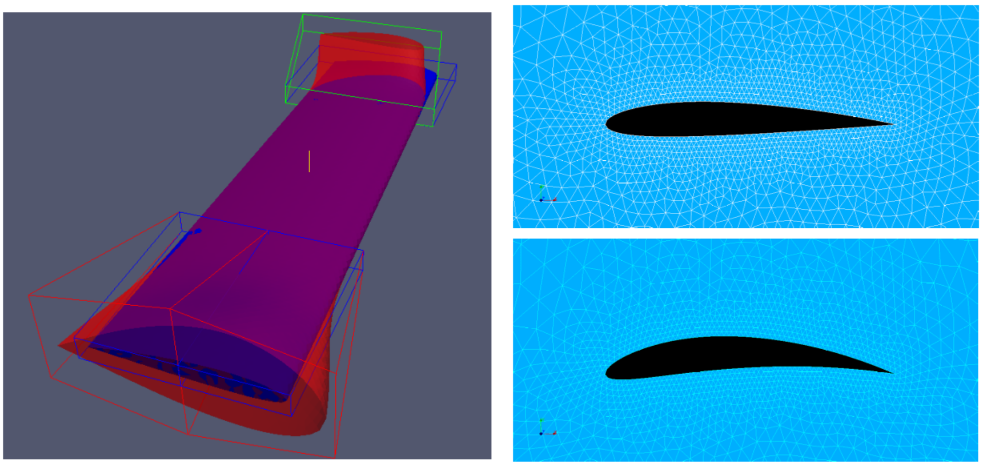


Figure 5: FFD in action: application to a CAD file (left) and a mesh (right) for a simple aeronautical geometry.

engineering problems. In Figure 5 we depict two applications of FFD corresponding to the two different approaches.

The choice of which strategy is the best depends mainly on the specific case. For example, if the high-fidelity software adopted takes as input the CAD file (and performs the mesh step automatically) and it is used as a black-box code, the FFD should be applied to the CAD file itself. Still, if one does not have any particular constraint, and can deeply modify the software, the strategy of applying FFD to the mesh is more efficient and also more suitable for the model reduction. Moreover, starting from this general paradigm, specialized shape parametrization techniques can be tailored for each application [5, 6].

4.3 From CAD to real-time evaluation: isogeometric solvers coupled with ROMs

One of the emerging fields in computational fluid dynamics is the use of isogeometric solvers. These present several features that are very attractive, especially for industrial problems. In fact, all the Computer Aided Design (CAD) geometries, the standard tool for industrial design, are nothing but NURBS patches. The idea is to employ the basis functions describing the geometry also for the analysis of the problem in a fully integrated framework [23]. Isogeometric analysis (IGA) offers also very interesting features from the geometrical parametrization point of view, thanks to its direct interface with CAD files structure. Unfortunately, real CAD files are very complex and contain patches with different parametrization of the geometry, trimmed surfaces and other features difficult to be treated efficiently. The full integration of geometry and anal-

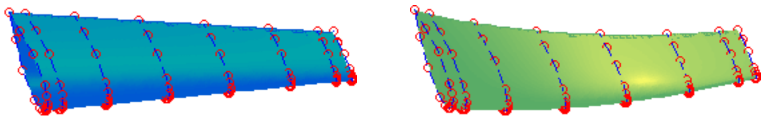


Figure 6: Isogeometric aircraft wings.

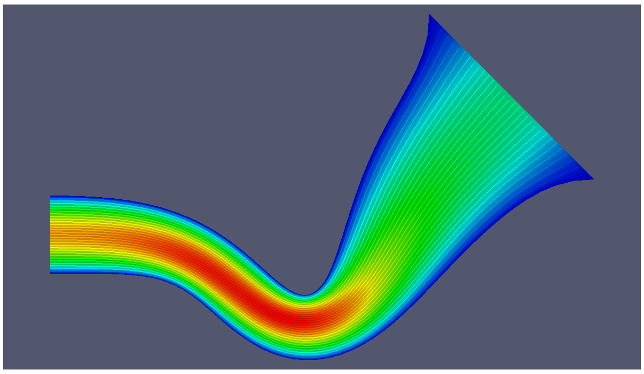


Figure 7: Example of an isogeometric exhaust gasses device.

ysis of the problem is still an open problem, although much work has been done to overcome this gap. See, for instance, Figures 6 and 7 for the case of an isogeometric aircraft wing and exhaust gasses device.

The integration of IGA with reduced order methods is a challenge that can bring together all the aspect of design procedures in an easier way: from the modification of the CAD file to the real time evaluation of the solution for the new configuration.

