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# MODEL AND INVESTIGATION OF DYNAMICS OF SOLID SYSTEM WITH TWO MASSIVE ECCENTRICS ON A ROUGH PLANE

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**Abstract.** *In this paper we investigate the solid system with internal rotating eccentrics as a movers. The system moves in plane with dry friction. Massive eccentrics rotating with variable angular velocity convert rotational motion into uneven translational. The motion is caused by inertia forces and friction between the system and the rough surface and occurs in short steps or slides. The equations of motion are derived under assumption of decreasing linear dependence of electric motor torque on angular velocity. We consider the motion of the platform and the pendulum, and then the resulting system is presented in a convenient form for numerical integration in MatLab. Runge–Kutta fourth-order method is used. Tracking of labels applied to the system is performed using time-lapse video and Adobe After Effects. The results of numerical integration of math model are presented and compared with experimental data obtained using a tracking program.*

## 1 INTRODUCTION

An interest in solid systems driven by inner movements of masses without outer movers, such as wheels, chain tracks, or legs, arises in last decade. A new class of mechanisms (robots, mechatronic systems) able to move in a resisting medium without external movers due to movement of internal bodies attracts attention [1-12].

In study [1] there is a very simple mechanical model: an absolutely rigid body that can move along a horizontal line; inside the body, there is a movable mass that also moves horizontally. Such a motion realized as a result of the specific periodic relative motions of a material point inside the body. The internal motion parameters providing for a maximum average velocity of the system as a whole are determined.

In paper [2] they deal with a class of vibration-driven models that simulate the dynamics of vibration-driven mobile robots, which mean autonomous mechanisms consisting of a body and movable internal masses. Moving under the action of drives, the internal masses interact with the robot body and the body, in turn, interacts with the external environment. By controlling the motion of internal masses, they can control the force of reaction of the external environment acting on the robot body, providing its motion in the desired direction and regulating the velocity. Vibration-driven robots are simple in design and do not require special limbs, such as wheels, chain tracks, or legs. In particular, this makes them promising for movement not only on surfaces but also within dense media resisting pressure (e.g., in soil), as well as in pipes. Presumably, micro robots of this type can be applied in medicine in order to deliver drugs or a diagnostic sensor to an affected area.

In work [3] the controlled horizontal motion of a body in the presence of dry friction forces is investigated. Control is accomplished by means of a movable mass that can move within the body in a bounded range.

In study [4] for three forms of resistance forces they have calculated the periodic translational motion of a two-mass system consisting of the main body (the container), interacting with the medium, and an internal moving mass. They have considered simple motions of the internal mass with respect to the container, called two-phase motion, and a corresponding piecewise-constant relative velocity of the internal mass. They have shown that as a result of simple motions of the internal mass with respect to the body, the system is displaced as a whole. The principle of motion considered in this paper was realized in a number of experimental models. The internal displacements were obtained by means of a pendulum system and a rotating mass. Mobile mini-robots, moving inside a tube, have been designed and successfully tested. Experiments have confirmed the practical realizability of the principle described above for displacing bodies in resistant media.

In paper [5] the dynamical system consisting of a rigid body and an internal mass is investigated. The rigid body can move along a rough horizontal plane with a piece-wise linear resistance law due to the special periodic internal motions. A three-phase control is constructed and the motion of the body relative to the environment is controlled by varying the relative acceleration of the internal mass. The relationships between the control parameters for the realization of a directed velocity-periodic motion of the rigid body are established through both theoretical and numerical methods. Optimal and practical parameters of the internal motion, at which the maximal mean velocity of the body is reached, are determined.

In work [6] a periodic motion of the internal body relative to the main body, which generates the motion of the main body with periodically changing velocity and the maximum displacement for the period, is constructed for a wide class of laws of resistance of the environment to the motion of the main body. The principle of motion considered is appropriate for mobile mini- and micro-robots. The body (housing) of such robots can be hermetically sealed and smooth, without protruding parts, which enables these robots to be used for the non-destructive inspection of miniature engineering structures such as thin pipe-lines, as well as in medicine.

In study [7], optimal control problems are solved for two types of vibrationally excited mobile systems. First, a three-body articulated system moving in a horizontal plane is considered. The system consists of a rigid body (main body) to which two links are attached by revolute joints. All three bodies interact with the environment with the forces depending on the velocity of motion of these bodies relative to the environment. The system is controlled by highfrequency periodic angular oscillations of the links relative to the main body. This system models the swimming of some animals. The equation of motion of this system is derived and analyzed. Second, a two-body system moving along a horizontal line in a nonlinear resistive medium is considered. One of the bodies (the main body) interacts with the environment and with the other body (internal body), which interacts with the main body but does not interact with the environment. The environment resists the motion of the main body with a force that depends on the velocity of the main body relative to the environment. A periodic optimal motion of the internal body relative to the main body is defined by solving an optimal control problem.

