ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

NON-REGULAR VEHICLE DYNAMICS. APPLICATION TO COLLISION

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Keywords: Non-Regular, Dynamic, Vehicle, Linear Model, Holding Contact, Collision, Fixed Obstacle.

Abstract. *The principal aim of this paper is to evaluate the non-regular dynamic behavior of a vehicle involved in a collision with a fixed obstacle. Firstly, the model used for the wheels/ground interaction is the linear model. In addition, we established the constraints equations relating to holding contact with the ground and those concerning conditions of Ackermann. The obtained results had been compared with those given by the Pc-Crash software. We noted a good correlation for various scenarios. After validation, we used this model in the case of collision vehicle/fixed obstacle. A comparison of the simulation and experimental results confirmed the good agreement between the model and the real system of Crash-Test.*

1 INTRODUCTION

The modeling and simulation of the dynamic behavior of vehicle today are a scientific theme that has progressed rapidly under the impetus of vehicles industries. Thanks to their potential, this discipline has become an indispensable tool for improving the performance of vehicle.
However, in some situations, particularly those characterized by a non-regular vehicles dynamics (rollover, crash, sudden change in certain parameters of the roadway, demining operation by an intervention vehicle ...), the equations governing the movement of system lead to differential and algebraic equations are difficult to solve.

Most mechanical systems subjected to non-permanent excitations can be modeled by non-smooth dynamical systems. This non-regularity can originate in the batch-term control, in the interaction of the environment with the system and in problems of sudden change in system parameters. This loss of regularity, in the late 1970s, Michelle Schatzman [1] and Jean Jacques Moreau [2] started researching a mathematically consistent formulation for modeling the dynamic behavior of non-regular mechanisms.

In the case of vehicle, the non-regular dynamic situations can be classified into three categories:

i) Non-regular dynamic situations caused by discontinuities of control parameters (sudden change of steering angle, sudden braking action...).

ii) Non-regular dynamic situations caused by discontinuities in the system/environment interaction (collision vehicle/fixed barrier, collision vehicle/vehicle, rollover, skidding, ground roughness, slope change in a way, sudden change in the parameters road…).

iii) Non-regular dynamic situations caused by discontinuities of some parameters of the vehicle (sudden change of the damping coefficient or stiffness of the suspension system ...).

The objective of this work is the development of a comprehensive approach to dynamic behavior modeling of the vehicle taking into account the problems of irregular dynamics in collision between the vehicle and a fixed obstacle. In addition, the validation of the results of this approach compared with those given by appropriate software.

2 VEHICLE DYNAMIC MODEL

2.1 Geometric description of the system

In the present study, the structure of the considered mobile robot is composed of a vehicle (Figure 1) which is supposed rigid body fixed to the chassis and suspended via the suspension system composed of a spring and damper (*ki* is the stiffness and *bi* is the damping coefficient). Unsuspended masses are negligible compared to the suspended masse. In addition, the caster, toe and tilt angles of the kingpin are not considered. The anti-roll bar is not modeled. The braking is applied to all wheels and the load transfer is considered.

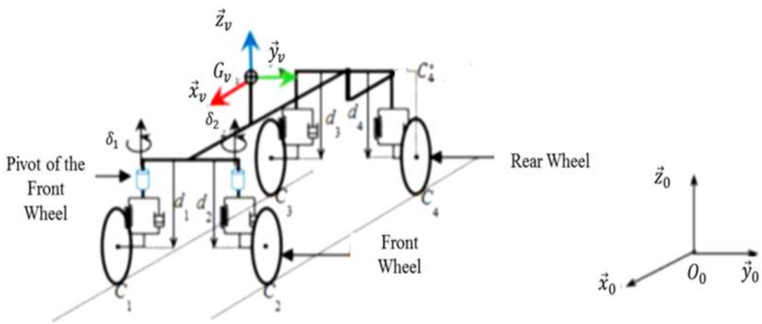


Figure 1: Vehicle model.