The work carried out in this field, is intrinsically connected to what has been proposed in section 4.2: both methods treat, from a different point of view, the direct interface with complex geometries, and these two steps can be coupled together to form a complete pipeline from CAD file to ROM solution. For some applications to the ROM-IGA coupling, see [83, 84, 53, 25].

4.4 New opportunities in parameter studies: active subspaces

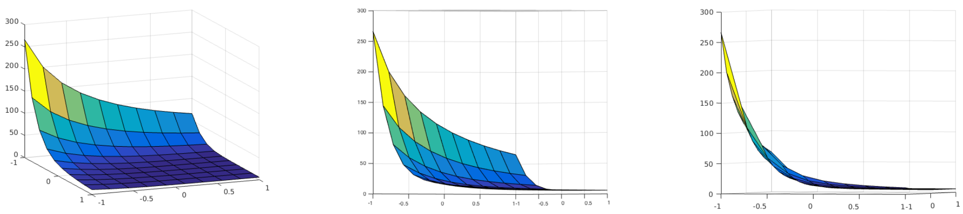
In many cases the dimension of the parametrized problem is only artificially high. To tackle this problem there are several techniques. One of them is manifold learning technique, that is a general cap for many different method (see [46]). They try to represent the manifold of the snapshots from the high fidelity solver in a reduced way, for instance unfolding the manifold along particular directions. Manifold Learning can be thought of as an attempt to generalize linear frameworks like Principal Component Analysis to be sensitive to non-linear structure in data. Though supervised variants exist, the typical manifold learning problem is unsupervised: it learns the high-dimensional structure of the data from the data itself, without the use of predetermined classifications. Engineering applications include manufacturing processes (see [45], where a manifold walking algorithm is used, and [76]) and mechanical tests [56], and structural optimization problems [75]. Among other we mention its application in natural science, for instance [3].

Another emerging idea is the active subspaces method. The active subspaces approach represents one of the emerging ideas for dimension reduction in the parameter studies. The concept was introduced by Constantine in [21] and employed in different real problems. We mention, among others, aerodynamic shape optimization [50], the parameter reduction for the HyShot II scramjet model [22] and active subspaces for integrated hydrologic model [42].

Several recent model reduction techniques attempt to compute comparable predictions at reduced costs for the same inputs [78]. However, using ROMs in a parameter study gives answers for the reduced order model instead of the actual model.

A characteristic of the active subspaces is that instead of identifying a subset of the inputs as important, they identify a set of important directions in the space of all inputs. This is done by exploiting the information given from the gradients of the output function. If the simulation's prediction does not change as the inputs move along a particular direction, then we can safely ignore that direction in the parameter study. We call subspace-based dimension reduction the dimension reduction with linear combinations of inputs. In Figure 8 it is possible to capture the main idea behind the active subspaces approach: we try to rotate the inputs domain in such a way lower dimension behavior of the output function are revealed. When you identify an active subspace for your problem of interest, then it is possible to perform different parameter studies such as response surfaces, integration, optimization and statistical inversion.

Moreover, this method seems to have great potentiality in the model reduction when the input space is very large, for instance with geometrical parameters (see section 4.2) or in shape optimization problem [27]. Moreover it can be coupled with many optimization problems linking together different but correlated outputs.



(a) Original output function (b) Intermediate rotation (c) Output function with respect to the active variable

Figure 8: Example of a bivariate output function (a), intermediate rotation of the domain (b), and the final state (c), where we can see the variation of the function along the active variable.

4.5 Towards multi-physics: an example of fluid-structure interaction problem

Coupled problems are often encountered in the industrial practice, involving different physical system each described by a set of PDEs. An example are fluid structure interaction (FSI) problems, where a mutual interaction between a fluid flow and a deformable structure occurs (see Figure 9). The structural motion or deformation caused by the fluid action is significant, so that it will in turn affect the fluid flow. Such a behavior has to be modeled with proper coupling conditions that need to be treated with adequate numerical techniques. The classic example of fluid structure interaction problem that is most often encountered in the scientific literature is the interaction between an incompressible fluid flow and an hyperelastic solid. Industrial problems, however, often require coupling to further equations to describe a complex multiphysics scenario, characterized for instance by turbulent flows (with boundary layers that need to be accurately solved), thermal analysis, multiphase flows, free surfaces or chemical reactions. Each additional coupled problem severely increases the computational time required for the simulation. Thus, even though complex multiphysics problems are the most challenging for reduced order methods, effective reduction techniques will pave the way to striking computational speedups [4].

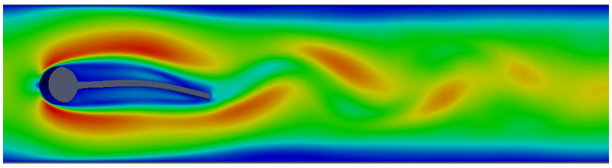


Figure 9: Snapshot taken from the simulation of the Hron and Turek FSI benchmark, proposed in [87].