In paper [8] they investigate two systems consisting of a spherical shell rolling without slipping on a plane and a moving rigid body fixed inside the shell by means of two different mechanisms. In the former case the rigid body is attached to the center of the ball on a spherical hinge. They show an isomorphism between the equations of motion for the inner body with those for the ball moving on a smooth plane. In the latter case the rigid body is fixed by means of a nonholonomic hinge. Equations of motion for this system have been obtained and new integrable cases found.

Paper [9] is concerned with the motion of a cubic rigid body (cube) with a rotor, caused by a sudden brake of the rotor, which imparts its angular momentum to the body. This produces an impulsive reaction of the support, leading to a jump or rolling from one face to another. The robot, called by them M-block, is 4 cm in size and uses an internal flywheel mechanism rotating at 20 000 rev/min. Initially the cube rests on a horizontal plane. When the brake is set, the relative rotation slows down, and its energy is imparted to the case.

This paper [10] deals with the problem of a spherical robot propelled by an internal omniwheel platform and rolling without slipping on a plane. The problem of control of spherical robot motion along an arbitrary trajectory is solved within the framework of a kinematic model and a dynamic model. A number of particular cases of motion are identified, and their stability is investigated. An algorithm for constructing elementary maneuvers (gaits) providing the transition from one steady-state motion to another is presented for the dynamic model. A number of experiments have been carried out confirming the adequacy of the proposed kinematic model.

In work [11] they consider the motion of a system consisting of a rigid body and internal movable masses on a rough surface. The possibility of rotation of the system around its center of mass due to the motion of internal movable masses is investigated. To describe the friction between the body and the reference surface, a local Amontons-Coulomb law is selected. To determine the normal stress distribution in the contact area between the body and the surface, a linear dynamically consistent model is used. As examples they consider two configurations of internal masses: a hard horizontal disk and two material points, which move parallel to the longitudinal axis of the body symmetry in the opposite way. Motions of the system are analyzed for selected configurations.

In work [12] the two-dimensional motion of a system consisting of a hollow rigid body, resting on a rough plane, and two internal mobile point masses, capable of moving parallel to the longitudinal axis of symmetry of the body is considered. Friction in the contact area is modelled by a local Amontons–Coulomb law. A dynamically matched linear model is used to describe the normal stress distribution. The possibility of achieving rotation of the system by a certain relative motion of internal masses is investigated. Two control laws for the movable masses are considered: piecewise-linear and harmonic, for which the equations of motion are integrated numerically for different values of the parameters of the control laws.

In our work we consider solid system consisting of a rigid nesting frame with two internal masses in the form of eccentrics (unbalanced massive wheels). Eccentrics rotate with variable angular velocity and have no contact with external medium. Nesting frame has a contact with a rough surface with dry friction and moves short steps (or slides) in plane due to rotating eccentrics.

## 2 EXPERIMENTS

We constructed a model of solid system with two unbalanced wheels in Moscow Institute of Physics and Technology at the Chair of Theoretical Mechanics.

The solid system was designed in SolidWorks program. From a practical point of view, we decided to use open-source building platform (Makeblock, mechanical parts and motors). The result of modeling is presented on Fig.1.