The system is moving on a road to which is attached a frame $\Re_0 = \left(O_0, \vec{x}_0, \vec{y}_0, \vec{z}_0\right)$. Let $\Re_v = \left(G_v, \vec{x}_v, \vec{y}_v, \vec{z}_v\right)$ be the mobile frame linked to the robot chassis, G_v the original center of gravity of the robot. The axis $\vec{x}_v$ is perpendicular to the plane of longitudinal symmetry of the vehicle, facing the left side, the axis is perpendicular to $\vec{y}_v$ and directed towards the front of the vehicle.

The configuration of the vehicle frame versus the fixed reference $\Re_0$ is defined by six parameters: three cartesian coordinates (x, y, z) of the center of gravity and the three angles (φ_1: roll, φ_2: pitching and φ_3: yaw) orientation of the $\Re_v$ with respect to reference $\Re_0$. The rotation matrix of $\Re_v$ relatively to $\Re_0$ is:

$$ {}^0P_v = \begin{bmatrix} c_2c_3 & s_1s_2c_3 - c_1s_3 & c_1s_2c_3 + s_1s_3 \\ c_2s_3 & s_1s_2s_3 + c_1c_3 & c_1s_2s_3 - s_1s_3 \\ -s_2 & s_1c_2 & c_1c_2 \end{bmatrix} \quad (1) $$

With: $c_i = \cos(\varphi_i), s_i = \sin(\varphi_i), i = 1..3$.

The kinematic wrenches of the vehicle frame relative to the reference, expressed in the vehicle frame are given by the following relationships:

$$ {}^v\vec{V}_{Gv} = {}^vP_0 \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} \text{ and } {}^v\vec{w}_v = \begin{pmatrix} w_{xv} \\ w_{yv} \\ w_{zv} \end{pmatrix} = \begin{bmatrix} 1 & 0 & -s_2 \\ 0 & c_1 & s_1c_2 \\ 0 & -s_1 & c_1c_2 \end{bmatrix} \begin{pmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{pmatrix} \quad (2) $$

In order to take into account the efforts of wheels/ground interaction, we attach to each wheel, a frame $\Re_i = \left(C_i, \vec{e}_{xi}, \vec{e}_{yi}, \vec{e}_{zi}\right)$ placed at the wheel/ground contact point. The unit vectors of $\Re_i$ are defined, according to the normal $\vec{n}_i$ of the wheel i contact with the ground plane and its steering angle, by the following expressions:

$$For\ \ i=1...4\quad \left\{ \begin{array}{c} {}^{0}\vec{e}_{zi} = {}^{0}\vec{n}_{i} \\ {}^{0}\vec{e}_{xi} = \dfrac{({}^{0}P_{v}\cdot {}^{v}\vec{v}_{i}) \wedge {}^{0}\vec{e}_{zi}}{\left|({}^{0}P_{v}\cdot {}^{v}\vec{v}_{i}) \wedge {}^{0}\vec{e}_{zi}\right|} \\ {}^{0}\vec{e}_{yi} = {}^{0}\vec{e}_{zi} \wedge {}^{0}\vec{e}_{xi} \end{array} \right\} \tag{3}$$

With $\vec{n}_i$ the vector normal to the wheel plane. These vectors are defined by ${}^{v}\vec{v}_i = \begin{pmatrix} -\sin\delta_i & \cos\delta_i & 0 \end{pmatrix}^T$ for the front wheels of the vehicle and ${}^{v}\vec{v}_i = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$ for the other wheels.

The transition matrix of the wheel/ground interaction frame ($\Re_i/\Re_0$) is defined by the following relationship:

$${}^{0}P_i = \left[{}^{0}\vec{e}_{xi} \mid {}^{0}\vec{e}_{yi} \mid {}^{0}\vec{e}_{zi} \right] \quad for\ \ i=1...4 \tag{4}$$

In this, the elongations of the springs are modeled by the variables d_i, for i = 1...4. The vector of the generalized coordinates $\vec{q}$ contains the kinematic variables of translation and orientation of the vehicle, the variables characterizing the elongation of the springs and the two front wheels steering angles δ_1 and δ_2. The vector $\vec{q}$ is thus defined as follows:

$$\vec{q} = \left(x, y, z, \varphi_1, \varphi_2, \varphi_3, d_1, d_2, d_3, d_4, \delta_1, \delta_2\right)$$

2.2 Constraints equations

The constraints equations of a vehicle are defined by the conditions linked to Ackermann steering system and the wheels/ground holding contact conditions of the vehicle. They are developed bellows.