As an additional source of complexity multiple regions, and thus multiple interfaces, can be present (see for example Figure 10). If multiple couplings have to be solved, it is seldom sufficient to simply concatenate them and solve each interaction sequentially. If the mutual interactions are strong, convergence will not be achieved, and a simultaneous solution of the different interfaces is required [16].

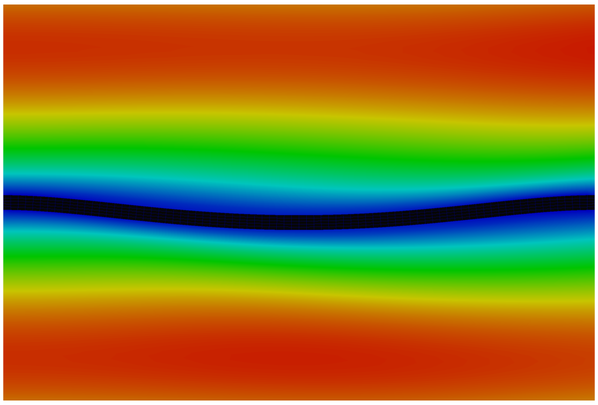


Figure 10: Example of multi-fields problem: a solid plate divides two fluid domains. In this particular example, computations were performed with a finite volume code [61].

At the high-fidelity level, the solution approaches to FSI problems can be classified in two broad categories: partitioned approach (flow and structure equations are solved in sequence) [85], and monolithic approach (the continuum is discretized and solved as a whole) [39]. Reduced order methods built on both approaches should be considered.

The partitioned approach is the most broadly used in the industrial context, because it allows the coupling of existing solvers, treating them as black boxes if the access to the code is not possible. Thus, the already optimized solvers, along with their preconditioners can be readily employed. A partitioned approach often requires to iterate the solution process within each time step of the simulation. This is due to the strength of the coupling as well as the known problem of the artificial added mass [31], that introduces further instabilities. This means that the systems of equations might have to be solved several times every time step. Thus, the computational costs can quickly become prohibitive unless efficient iterative methods to ensure a low number of iterations are employed. Quasi-Newton methods (for example [26]) have proven quite efficient to this regard. In this context, reduction techniques, prove very useful to make the iterative method as efficient and fast as possible. In particular, the reuse of information coming from the results at previous iterations (and previous time steps) has to be optimized. Reusing more iterations allows for a better approximation of the system's Jacobian, but at the expense of the conditioning. The interaction of a fluid and slender structures is an example of a case where the numerical treatment of the problem is very challenging [68]. Proper Orthogonal

Decomposition has been used to preserve the convergence rate of the iterative method when a lot of previous iteration are reused [14]. Singular Value Decomposition is also a very robust method to deal with the ill conditioning (and possible rank-deficiency) of the linear system to be solved in an iterative method [30]. In this latter case, the errors due to ill conditioning can be directly bounded by choosing how many singular values to retain from the decomposition (setting the smaller ones to a zero value).

Monolithic solvers, on the other hand, are more robust for strongly coupled problems. Generally, their performance, in terms of computational times, is superior to that of segregated solvers. Indeed, it has been shown that the monolithic approach can be competitive even for problems that feature a weak coupling, and thus would require a low number of iterations for the segregated approach [36]. For these reasons, preliminary investigation of monolithic reduced order models for FSI problem have been investigated in [9].

4.6 Biomedical applications: from patient-specific clinical data to real-time simulation

Computational fluid dynamics applications in biomedicine, in particular related to haemodynamics, are another example of fields where reduced order methods are required to propose a more widespread usage of mathematical modelling in the daily practice. Indeed, for instance in cardiovascular applications, several studies have shown that computational fluid dynamics indicators actually correlate with relevant medical questions, so that it is nowadays agreed that numerical simulation could, in principle, support medical decisions [49]. Two main requirements are usually identified by clinical partners in order for computational approaches to be really useful in the daily practice: integration with clinical data and real-time simulations. The former takes into account the personalization of the numerical simulation, starting from patient-specific clinical data on the disease (e.g. MRI or CT scans of the organ) or on flow conditions (e.g. flow rate measurements). The latter is dictated by the short time span available to take clinical decisions, especially in cases where the disease may impair the health of the patient. Under these constraints, it is natural to turn to reduced order modelling to provide fast and reliable simulations. We refer to [55, 33, 15, 63, 64, 65] for some applications of ROMs in a biomedical problems. A description of the full reduced order pipeline for the sensitivity analysis of patient-specific configurations of coronary artery bypass grafts (CABGs) is provided in [5, 6].

