ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

COMBINING FINITE ELEMENT ANALYSIS AND ANALYTICAL MODELLING FOR EFFICIENT SIMULATIONS OF NON-LINEAR GEAR DYNAMICS

Shadi Shweiki[1], Jakub Korta[1], Antonio Palermo[1], Rocco Adduci[1] and Domenico Mundo[1]

[1] Department of Mechanical, Energy and Management Engineering, Università della Calabria
Cubo 45C, 87036, Rende, Italy
{shadi.shweiki, jakub.korta, antonio.palermo, domenico.mundo}@unical.it

Keywords: gear dynamics, meshing stiffness, 1D modelling, finite element simulation

Abstract. *Although gears have been used in mechanical industry for a long time and first systematic research activities, aimed at understanding gear meshing, started in the '50s, still nowadays a lack of accurate description for several physical phenomena represents a limitation for transmission design. One of the main causes of such limitations is the three-dimensional (3D), local and non-linear nature of contact problems. Given the complexity of these problems, a variety of modelling techniques with different levels of fidelity and required computational effort have been set up. Among the available methodologies, analytical modelling, which has been developed for a vast variety of applications from the early days of gear dynamics research, still attracts the interest of researchers thanks to the low computational requirements and to the capability to efficiently describe specific phenomena.*
Among analytical models, one-dimensional (1D) description aims at studying the torsional gear vibrations around the rotational axes and can be used to simulate either gear whine or gear rattle phenomena.
The aim of this paper is to illustrate a numerical approach, based on 3D (Finite Element) FE simulations, to estimate the mass and inertia properties of a 1D gear pair model. The proposed approach is based on pre-strained modal analyses, carried out on the 3D FE model of a pair of meshing gears, from which the variable meshing stiffness and modal mass values in a given system configuration are derived and used to calibrate the equivalent 1D model. An application example of the proposed method is provided by analyzing a pair of identical spur gears, for which the 1D model is created and used to estimate the dynamic TE at different rotation speeds and under constant external load in steady-state conditions.

1 INTRODUCTION

In the field of mechanical transmissions, simulation tools are acquiring ever more importance in the industrial development process that aims at achieving products of high quality at affordable costs, as to stand the competitive pressure of globalized markets. The dynamic behavior of gears has a direct influence on the performance of the entire system in terms of acoustic emissions and durability. The recent trend for lightweight design stresses even more the necessity to include a proper dynamic behavior as one of the main design targets. The gears themselves bring different challenging problems and difficulties at the design stage [1]. Typical non-linear phenomena [2] due to contact stiffness and clearance has to be well predicted in the initial design phase in order to reduce the development time and increase product quality. Given the complexity of these phenomena, a variety of modelling techniques with different levels of fidelity and required computational effort have been developed in the last decades. Three are the main model categories which can be identified: analytical, Finite Element (FE) and Multibody. Analytical models have been developed for a vast variety of applications[3-6]. They started from the early days of gear dynamics research, with a single gear pair and a single degree of freedom (DoF), and evolved until today's more refined 3D formulations. Ongoing research is still exploring analytical models since they provide low computational requirements, thanks to their lumped parameter formulation, and can be tailored to efficiently describe specific applications.

For these reasons, analytical formulations can be usefully employed, especially for analyses involving long operational time, where the simulation time and cost required by more detailed simulations would be prohibitively high. To achieve an analytical formulation of gear meshing, a one-dimensional (1D) description can be used with the aim of studying the torsional gear vibrations around the rotational. Such a simplified representation can be used to simulate either gear whine or gear rattle phenomena. Rotating masses are lumped into inertia moments and associated to the related rotational DoFs. Connectivity between the DoFs can be set by means of translational spring-damper elements by expressing rotations as equivalent displacements along the line of action, which represents the so called Transmission Error (TE) between the meshing gears. A 1D analytical model, which considers both clearance-type nonlinearity due to backlash and angle-varying mesh stiffness was proposed by Blankenship and Kahraman [3] to address the contact loss phenomenon in steady-state forced response.