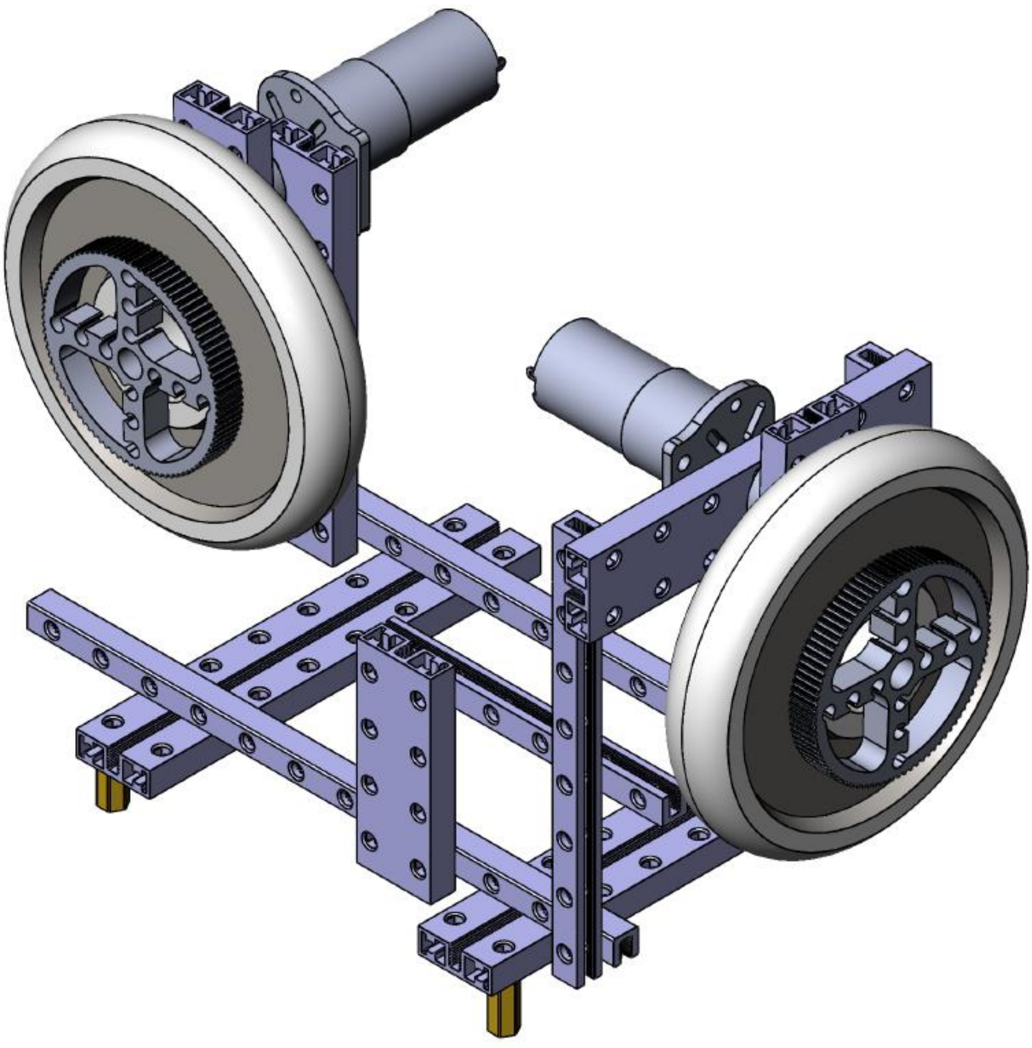

Figure 1: CAD-model of the solid system.

Assembly of the system has led to the result shown in the Fig. 2. Open-source microcontroller (Arduino) sets the velocities of motors rotating two eccentrics. Rigid nesting frame stays on four legs with rounded plastic foots.

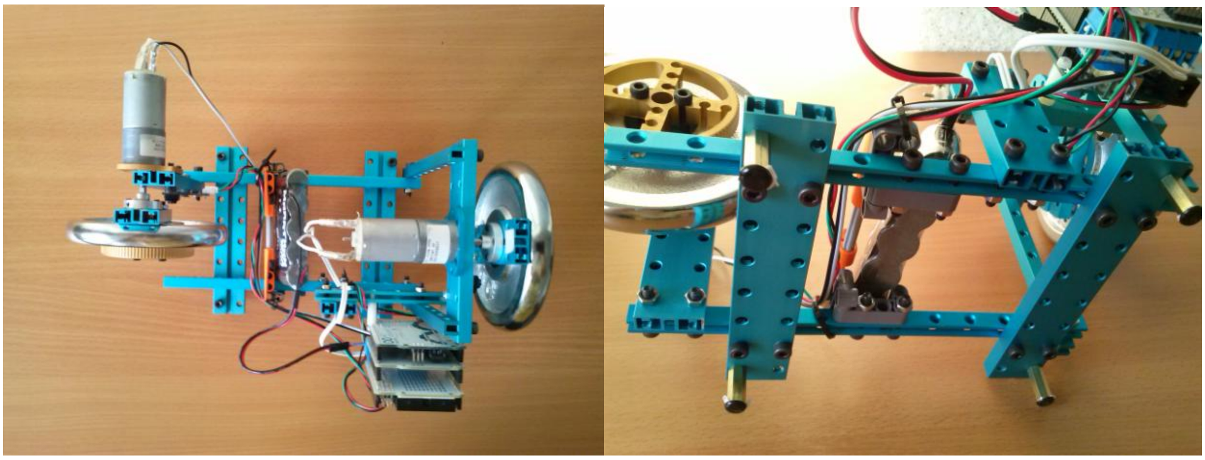

Figure 2: Experimental-model of the solid system (view from above, bottom view).