Moreover, a parametrized formulation of the biomedical problem allows clinicians to compare several different scenarios for what concerns both disease and medical treatment. For instance, ROMs can be a support in decision of whether to pursue medical treatment, by providing accurate indicators of the disease starting from clinical data (e.g. precise identification of degree of stenosis based on experimental pressure drop measurements in coronary arteries). Moreover, when coupled with shape parametrization techniques (Section 4.2), ROMs can also support the comparison of different surgical choices, which are usually reflected in geometrical variations of the computational domain. Figure 11 shows an example of a complete framework for a biomedical simulation. Finally, ROMs can also be used in the design of new medical treatment devices. The efficient treatment of multiphysics problems (Section 4.5) will enhance the applicability of ROM to a broader number of applications, e.g. considering reduced order simulations of fluid-structure interaction phenomena in large arteries or coupling of reduced CFD simulations to drug delivery treatments.

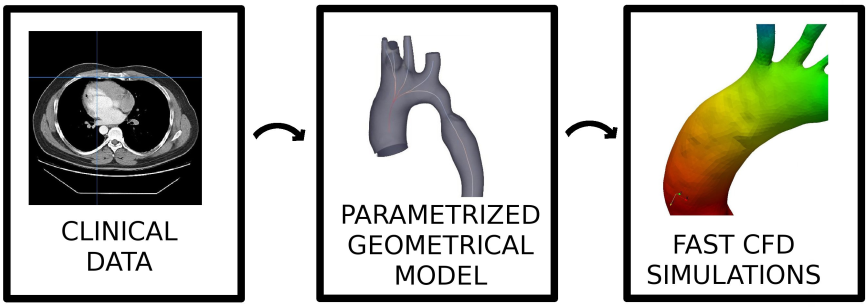


Figure 11: Example of a reduced order framework for biomedical applications.

5 Perspectives

We have provided examples of research activities in industrial and clinical projects where there is a certain degree of complexity and a certain need for the development of reduced models and methods. It is time to better integrate *Data, Modelling, Analysis, Numerics, Control, Optimization and Uncertainty Quantification* in a *new parametrized, reduced and coupled paradigm* to be able to face more and more complex problems, representing complex systems. We need to draw the attention to the fact that *Science and Industry advance with Mathematics* which is a *propeller for Innovation* and technological transfer, as shown in some examples taken from projects in naval and yachts engineering, as well as mechanical and biomedical engineering. Integration of CAD tools and geometrical parametrization, as well as medical data with geometrical reconstruction and exploration of parameter space are only few aspects of a much bigger framework, including proper integration of high order methods and low order ones (IGA-ROM is one of the possible examples).
Among other important issue under exploration, we mention the detection of flow bifurcations and the study of flow stability in CFD problems with ROM techniques [70], with also important applications in cardiovascular flows, such as the Coanda effect in Mitral valves [69]. Reduced order methods for parametrized CFD problems characterized by turbulent patterns are also under investigation for important industrial applications [48]. Last, but not least, we mention important recent advances in model order reduction for uncertainty quantification, see [19] and references therein for recent contributions and a wider framework.

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REFERENCES

[1] F. Alouges, A. DeSimone, and L. Heltai. Optimally swimming stokesian robots. *Discrete and Continuous Dynamical Systems - Series B (DCDS-B)*, 18(5):1189–1215, 2013.

[2] P. F. Antonietti, P. Pacciarini, and A. Quarteroni. A discontinuous Galerkin reduced basis element method for elliptic problems. ESAIM: M2AN, in press, 2015.

[3] M. Arroyo, L. Heltai, D. Millán, and A. DeSimone. Reverse engineering the euglenoid movement. *Proceedings of the National Academy of Sciences*, 109(44):17874–17879, 2012.

[4] M. Balajewicz and C. Farhat. Reduction of nonlinear embedded boundary models for problems with evolving interfaces. *Journal of Computational Physics*, 274:489–504, 2014.

[5] F. Ballarin. *Reduced-order models for patient-specific haemodynamics of coronary artery bypass grafts*. PhD thesis, Department of Mathematics, Politecnico di Milano, 2015. http://hdl.handle.net/10589/102804.

[6] F. Ballarin, E. Faggiano, S. Ippolito, A. Manzoni, A. Quarteroni, G. Rozza, and R. Scrofani. Fast simulations of patient-specific haemodynamics of coronary artery bypass grafts based on a POD–Galerkin method and a vascular shape parametrization. http://urania.sissa.it/xmlui/handle/1963/34623, submitted, SISSA preprint 45/2015/MATE, 2015.

[7] F. Ballarin, A. Manzoni, A. Quarteroni, and G. Rozza. Supremizer stabilization of POD–Galerkin approximation of parametrized steady incompressible Navier–Stokes equations. *International Journal for Numerical Methods in Engineering*, 102(5):1136–1161, 2015.