When a 1D modeling approach for gear meshing analysis is used, simulation results can be found in a significantly shorter simulation time, as compared to the detailed FE simulations, but a higher approximation level in the description of the dynamic behavior of the engaging gears is paid. This means that, in order to use in a proficient way an analytical model, an a-priori knowledge of the system is required to prevent from neglecting important effects [7] or wasting efforts in modelling irrelevant phenomena.

The dynamic behavior of two engaging gears is strongly related to the variable meshing stiffness of the teeth in contact[8, 9]: a poorly approximated representation of the meshing stiffness in the simulation model causes poor results in terms of accurate prediction of the real behavior. In particular, an inaccurate representation of the meshing stiffness can result in a poor prediction of the dynamic response excited by orders coexisting with the fundamental meshing order in the system.

The purpose of this works is not to develop new analytical models but to present an alternative approach to improve the predictive accuracy of an existing 1D model without increasing its computational cost. From the simulation point of view, FE formulations proved a reliable tool in predicting the static and dynamic behavior of mechanical system in a detailed manner, at the cost of high computational times. For this reason, the idea behind this work is to combine 3D FE modeling with a one DoF analytical model for dynamic simulation of gear meshing. This way, computationally demanding FE simulations are used in a model preparation phase to derive the gear pair meshing stiffness and resonance frequencies with high accuracy for a discrete number of different positions of the gears along the meshing cycle. A modal mass value is also then derived from the previous estimated quantities for each analyzed position along the meshing cycle. Model data derived from FE simulation is then used to define the parameters (equivalent inertia and instantaneous meshing stiffness) defining the 1D analytical model. The resulting 1D model is then used to simulate the dynamic behavior of a pair of identical gears in different operating conditions.

2 ANALYTICAL MODEL

One of the simplest models that allow predicting the dynamic torsional behavior of a pair of cylindrical gears is proposed in [3] and shown in Figure 1.

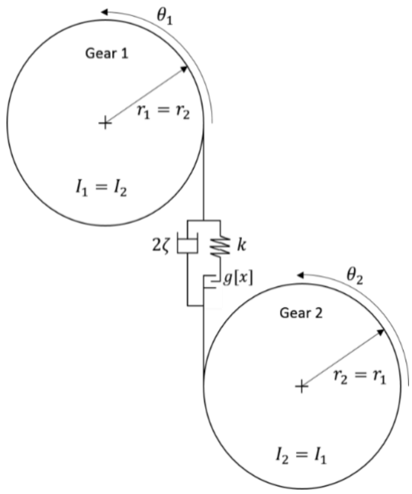


Figure 1: Dynamic representation of the 1D model of meshing cylindrical gears.

The dynamic behavior of this one DoF system, which considers a pair of identical gears, is ruled by the following equation of motion:

$$\ddot{x} + 2\zeta\dot{x} + k[POS(t)]g[x(t)] = F(t) \tag{1}$$

where x is the linear relative displacement of the contact points on the two meshing teeth along the line of action and represents the dynamic Transmission Error (*DTE*) in the system, $\dot{x}$ and $\ddot{x}$ are its first and second time derivatives; ζ is a damping coefficient introduced in the model in order to take into account friction losses between the teeth; the model parameter $k[POS(t)]$ is intended to represent the variable meshing stiffness as function of the instantaneous position of the teeth in the meshing cycle *POS(t)*, which is computed *a priori* with non-linear FE simulations and stored in a mono dimensional look-up table with the corresponding position in the meshing cycle[10, 11].

With this formulation it is also possible to model the backlash b between the gears introducing the restoring function $g[x(t)]$ which is the following function of the relative displacement between the teeth:

$$g[x(t)] = \begin{cases} x(t) - b, & x(t) \ > b, \\ 0, & |x(t)| \leq b, \\ x(t) + b, & x(t) \ < -b. \end{cases} \tag{2}$$