One of the wheels is placed along the longitudinal axis of symmetry, another one – in transversal position. The main parameters of the model are presented in Table 1.

| Characteristics | Value |
|---|---|
| Total weight | 1.825 kg |
| Weight of wheels | 0.5 kg |
| Average angular speed of the wheels | 125 rpm |
| Eccentricity of longitudinal wheel | 3.5 cm |
| Eccentricity of transversal wheel | 2.5 cm |
| The longitudinal distance between the points of contact with the surface | 13.1 cm |
| The transversal distance between the points of contact with the surface | 11.3 cm |

Table 1: Parameters of the model.

First part of experiment was devoted to testing of the system on different rough surfaces. Characteristics of surfaces are presented in Table 2.

| № | Type | Characterisation | Coef. of friction |
|---|---|---|---|
| 1 | Laminated chipboard 1 | Smooth, with microscopic uniform embossing | 0.31 |
| 2 | Glass | Smooth | 0.52 |
| 3 | Laminated chipboard 2 | Smooth, with the moth-eye pattern "under the tree" | 0.35 |
| 4 | Plywood | Very rough, non-uniform | 0.34 |
| 5 | Plastic | Slightly rough, uniform | 0.32 |

Table 2: Characteristics of surfaces.

Two series of experiments were conducted. First, the longitudinal motion of the body was studied (the transversal wheel does not rotate) for two types of rotation (back and forth). The results are shown in Fig. 3, where all measurements were made for 11 revolutions of the wheel with average angular velocity 125 rpm. In the second series, the rotational motion was experienced (only the transversal wheel is rotating) in similar conditions. The results are presented in Fig. 4.

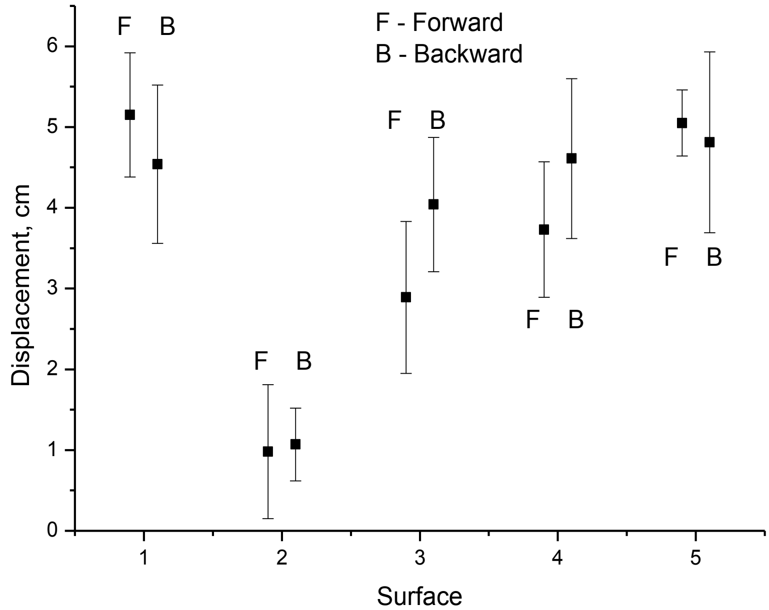


Figure 3: Longitudinal displacement of the body (back and forth) on different surfaces.

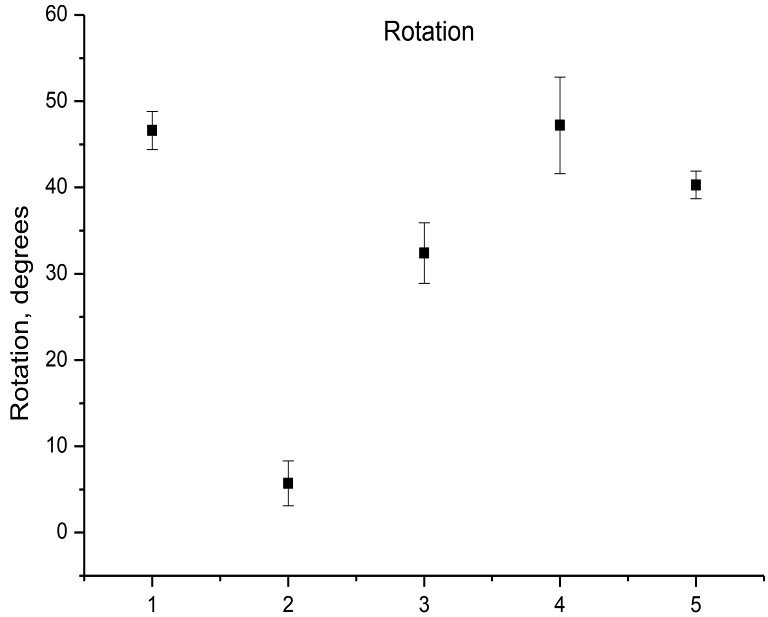


Figure 4: Rotational motion of the body on different surfaces.