[8] F. Ballarin, A. Manzoni, G. Rozza, and S. Salsa. Shape optimization by Free-Form Deformation: existence results and numerical solution for Stokes flows. *Journal of Scientific Computing*, 60(3):537–563, 2014.

[9] F. Ballarin and G. Rozza. POD–Galerkin monolithic reduced order models for parametrized fluid-structure interaction problems. Submitted, SISSA preprint 07/2016/MATE, 2016.

[10] F. Ballarin, A. Sartori, and G. Rozza. RBniCS – reduced order modelling in FEniCS, 2015. http://mathlab.sissa.it/rbnics.

[11] W. Bangerth, R. Hartmann, and G. Kanschat. deal.II – a general purpose object oriented finite element library. *ACM Transactions on Mathematical Software*, 33(4):24/1–24/27, 2007.

[12] W. Bangerth, T. Heister, L. Heltai, G. Kanschat, M. Kronbichler, M. Maier, B. Turcksin, and T. D. Young. The deal.II Library, Version 8.2. Technical report, 2015.

[13] M. Barrault, Y. Maday, N. C. Nguyen, and A. T. Patera. An empirical interpolation method: application to efficient reduced-basis discretization of partial differential equations. *C. R. Math. Acad. Sci. Paris*, 339(9):667–672, 2004.

[14] E. Bogaers, S. Kok, and T. Franz. Strongly coupled partitioned fsi using proper orthogonal decomposition. In *8th South African Conference on Computational and Applied Mechanics (SACAM 2012)*. SAAM, 2012.

[15] M. Boulakia, E. Schenone, and J.-F. Gerbeau. Reduced-order modeling for cardiac electrophysiology. Application to parameter identification. *International Journal for Numerical Methods in Biomedical Engineering*, 28(6-7):727–744, 2012.

[16] H.-J. Bungartz, F. Lindner, M. Mehl, and B. Uekermann. A plug-and-play coupling approach for parallel multi-field simulations. *Computational Mechanics*, 55, 2015.

[17] Y. Cao, J. Zhu, Z. Luo, and I. Navon. Reduced-order modeling of the upper tropical pacific ocean model using proper orthogonal decomposition. *Computers & Mathematics with Applications*, 52(8–9):1373–1386, 2006. Variational Data Assimilation and Optimal Control.

[18] E. Chahine, P. Laborde, and Y. Renard. A reduced basis enrichment for the extended finite element method. *Mathematical Modelling of Natural Phenomena*, 4(01):88–105, 2009.

[19] P. Chen, A. Quarteroni, and G. Rozza. Multilevel and weighted reduced basis method for stochastic optimal control problems constrained by stokes equations. *Numerische Mathematik, in press*, pages 1–36, 2015.

[20] F. Chinesta, A. Huerta, G. Rozza, and K. Willcox. Model order reduction. Encyclopedia of Computational Mechanics 2016, in press, Elsevier. SISSA preprint 55/2015/MATE.

[21] P. Constantine. Active subspaces: Emerging ideas for dimension reduction in parameter studies, volume 2 of SIAM spotlights. *Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA*, 2015.

[22] P. Constantine, M. Emory, J. Larsson, and G. Iaccarino. Exploiting active subspaces to quantify uncertainty in the numerical simulation of the HyShot II scramjet. *Journal of Computational Physics*, 302:1–20, 2015.

[23] J. A. Cottrell, T. J. Hughes, and Y. Bazilevs. *Isogeometric analysis: toward integration of CAD and FEA*. John Wiley & Sons, 2009.

[24] F. Dassi, A. Mola, and H. Si. Curvature-Adapted Remeshing of CAD Surfaces. In *Proceedings of the 23rd International Meshing Roundtable, Procedia Engineering,*, 2014.

[25] L. Dedè, A. Quarteroni, and S. Zhu. Isogeometric analysis and proper orthogonal decomposition for parabolic problems. Technical report EPFL MATHICSE 44.2014, 2014.

[26] J. Degroote, K.-J. Bathe, and J. Vierendeels. Performance of a new partitioned procedure versus a monolithic procedure in fluid–structure interaction. *Computers and Structures*, 87:793–801, 2009.

[27] M. Diez, E. F. Campana, and F. Stern. Design-space dimensionality reduction in shape optimization by Karhunen–Loève expansion. *Computer Methods in Applied Mechanics and Engineering*, 283:1525–1544, 2015.

[28] M. Drohmann, B. Haasdonk, and M. Ohlberger. Reduced basis approximation for nonlinear parametrized evolution equations based on empirical operator interpolation. *SIAM Journal on Scientific Computing*, 34(2):A937–A969, 2012.