The restoring function acts as a switch for the meshing stiffness, by bringing its value to zero when the relative displacement is below the backlash threshold. This situation means that there is no contact between the teeth and consequently no elastic meshing force. When the instantaneous relative displacement assumes a value higher than the imposed backlash, the restoring function allows to consider only the effective penetration between the teeth in contact, subtracting the backlash from the estimated relative displacement.
The external force $F(t)$ in equation (1) represents the loading force in the system. In general, a time-varying load can be considered in the model, but it is considered to be constant in the work presented in this paper, based on the assumption that the imposed value of torque transmitted by the gears is constant.
The instantaneous acceleration value $\ddot{x}$ is computed numerically by using a Runge-Kutta algorithm, implemented in a Simulink model, that solves the following equation:

$$F - (kg + c\dot{x}) = M_{eq}\ddot{x} \tag{2}$$

The relative gear velocity and displacement are obtained by integrating acceleration and velocity respectively. The terms on the left side of equation 2 represent the force contribution due to external excitation, elastic force and damping force. The values of the last two contributions are calculated at each time step by multiplying the relative speed and penetration by the damping coefficient, assumed constant, and by the actual value of meshing stiffness respectively, which instead is dependent on the actual position of the gears along the meshing cycle. The latter is calculated, along with the equivalent mass M_{eq}, through a series of FE dynamic analyses of the two gears in different positions along the meshing cycle, as it will be illustrated in section 3.

3 FE MODEL

In order to enable the use of the analytical model described in section 2 and illustrated in Figure 1, the values of model parameters, such as meshing stiffness, damping coefficient and equivalent mass must be determined before the simulation starts.
The approach proposed in this work to obtain an estimate of these parameters is based on detailed static and dynamic FE simulations[12].
As illustration example, a simple mechanical transmission, consisting of a pair of identical gears already analyzed in [3] and with main geometric properties listed in table 1, is analyzed.

Parameter	Value
Normal module	3 mm
Normal pressure angle	20 deg
Pitch diameter	150 mm
Root diameter	142.5 mm
Facewidth	25.4 mm

Table 1: Gears specifications.

The proposed approach is based on a pre-processing phase in which the variable stiffness, and equivalent mass are computed from FE simulations, for different positions along the meshing stiffness. The 3D gear geometry was generated according to the geometrical parameters listed in table 1 and discretized with hexagonal elements for the FE analyses [13], as shown in figure 2. Data needed for 1D simulation are calculated using the pre-strained modal approach, carried out on a series of models, which represent the gears in discrete angular positions along the meshing cycle. In every position, a two-step FE simulation is carried out, starting from calculating teeth static deflection resulting from the application of a 300 Nm torque on the driving gear and by fixing all the DoFs of the center of rotation of the driven gear. This way, the static deflection of the teeth for different gear pair configurations is obtained, which is later used in combination with the results of a modal analysis, aimed at calculating the fundamental rotational resonant frequency of the system, to estimate the 1D model parameters.

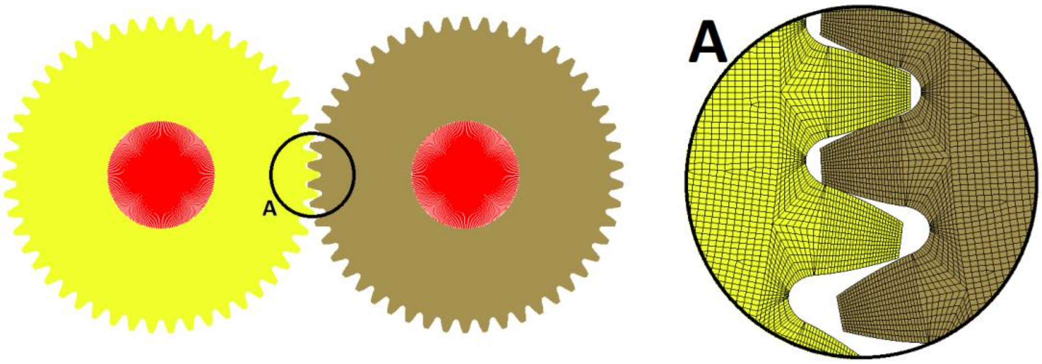


Figure 2: FE model of the analyzed gear pair, used to derive parameter values to be used in the 1D model.