It can be seen that both displacement and rotation angle are diminishing with increasing of the coefficient of friction. Note that if this coefficient were zero, no displacement would occur.

In this series of experiments the glass was the most difficult material in terms of repeatability – relative measurement error was the highest compared to other materials. Conversely, the most predictable was the laminated chipboard surface. That is why in the second part of experiments, we decided to use laminated chipboard only.

Second part of experiment was devoted to tracking of the system on the rough surface. The longitudinal motion of the body was studied (the transversal wheel does not rotate) for one type of rotation (forth).

Tracking of labels applied to the system is performed using time-lapse video and Adobe After Effects program. Video frame rate was 60 frames per second. The results of tracking experiments are presented on Fig. 5 (displacement of the system body in meters, time in seconds). Observation of the movement is accomplished by means of contrasting markers on the system components. Recording is done on camera. The resulting video is processed in the software. It uses the standard function "Tracking". In the case of tracking robot position output data are contingent pixels that are converted into millimeters (60 px = 10 mm). To control the time periods and synchronization in the experiment it was used a stopwatch accurate to hundredths of a second.

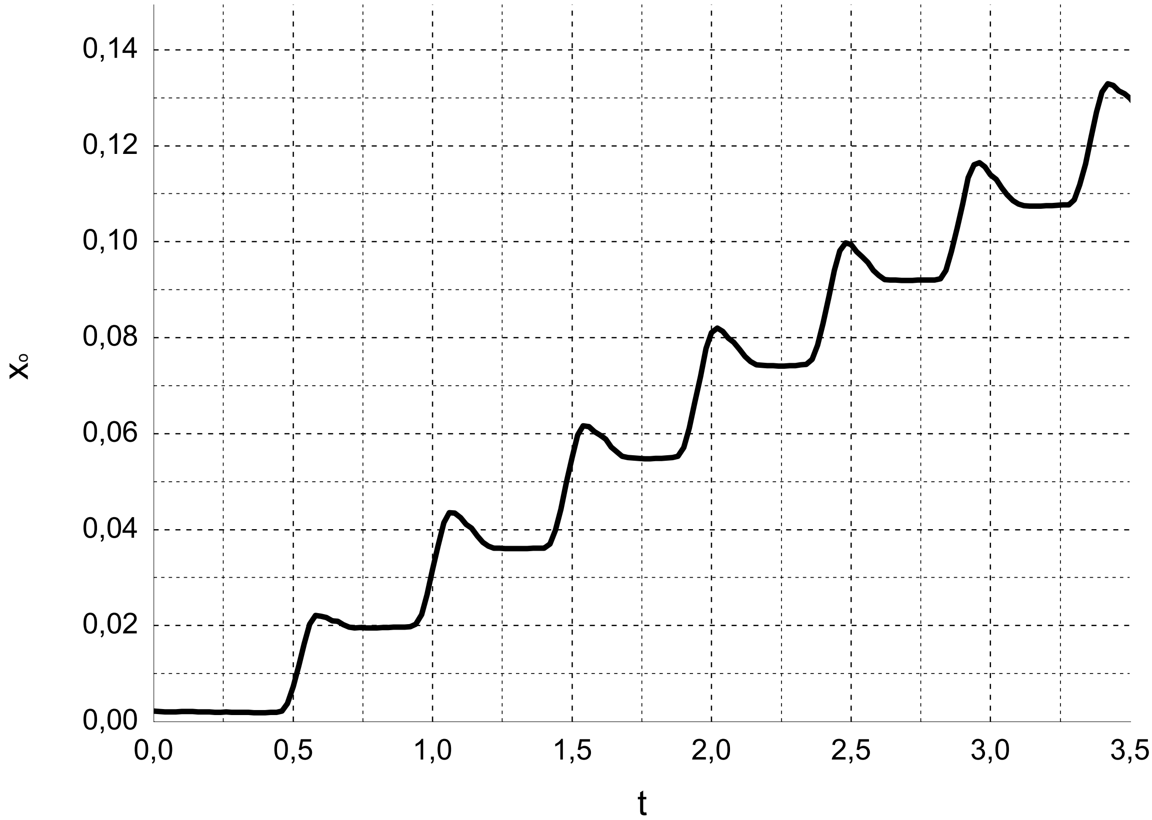


Figure 5: Experimental data obtained using a tracking system in Adobe After Effects program, displacement of body in dependence of time

As we can see, the movement takes place at a constant average speed. Approximately periodic movement occurs sliding forward with some setback.