[29] J. Du, J. Zhu, Z. Luo, and I. M. Navon. An optimizing finite difference scheme based on proper orthogonal decomposition for CVD equations. *International Journal for Numerical Methods in Biomedical Engineering*, 27(1):78–94, 2011.

[30] H.-R. Fang and Y. Saad. Two classes of multisecant methods for nonlinear acceleration. *Numerical linear algebra with applications*, 16:197–221, 2009.

[31] C. Förster, W. A. Wall, and E. Ramm. Artificial added mass instabilities in sequential staggered coupling of nonlinear structures and incompressible viscous flows. *Computational Methods in Applied Mechanical Engineering*, 196:1278–1293, 2007.

[32] M. A. Grepl and A. T. Patera. A posteriori error bounds for reduced-basis approximations of parametrized parabolic partial differential equations. *ESAIM: M2AN*, 39(1):157–181, 2005.

[33] R. Guibert, K. McLeod, A. Caiazzo, T. Mansi, M. A. Fernández, M. Sermesant, X. Pennec, I. E. Vignon-Clementel, Y. Boudjemline, and J.-F. Gerbeau. Group-wise construction of reduced models for understanding and characterization of pulmonary blood flows from medical images. *Medical Image Analysis*, 18(1):63–82, 2014.

[34] B. Haasdonk and M. Ohlberger. Reduced basis method for finite volume approximations of parametrized linear evolution equations. *ESAIM: Mathematical Modelling and Numerical Analysis-Modélisation Mathématique et Analyse Numérique*, 42(2):277–302, 2008.

[35] B. Haasdonk, M. Ohlberger, and Rozza. A reduced basis method for evolution schemes with parameter-dependent explicit operators. *Electronic Transactions on Numerical Analysis*, 32:145–161, 2008.

[36] M. Heil, A. L. Hazel, and J. Boyle. Solvers for large displacement fluid-structure interaction problems: segregated versus monolithic approaches. *Computational Mechanics*, 43(1):91–101, 2008.

[37] J. S. Hesthaven, G. Rozza, and B. Stamm. *Certified Reduced Basis Methods for Parametrized Partial Differential Equations*. Springer Briefs in Mathematics, 2015.

[38] A. C. Hindmarsh, P. N. Brown, K. E. Grant, S. L. Lee, R. Serban, D. E. Shumaker, and C. S. Woodward. Sundials: Suite of nonlinear and differential/algebraic equation solvers. *ACM Transactions on Mathematical Software*, 31(3):363–396, 2005.

[39] J. Hron and S. Turek. A monolithic FEM/multigrid solver for an ALE formulation of fluid-structure interaction with applications in biomechanics. In *Fluid-Structure Interaction. Modelling, Simulation, Optimisation*, pages 146–170. Springer Berlin Heidelberg, 2006.

[40] L. Iapichino, A. Quarteroni, and G. Rozza. A reduced basis hybrid method for the coupling of parametrized domains represented by fluidic networks. *Computer Methods in Applied Mechanics and Engineering*, 221–222:63–82, 2012.

[41] L. Iapichino, A. Quarteroni, and G. Rozza. Reduced basis method and domain decomposition for elliptic problems in networks and complex parametrized geometries. *Computers & Mathematics with Applications*, 71(1):408–430, 2016.

[42] J. Jefferson, J. Gilbert, P. Constantine, and R. Maxwell. Reprint of: Active subspaces for sensitivity analysis and dimension reduction of an integrated hydrologic model. *Computers & Geosciences*, 2015.

[43] S. Kaulmann, M. Ohlberger, and B. Haasdonk. A new local reduced basis discontinuous galerkin approach for heterogeneous multiscale problems. *Comptes Rendus Mathematique*, 349(23):1233–1238, 2011.

[44] T. Lassila and G. Rozza. Parametric free-form shape design with PDE models and reduced basis method. *Comput. Meth. Appl. Mech. Engrg.*, 199(23-24):1583–1592, 2010.

[45] G. Le Quilliec, B. Raghavan, and P. Breitkopf. A manifold learning-based reduced order model for springback shape characterization and optimization in sheet metal forming. *Computer Methods in Applied Mechanics and Engineering*, 285:621–638, 2015.

[46] J. A. Lee and M. Verleysen. *Nonlinear dimensionality reduction*. Springer Science & Business Media, 2007.

[47] A. Logg, K.-A. Mardal, G. N. Wells, et al. *Automated Solution of Differential Equations by the Finite Element Method*. Springer, 2012.

[48] S. Lorenzi, A. Cammi, L. Luzzi, and G. Rozza. POD-Galerkin method for finite volume approximation of Navier-Stokes and RANS equations. Submitted. SISSA preprint 2016/MATE, 2016.