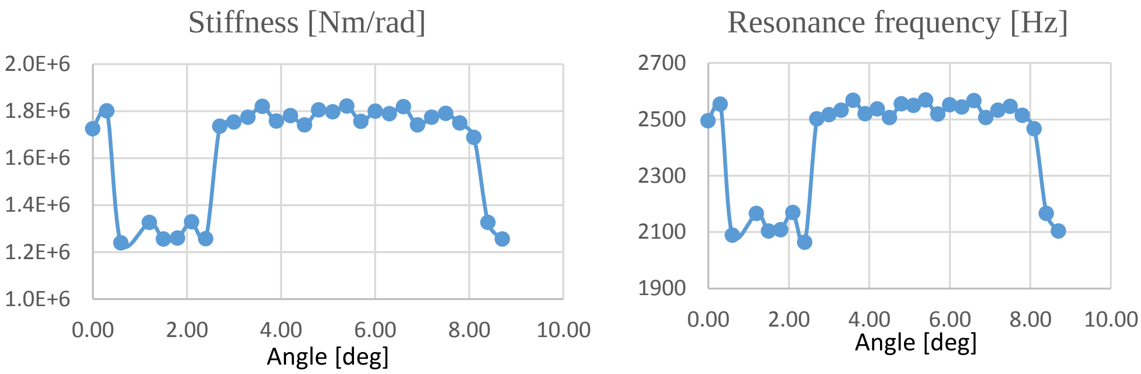
The rotational (torsional) stiffness of the 1D model is calculated according as:

$$k_{\emptyset} = \frac{T}{\emptyset} \tag{3}$$

where T is the applied torque and $\emptyset$ the angular displacement of the statically loaded gear along the rotational axis. Another parameter derived from the simulation results is the equivalent mass, which is calculated as modal mass in the 1D model by using the following equation:

$$M_{eq} = \frac{k_{\emptyset}}{\omega^2} \tag{4}$$

where ω^2 is the square of fundamental torsional frequency estimated for the 3D model. Using this expression, M_{eq} and $k_{\emptyset}$ have been computed, which are used as model parameter data for the 1D model at the later stage. Figure 3 shows the calculated rotational stiffness and first resonant frequency estimated for the gear pair model in different angular positions along the meshing cycle. From these simulation results, it appears clear that the static and dynamic characteristics of a meshing gear pair depend very strongly on the instantaneous configuration. Figure 4 shows the first mode shape of the gears in two different angular positions, in which one and two teeth pairs are in contact. The figure shows also the frequency values of the first resonance in the two configurations, which were identified for these models through a pre-strained modal analysis. It can be seen that the difference is as high as 400 Hz, which is about 20% of the lower value.



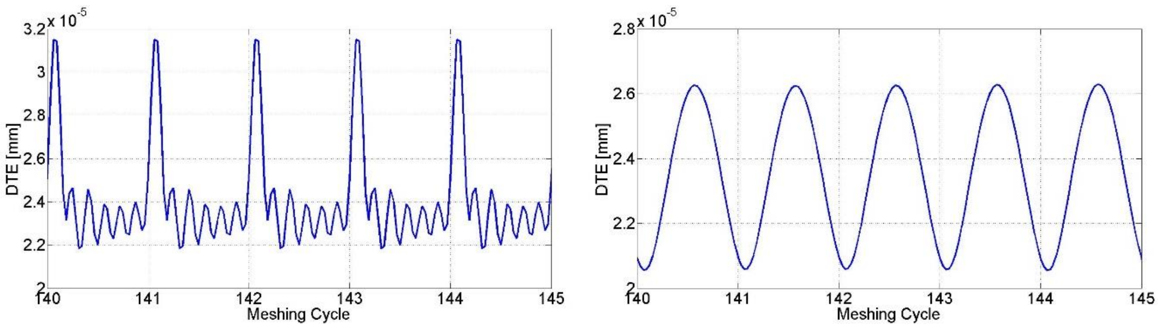
a b

Figure 3. Rotational stiffness (a) and fundamental frequency (b) values for the analyzed gear pair.