Such movement can be explained qualitatively as follows. Wheel rotates with variable angular velocity. There are two phases: first, when wheel moves same direction as acceleration of gravity; and the second, vice versa. In first case wheel is accelerated, in second – slows down. Uneven rotational movement of the wheel is converted into a non-uniform translational movement of the base frame, since there are unequal movement pulses differently compensated by friction forces between foots and surface.

The nature of the movement is in good agreement with the results of mathematical modeling, which are presented in the next section.

## 3 MATHEMATICAL MODELING

First consider the simplified system, shown in Fig.6. A hollow case of mass $m$ with flat basement carries a mathematical pendulum of mass $m_1$ and length $a$, suspended at the point $O'$. The external forces include normal reaction $N$, friction force $F$, and moment $M$, applied to the pendulum. Note that while the reaction forces $N$ and $F$ have passive character, the moment $M$ is created by a motor, attached to the case, and ensures the movement of the system.

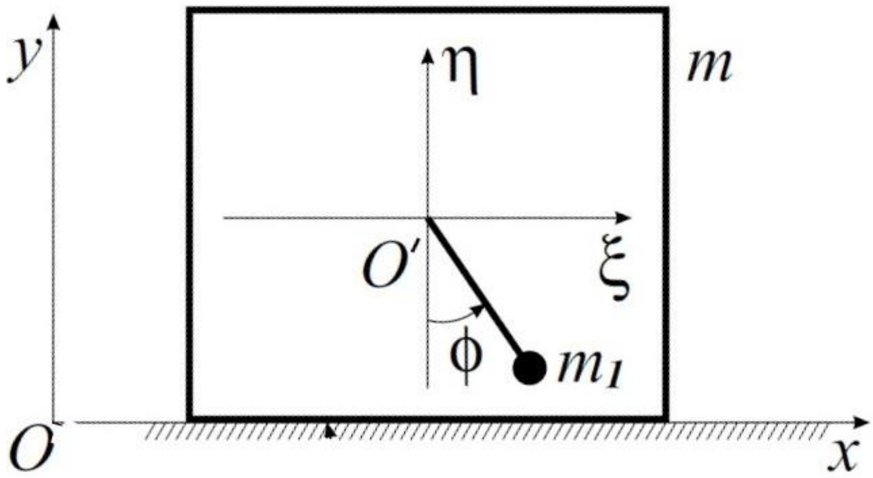


Figure 6: Mechanical model of the system with single unbalanced wheel.

We set coordinates $x$ and $y$ of point $O'$ and angle $\phi$ between the pendulum axle and vertical as generalized coordinates. The equations of motion in the Lagrangian form are:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right)=F,\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right)=N-(m+m_1)g,\quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right)-\frac{\partial T}{\partial \phi}=-\frac{\partial \Pi}{\partial \phi}+M,$$
$$T=\frac{1}{2}(m+m_1)\left(\dot{x}^2+\dot{y}^2\right)+\frac{1}{2}m_1\left(a^2\dot{\phi}^2+2a\dot{x}\dot{\phi}\cos\phi+2a\dot{y}\dot{\phi}\sin\phi\right), \tag{1}$$
$$\Pi=-m_1 ag\cos\phi,\quad F=-\mu N \operatorname{sign}\dot{x}.$$

Since $y=const$, the second equation (1) follows:

$$N=(m+m_1)g+am_1\left(\ddot{\phi}\sin\phi+\dot{\phi}^2\cos\phi\right). \tag{2}$$

A physically plausible form of system (1) is:

$$\left(m+m_1\right)\frac{dv_0}{dt}=F,\quad \frac{d}{dt}(T+\Pi)=F\dot{x}+M\dot{\phi},\quad v_0=\dot{x}+a\xi\dot{\phi}\cos\phi,\quad \xi=\frac{m_1 a}{m+m_1}, \tag{3}$$

where $v_0$ is velocity of the centre of gravity.

We will make the assumption of a decreasing linear relationship between torque of the motor and the angular velocity of its rotation (Fig. 7):

$$M=M_0-k|\omega|,\quad M_0,k=const,\quad M_0,k>0. \tag{4}$$

This kind of dependence usually takes place in direct current motors as in our work.