[49] F. Loth, P. F. Fischer, and H. S. Bassiouny. Blood flow in end-to-side anastomoses. *Annual Review of Fluid Mechanics*, 40:367–393, 2008.

[50] T. Lukaczyk, F. Palacios, J. Alonso, and P. Constantine. Active subspaces for shape optimization. In *10th AIAA Multidisciplinary Design Optimization Conference*, 2014.

[51] Z. Luo, J. Chen, J. Zhu, R. Wang, and I. M. Navon. An optimizing reduced order FDS for the tropical pacific ocean reduced gravity model. *International Journal for Numerical Methods in Fluids*, 55(2):143–161, 2007.

[52] Z. Luo, H. Li, P. Sun, J. An, and I. M. Navon. A reduced-order finite volume element formulation based on POD method and numerical simulation for two-dimensional solute transport problems. *Mathematics and Computers in Simulation*, 89:50–68, 2013.

[53] A. Manzoni, F. Salmoiraghi, and L. Heltai. Reduced Basis Isogeometric Methods (RB-IGA) for the real-time simulation of potential flows about parametrized NACA airfoils. *Computer Methods in Applied Mechanics and Engineering*, 284:1147–1180, 2015.

[54] I. Martini, G. Rozza, and B. Haasdonk. Reduced basis approximation and a-posteriori error estimation for the coupled Stokes-Darcy system. *Advances in Computational Mathematics*, 41(5):1131–1157, 2015.

[55] K. McLeod, A. Caiazzo, M. Fernández, T. Mansi, I. Vignon-Clementel, M. Sermesant, X. Pennec, Y. Boudjemline, and J.-F. Gerbeau. Atlas-based reduced models of blood flows for fast patient-specific simulations. In O. Camara, M. Pop, K. Rhode, M. Sermesant, N. Smith, and A. Young, editors, *Statistical Atlases and Computational Models of the Heart*, volume 6364 of *Lecture Notes in Computer Science*, pages 95–104. Springer Berlin / Heidelberg, 2010.

[56] L. Meng, P. Breitkopf, B. Raghavan, G. Mauvoisin, O. Bartier, and X. Hernot. Identification of material properties using indentation test and shape manifold learning approach. *Computer Methods in Applied Mechanics and Engineering*, 297:239–257, 2015.

[57] A. Mola and L. Heltai. The step-54 tutorial program. `http://dealii.org/developer/doxygen/deal.II/step_54.html`, 2014.

[58] A. Mola, L. Heltai, and A. DeSimone. A stable and adaptive semi-lagrangian potential model for unsteady and nonlinear ship-wave interactions. *Engineering Analysis with Boundary Elements*, 37(1):128 – 143, 2013.

[59] A. Mola, L. Heltai, and A. Desimone. A fully nonlinear potential model for ship hydrodynamics directly interfaced with CAD data structures. In *The 24th International Ocean and Polar Engineering Conference*, 2014.

[60] A. Mola, L. Heltai, and A. Desimone. Ship Sinkage and Trim Predictions Based on a CAD Interfaced Fully Nonlinear Potential Model, submitted. In *The 26th International Ocean and Polar Engineering Conference*, 2016.

[61] F. Moukalled, L. Mangani, and M. Darwish. *The finite volume method in computational fluid dynamics*. Springer international publishing Switzerland, 2016.

[62] N.-C. Nguyen, G. Rozza, and A. Patera. Reduced basis approximation and a posteriori error estimation for the time-dependent viscous Burgers' equation. *Calcolo*, 46:157–185, 2009.

[63] S. Niroomandi, I. Alfaro, E. Cueto, and F. Chinesta. Real-time deformable models of non-linear tissues by model reduction techniques. *Computer Methods and Programs in Biomedicine*, 91(3):223–231, 2008.

[64] S. Niroomandi, I. Alfaro, D. Gonzlez, E. Cueto, and F. Chinesta. Real-time simulation of surgery by reduced-order modeling and X-FEM techniques. *International Journal for Numerical Methods in Biomedical Engineering*, 28(5):574–588, 2012.

[65] S. Niroomandi, D. Gonzlez, I. Alfaro, F. Bordeu, A. Leygue, E. Cueto, and F. Chinesta. Real-time simulation of biological soft tissues: a PGD approach. *International Journal for Numerical Methods in Biomedical Engineering*, 29(5):586–600, 2013.

[66] Open Cascade S.A.S., OpenCASCADE technology, 2010. `http://www.opencascade.org`.

[67] P. Pacciarini. *Discontinuous Galerkin Reduced Basis Element methods for Parametrized Partial Differential Equations in Partitioned Domains*. PhD thesis, Department of Mathematics, Politecnico di Milano, 2016.