Figure 4. First resonant frequency and mode shape for a gear pair in two different angular configurations

4 RESULTS

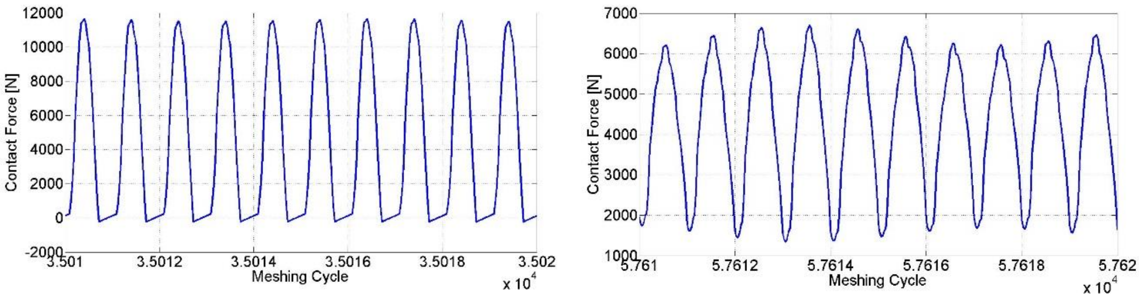
Once the 1D model parameters are calculated, different analyses can be carried out by solving equation 2 and predict the dynamic behavior of the gear pair. In this paper we show the results of analyses that have been performed by operating the input gear with a constant speed until steady state was obtained. This process was repeated for different increasing and decreasing rotational velocities. Figure 5 shows a comparison between the DTE computed for an angular velocity of 600 rpm (a) and of 4200 rpm (b). The different shape of the two curves proves that this model is able to catch dynamic effects of gear meshing on DTE, which are not relevant at low rotational speed. The figure shows also that when the meshing frequency is close to the resonant frequency of the system, as in the case of figure 5-b, the DTE signal is dominated by the meshing order. In this case the first order of the rotational speed corresponds to the natural frequency of the gear pair.



a b

Figure 5. Dynamic transmission error (DTE) for different angular speeds: 600 rpm (a) and 4200 rpm (b)

Effects of system non-linearity due to backlash between the meshing teeth is visible in figure 6, where the contact force along different meshing cycle is plotted. Contact losses between the teeth are shown by values of the meshing force close to 0 in figure 6-a, where elastic force is imposed to be zero by to the restoring function, while the damping force is still acting. Figure 6-b shows the trend of the contact force when no contact loss occurs, and minimum values are far away from zero.



a b

Figure 6: Dynamic meshing force for in case of contact loss a) and without contact loss b)

Other effects that can be analyzed using this proposed simulation approach are the jump phenomena in the RMS value of the computed DTE, due to the clearance type non-linearity (Figure 7).

Due to the backlash between the gear teeth, different branches of RMS curves for the computed DTE are obtained when the rotational velocity changes, corresponding to different states of the system. The lower branch is obtained for slightly increasing speed while the upper branch is obtained for decreasing speed.

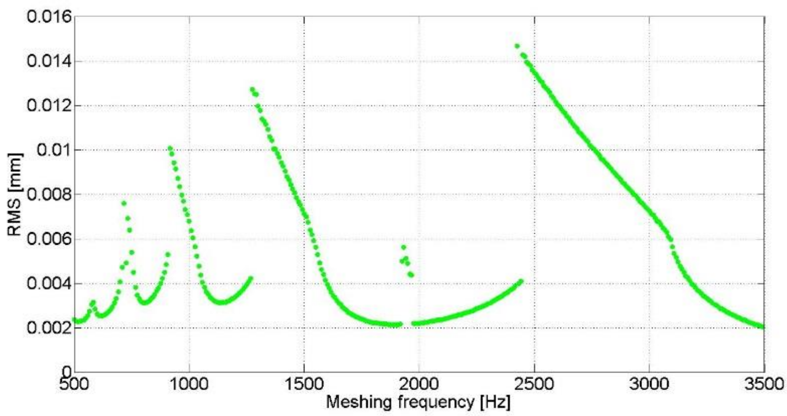


Figure 7: RMS of DTE plotted against meshing frequency

By considering the curves reported in figure 7 it is possible to recognize an almost-linear trend in the regions distant from the jumps frequency. The onset of contact loss causes a knee in the curve, recognizable as a softening effect compared to the almost-linear trend, which is a well-known behavior described in literature [3].