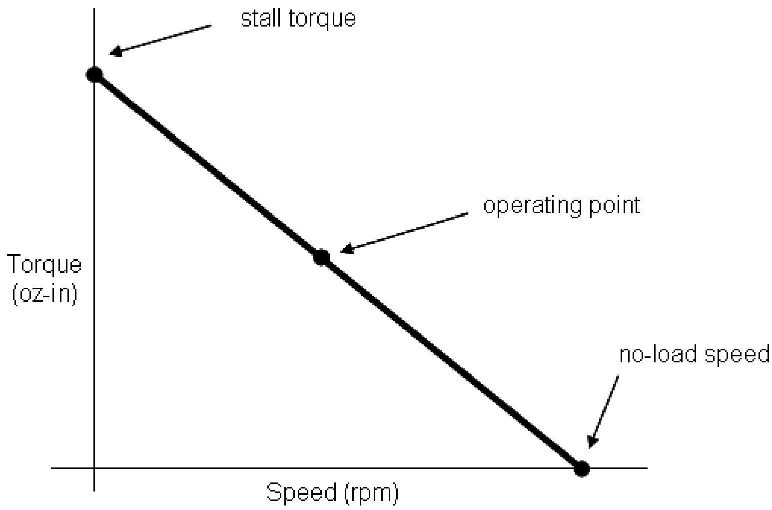


Figure 7: The dependence of the torque of the motor from its angular velocity.

As a result, we obtain the following system of equations for the motion of the platform and the pendulum:

$$
\begin{gathered}
(m+m_1)\dot{\upsilon}+am_1\left(\ddot{\varphi}\cos\varphi-\dot{\varphi}^2\sin\varphi\right)=-\mu N\frac{\upsilon}{|\upsilon|},\\
N=(m+m_1)g+m_1a\left(\ddot{\varphi}\sin\varphi+\dot{\varphi}^2\cos\varphi\right),\\
m_1a^2\ddot{\varphi}+m_1a\dot{\upsilon}\cos\varphi=M_0-k|\dot{\varphi}|-m_1ag\sin\varphi.
\end{gathered}
\tag{5}
$$

The system is represented as (if $\upsilon\neq 0$):

$$
\dot{\upsilon}=\frac{\Delta_1}{\Delta},\quad \ddot{\varphi}=\frac{\Delta_2}{\Delta},
\tag{6}
$$

where:

$$
\begin{gathered}
\Delta=mm_1a^2+a^2m_1^2\sin^2\varphi-\mu a^2m_1^2\sin\varphi\cos\varphi\frac{\upsilon}{|\upsilon|},\\
\Delta_1=\left(-\mu(m+m_1)g\frac{\upsilon}{|\upsilon|}+am_1\dot{\varphi}^2\left(\sin\varphi-\mu\cos\varphi\frac{\upsilon}{|\upsilon|}\right)\right)m_1a^2-\\
-am_1\left(\cos\varphi+\mu\sin\varphi\frac{\upsilon}{|\upsilon|}\right)\left(M_0-k|\dot{\varphi}|-m_1ag\sin\varphi\right),\\
\Delta_2=(m+m_1)\left(M_0-k|\dot{\varphi}|-m_1ag\sin\varphi\right)-\\
-\left(-\mu(m+m_1)g\frac{\upsilon}{|\upsilon|}+am_1\dot{\varphi}^2\left(\sin\varphi-\mu\cos\varphi\frac{\upsilon}{|\upsilon|}\right)\right)m_1a\cos\varphi.
\end{gathered}
\tag{7}
$$

Then the system (6, 7) (taking into account also the case $\upsilon=0$) is solved numerically in MatLab program. Runge–Kutta fourth-order method is used. The results are shown in Fig. 8.