[68] M. Paidoussis. *Fluid-Structure Interactions: Slender Structures and Axial Flow*. Academic Press, 2014.

[69] G. Pitton, A. Quaini, and G. Rozza. Computational reduction strategies for bifurcations and stability analysis in fluid-dynamics: applications to Coanda effect in cardiac flows. Submitted, 2016.

[70] G. Pitton and G. Rozza. A reduced basis method for bifurcation problems in incompressible fluid dynamics. Submitted. SISSA preprint 55/2015/MATE, 2015.

[71] C. Prud'homme, D. V. Rovas, K. Veroy, L. Machiels, Y. Maday, A. T. Patera, and G. Turinici. Reliable real-time solution of parametrized partial differential equations: Reduced-basis output bound methods. *Journal of Fluids Engineering*, 124(1):70–80, 2002.

[72] A. Quarteroni and G. Rozza. Numerical solution of parametrized Navier–Stokes equations by reduced basis methods. *Numerical Methods for Partial Differential Equations*, 23(4):923–948, 2007.

[73] A. Quarteroni and G. Rozza. *Reduced Order Methods for Modeling and Computational Reduction*, volume 9. Springer Milano, MS&A Series, 2014.

[74] A. Quarteroni, G. Rozza, and A. Manzoni. Certified reduced basis approximation for parametrized partial differential equations and applications. *Journal of Mathematics in Industry*, 1(1):1–49, 2011.

[75] B. Raghavan, P. Breitkopf, Y. Tourbier, and P. Villon. Towards a space reduction approach for efficient structural shape optimization. *Structural and Multidisciplinary Optimization*, 48(5):987–1000, 2013.

[76] B. Raghavan, G. Le Quilliec, P. Breitkopf, A. Rassineux, J.-M. Roelandt, and P. Villon. Numerical assessment of springback for the deep drawing process by level set interpolation using shape manifolds. *International Journal of Material Forming*, 7(4):487–501, 2014.

[77] O. Roderick, M. Anitescu, and Y. Peet. Proper orthogonal decompositions in multifidelity uncertainty quantification of complex simulation models. *International Journal of Computer Mathematics*, 91(4):748–769, 2014.

[78] C. Rowley. Model reduction for fluids, using balanced proper orthogonal decomposition. *International Journal of Bifurcation and Chaos*, 15(03):997–1013, 2005.

[79] G. Rozza. Reduced basis approximation and error bounds for potential flows in parametrized geometries. *Comm. Comput. Phys.*, 9:1–48, 2011.

[80] G. Rozza, D. Huynh, and A. Manzoni. Reduced basis approximation and a posteriori error estimation for Stokes flows in parametrized geometries: roles of the inf-sup stability constants. *Numer. Math.*, 125(1):115–152, 2013.

[81] G. Rozza, D. B. P. Huynh, and A. T. Patera. Reduced basis approximation and a posteriori error estimation for affinely parametrized elliptic coercive partial differential equations. *Archives of Computational Methods in Engineering*, 15:1–47, 2008.

[82] G. Rozza, A. Koshakji, and A. Quarteroni. Free Form Deformation techniques applied to 3D shape optimization problems. *Communications in Applied and Industrial Mathematics*, 2013.

[83] F. Salmoiraghi. Reduced order models for potential flows past parametrized NACA airfoils based on an isogeometric boundary element method. Master thesis, Aerospace Engineering, Politecnico di Milano. `https://home.aero.polimi.it/quadrio/it/Tesi/salmoiraghi/tesi-salmoiraghi.pdf`, 2014.

[84] F. Salmoiraghi, F. Ballarin, L. Heltai, and G. Rozza. Isogeometric analysis-based reduced order modelling for incompressible linear viscous flows in parametrized shapes. Submitted. SISSA preprint 06/2016/MATE.

[85] M. Schäfer, M. Heck, and S. Yigit. An implicit partitioned method for the numerical simulation of fluid-structure interaction. In *Fluid-Structure Interaction. Modelling, Simulation, Optimisation*, pages 171–194. Springer Berlin Heidelberg, 2006.

[86] T. W. Sederberg and S. R. Parry. Free-form deformation of solid geometric models. In *ACM SIGGRAPH computer graphics*, volume 20, pages 151–160. ACM, 1986.

[87] S. Turek and J. Hron. Proposal for numerical benchmarking of fluid-structure interaction between an elastic object and laminar incompressible flow. In *Fluid-Structure Interaction. Modelling, Simulation, Optimisation*, pages 371–385. Springer Berlin Heidelberg, 2006.

[88] K. Veroy and A. Patera. Certified real-time solution of the parametrized steady incompressible Navier-Stokes equations: rigorous reduced-basis a posteriori error bounds. *International Journal for Numerical Methods in Fluids*, 47(8):773–788, 2005.