2.2.1 Ackermann Conditions

During the movement of the vehicle along a curve, the steering angles are linked to the existing of yaw speed w_{zv} and the radius of curvature of this curve by the following equations:

$$\begin{array}{l} if\ \ w_{zv} = 0 \Rightarrow \left\{ \begin{array}{l} \delta_1 = 0 \\ \delta_2 = 0 \end{array} \right. \\ if\ \ w_{zv} \neq 0 \Rightarrow \left\{ \begin{array}{l} \delta_1 = \tan^{-1}\left(\dfrac{L_v}{R + L_w/2} \right) \\ \delta_2 = \tan^{-1}\left(\dfrac{L_v}{R - L_w/2} \right) \end{array} \right. \end{array} \tag{5}$$

With: L_v : vehicle wheelbase; L_w : rear track; $R = V_{xv}/w_{zv}$ The curvature radius calculated on the rear track of the robot center and V_{xv} : longitudinal velocity.

2.2.2 Wheels/ground holding contact conditions (WGHCC)

WGHCCs of a vehicle are defined by the following algebraic equations:

$$\left({}^{0}\overrightarrow{O_0C_i} - {}^{0}\overrightarrow{O_0G_v} - {}^{0}\overrightarrow{G_vC_i}^{*} + r_w\,{}^{0}\vec{n}_i\right)^{0}\vec{z}_v + d_i = 0 \quad for\ i = 1...4 \tag{6}$$

With r_W: the radius of the wheels.
The conditions (6), derived twice with respect to time, can be expressed as follows:

$$\left[J_v\right]\cdot\vec{\ddot{X}} + \overrightarrow{B_{cv}} = \vec{0} \tag{7}$$

With: $\left[J_v\right]$ is the Jacobean matrix inherent to the **WGHCC** of the vehicle;

$\overrightarrow{B_{cv}}$ are the acceleration constraints vector of **WGHCC** of the vehicle.

$$\vec{\ddot{X}} = \left(\ddot{x}, \ddot{y}, \ddot{z}, \dot{w}_{xv}, \dot{w}_{yv}, \dot{w}_{zv}, \ddot{d}_1, \ddot{d}_2, \ddot{d}_3, \ddot{d}_4\right)^T \tag{8}$$

$$\left[J_v\right] = \begin{bmatrix} {}^{0}\vec{n}_1^{\,T} & -{}^{0}\vec{n}_1^{\,T}\left({}^{0}P_v\left[*\,{}^{v}\overrightarrow{G_vC_1}\right]\right) & -{}^{0}\vec{n}_1^{\,T}\,{}^{0}\vec{z}_v & 0 & 0 & 0 \\ {}^{0}\vec{n}_2^{\,T} & -{}^{0}\vec{n}_2^{\,T}\left({}^{0}P_v\left[*\,{}^{v}\overrightarrow{G_vC_2}\right]\right) & 0 & -{}^{0}\vec{n}_2^{\,T}\,{}^{0}\vec{z}_v & 0 & 0 \\ {}^{0}\vec{n}_3^{\,T} & -{}^{0}\vec{n}_3^{\,T}\left({}^{0}P_v\left[*\,{}^{v}\overrightarrow{G_vC_3}\right]\right) & 0 & 0 & -{}^{0}\vec{n}_3^{\,T}\,{}^{0}\vec{z}_v & 0 \\ {}^{0}\vec{n}_4^{\,T} & -{}^{0}\vec{n}_4^{\,T}\left({}^{0}P_v\left[*\,{}^{v}\overrightarrow{G_vC_4}\right]\right) & 0 & 0 & 0 & -{}^{0}\vec{n}_4^{\,T}\,{}^{0}\vec{z}_v \end{bmatrix} \tag{9}$$

$$\overrightarrow{B_{cv}}(i) = {}^{0}\vec{n}_i^{\,T}\left[{}^{0}P_v\left(2\cdot{}^{v}\vec{w}_v \wedge \left(-\dot{d}_i\cdot{}^{v}z_v\right) + {}^{v}\vec{w}_v \wedge \left({}^{v}\vec{w}_v \wedge {}^{v}\overrightarrow{G_vC_i}\right)\right)\right] \quad for\ i = 1...4 \tag{10}$$

$\left[*\,{}^{0}\overrightarrow{G_vC_i}\right]$: The operator of the vector cross product.