5 CONCLUSIONS

This work shows the possibility to use analytical model to describe complex phenomena related to gear dynamics. In the proposed approach the lack of accuracy of analytical and semi-analytical meshing stiffness formulations is covered by the use of FE

simulations which allow to have a realistic description of the meshing process. A high computational efficiency is reached by limiting the FE simulations in a preparation phase. The simplicity of the analytical formulation makes this model suitable for computationally efficient analyses, in those cases where the numerical complexity of FE-based models requires unaffordable computational time.
Classical jump-up phenomena are predicted by the model and agree with the classical trend shown in literature.
The 1D model described in this work takes into account only the relative rotation between the two gears bodies, under the assumption that the other relative displacements can be neglected during the simulation. The flexibility of the proposed approach for FE-based estimation of model parameters makes it applicable also to more complex analytical models for gear dynamic simulations, which allow taking into account additional relative displacements between the gears.

6 ACKNOWLEDGMENT

The research activities described in this paper were funded within the People Programme (Marie Curie Actions) of the 7th Framework Programme of the European Union FP7/2007-2013, under the contract with the Research Executive Agency (REA) n. 324336 and the related research project DEMETRA Design of Mechanical Transmissions: Efficiency, Noise and Durability Optimization (www.fp7demetra.eu).

7 REFERENCES

[1] S. Shweiki, A. Palermo, A. Toso, D. Mundo, and W. Desmet, "Effects of center distance and microgeometry on the dynamic behaviour of a spur gear pair," in Proceedings of the International Conference on Engineering Vibration (ICOEV 2015) September 7–10, 2015, Ljubljana, Slovenia, 2015, pp. 119–130.
[2] K. L. Johnson and K. L. Johnson, Contact mechanics. Cambridge university press, 1987.
[3] G. W. Blankenship, A. Kahraman, Steady state forced response of a mechanical oscillator with combined parametric excitation and clearance type non-linearity, Journal of Sound and Vibration, 185, 5 (1995) 743-765.
[4] H. N. Ozguven and D. R. Houser, "Dynamic analysis of high speed gears by using loaded static transmission error," Journal of Sound and Vibration, vol. 125.1, 1988, pp. 71–83, ISSN: 0022-460X.
[5] G. Liu and R. G. Parker, "Dynamic modeling and analysis of tooth profile modification for multimesh gear vibration," Journal of Mechanical Design, vol. 130, no. 12, 2008, p. 121402.
[6] D. R. Houser, J. Harianto, and Y. Ueda, "Determining the source of gear whine noise,"Gear Solutions, vol. 1623, 2004, pp. 17 – 22.
[7] J. D. Smith, Gear noise and vibration. CRC Press, 2003.
[8] R. G. Parker, S. M. Vijayakar, and T. Imajo, "Non-linear dynamic response of a spur gear pair: modelling and experimental comparisons," Journal of sound and vibration, vol. 237, no. 3, 2000, pp. 435–455.
[9] V. K. Tamminana, A. Kahraman, and S. Vijayakar, "A study of the relationship between the dynamic factors and the dynamic transmission error of spur gear pairs," Journal of Mechanical Design, vol. 129, 2007,pp. 75–84.

[10] A. Palermo, D. Mundo, R. Hadjit, and W. Desmet, “Multibody element for spur and helical gear meshing based on detailed three-dimensional contact calculations,” Mechanism and Machine Theory, vol. 62, 2013, pp. 13–30.
[11] A. Palermo, D. Mundo, R. Hadjit, and Desmet, “Effects of variable loads and misalignments on gear noise and vibration through multibody simulation based on transmission error,” in Proceedings of the International Conference on Noise and Vibration Engineering (ISMA2012) and 4th International Conference on Uncertainty in Structural Dynamics (USD2012) September 17–19, 2012, Leuven, Belgium, 2012, pp. 3995–4006.
[12] R. Version, “5.1, radioss theory manual, altair engineering,” Inc., Troy,MI, 2008.
[13] F. L. Litvin and F. A., Eds., Gear geometry and applied theory.Cambridge University Press, 2004, ISBN: 0-521-81517-7.