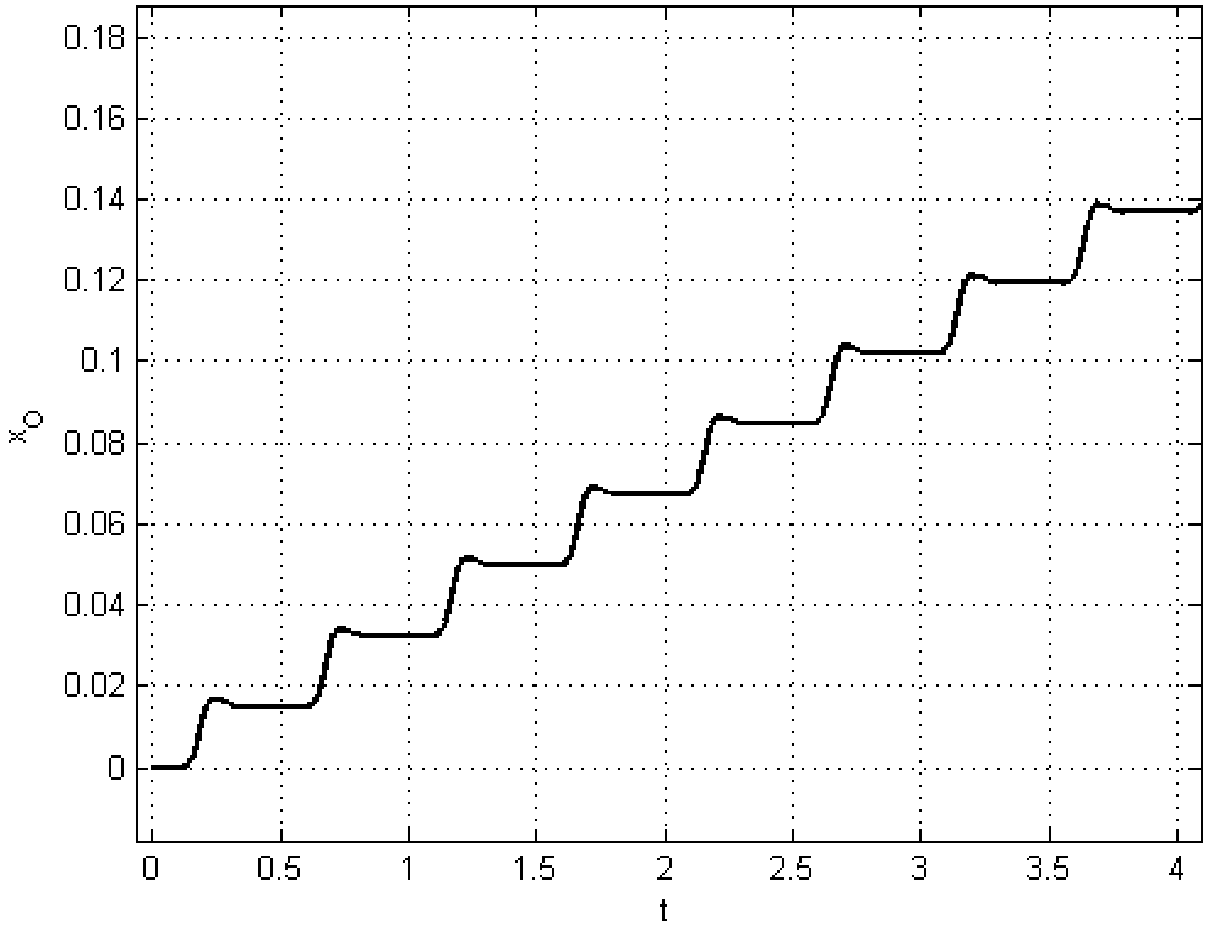


Figure 8: Numerical integration in MatLab, displacement of body in dependence of time.

Graph of body diplacement $x$ (meters) from the time $t$ (seconds) in Fig. 8 is obtained by numerical integration at the following conditions (Table 3):

| Parameter | Value |
|---|---|
| The reduced length of the pendulum | 0.06 m |
| Body weight (without pendulum) | 1.325 kg |
| The pendulum weight | 0.5 kg |
| The coefficient of friction | 0.3 |
| Acceleration of gravity | 9.8 m/s$^2$ |
| $M_0$ | 0.341 N·m |
| k | 0.020 N·m·s |

Table 3: Parameters in numerical integration.

Though this model can move, it has visible shortcomings. First of all, the rectilinear trajectory is unstable: due to unaccounted disbalances the body will deviate. Obviously, in practice, it is impossible to attach a cargo to the body, keeping ideal symmetry. Further, the practical needs are not restricted by rectilinear motion: usually mobile devises move along a prescribed curve. Our aim is to create a robot which could perform stable rectilinear or curvilinear motion. The key problem is to let the body move more accurately and to adapt to the surfaces with different coefficients of friction. This problem can be described in next paper.

## 4 CONCLUSIONS

- The motion of body with rotating internal wheels in interaction with external environment in form of rough surface can be used as the principle for the motion of mobile robots.
- Robotic mechanisms of this class, being isolated from external environment in shell, can be used in extreme conditions, with strong pollution of surrounding space, in aggressiveness of environment (for example, on another planet or in chemically active environment).
- The movement of the system is studied with the help of mathematical modeling and numerical integration and is compared with experimental data obtained by tracking method.

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