2.3 Motions equations

The main forces acting on a vehicle in motion are wheels/ground interaction forces, the dynamic forces, gravity and the aerodynamic forces. In this model, we consider only the dynamic forces, gravity, and wheel/ground interaction efforts for simplification reasons.

Interaction efforts applied to each wheel of the system are represented in the reference related to wheels by the vector $\left(F_{xi} \quad F_{yi} \quad F_{zi}\right)$. These three components are linked together by a dependency relationship defined by the wheels/ground interaction efforts of the linear model [3] [4] [5] [10] [22]. Taking account of the forces applied on the vehicle, the twelve following equations of motion is obtained as follows:

$$\begin{cases} m\begin{pmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{pmatrix} = m\cdot{}^{0}\vec{g} + \sum_{i=1}^{4}{}^{0}\vec{F}_i + {}^{0}\vec{F}_{out} \\ {}^{v}\mathbb{I}_{Gv}\,{}^{v}\vec{\dot{w}}_v + {}^{v}\vec{w}_v \wedge \left({}^{v}\mathbb{I}_{Gv}\,{}^{v}\vec{w}_v\right) = \sum_{i=1}^{4}{}^{v}\overrightarrow{G_vC_i} \wedge {}^{v}\vec{F}_i + {}^{v}\overrightarrow{M}_{out} \end{cases} \tag{11}$$

With ${}^{0}\vec{g}$: the gravitational acceleration; ${}^{v}\vec{\dot{w}}_v$: Angular acceleration of the vehicle expressed in $\Re_v$ frame; ${}^{v}\mathbb{I}_{Gv}$ is the tensor of inertia of the robot; m_v is the mass.
and:

$$\left({}^{v}P_i\cdot{}^{i}\vec{F}_i\right)\cdot{}^{v}\vec{z}_v = -k_i\left(d_i - L_{0i}\right) - b_i\cdot\dot{d}_i \qquad for\, i = 1...4 \tag{12}$$

With L_{0i}: Initial spring length of the wheel *i*.
By grouping the various equations of constraints and motion of the system (car-like robot), we obtain the following differential-algebraic system describing the dynamics of our system:

$$[A]\cdot\ddot{\vec{X}}=\vec{B} \tag{13}$$

$$[A]=\begin{bmatrix}[M_v]_{6\times10}\\ [J_v]_{4\times10}\end{bmatrix} with:[M_v]=\begin{bmatrix} m\cdot I_{3\times3} & 0_{3\times3} & 0_{3\times4}\\ 0_{3\times3} & {}^{v}\mathbb{I}_{Gv} & 0_{3\times4}\end{bmatrix}$$

$$\vec{B}=\begin{pmatrix}\vec{B}_{mv}\\ -\vec{B}_{cv}\end{pmatrix},\ \text{with}:\vec{B}_{mv}=\begin{pmatrix} m\cdot\vec{g}+\sum_{i=1}^{4}\left({}^{0}P_i\cdot{}^{i}\vec{F}_i\right)+{}^{0}\vec{F}_{out}\\ -{}^{v}\vec{w}_v\wedge\left({}^{v}\mathbb{I}_{Gv}\cdot{}^{v}\vec{w}_v\right)+\sum_{i=1}^{4}{}^{v}\overrightarrow{G_vC_i}\wedge\left({}^{0}P_i\cdot{}^{i}\vec{F}_i\right)+{}^{v}\vec{M}_{out}\end{pmatrix}$$

The numerical integration equations, with the consideration of the initial conditions, describe the dynamic behavior of the vehicle for a given time history of acceleration (or braking) and steering data. In this work, we used the Euler method for integrating these differential equations.

3 RESULTS

To discuss the reliability of obtained results we will validate them by comparison with those given by Pc-crash® accidents simulator [21]. Then, we will present some examples of applications of the modeling approach for different complex scenarios. In these examples, the applied geometrical and inertial parameters of the vehicle are summarized in Table 1.

Parameters	**Car like robot**
Mass *m*	950 *Kg*
Moment of inertia *Ixx*	1072 *Kg/m²*
Moment of inertia *Iyy*	1342 *Kg/m²*
Moment of inertia *Izz*	1342 *Kg/m²*
Height of center of gravity	0.50 *m*
Front track	1.41 *m*
Rear track	1.41 *m*
front wheelbase	1.04 *m*
Rear wheelbase	1.50 *m*
Front suspension stiffness	9.97 *KN/m*
Rear suspension stiffness	6.79 *KN/m*
Front dumping	897.28 *N.s/m*
Rear dumping	611.01 *N.s/m*
Coefficient of friction	0.70
Radius wheel	0.25 m

Table 1: Geometrical and inertial parameters of the vehicle.

Scenario 1: Acceleration and deceleration on a straight path:

This scenario is described as follows (Figure 2): *i)* an acceleration phase for 4 seconds; *ii)* a constant speed phase for 2 seconds and *iii)* a deceleration for 4 seconds (the braking is distributed on the front and rear wheels). The simulation results by the developed application and those given by PC-Crash are illustrated in figures 3 and 4. These figures show a good correlation between the results obtained by the dynamic model developed and those given by the software of Pc-Crash.

Scenario 2: Dynamics of the robot during the course of a turn:

This scenario is described as follows: *i)* linear movement for 2 seconds at constant speed *S = 15 m/s* and *ii)* turning for 8 seconds with 100*m* of curvature radius. The obtained simulation results are illustrated in figures 5 to 6. The curves plotted in these figures show a very satisfactory correlation between the results of our developed model and those given by Pc-Crash.

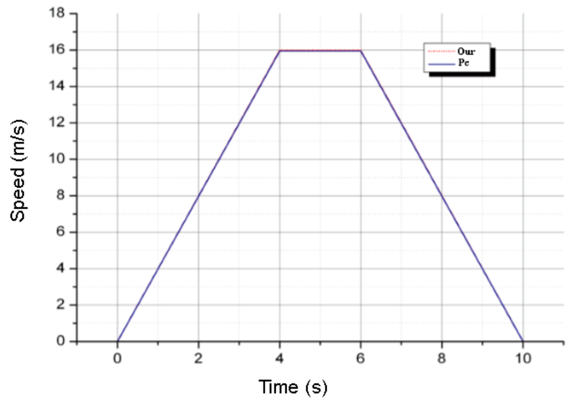


Figure 2: Speed of the vehicle.

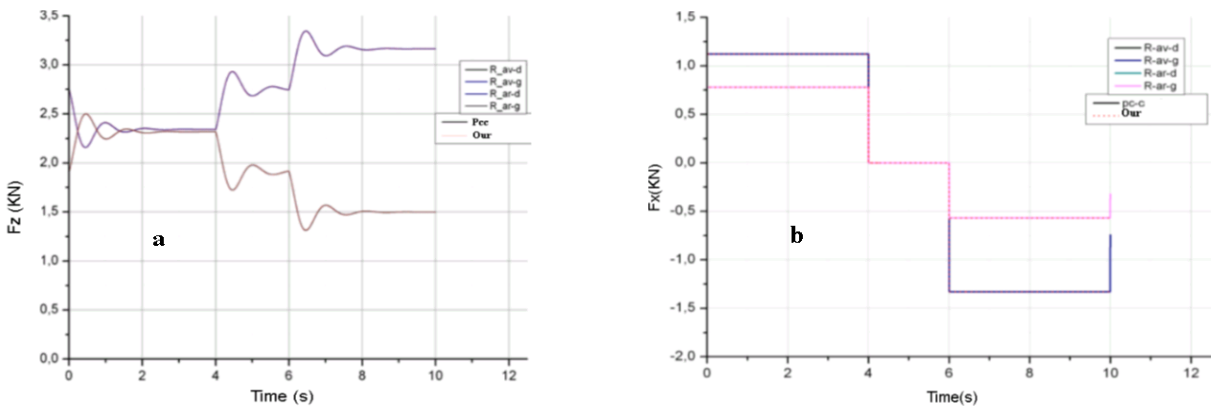


Figure 3: (a) Vertical and (b) longitudinal forces wheels

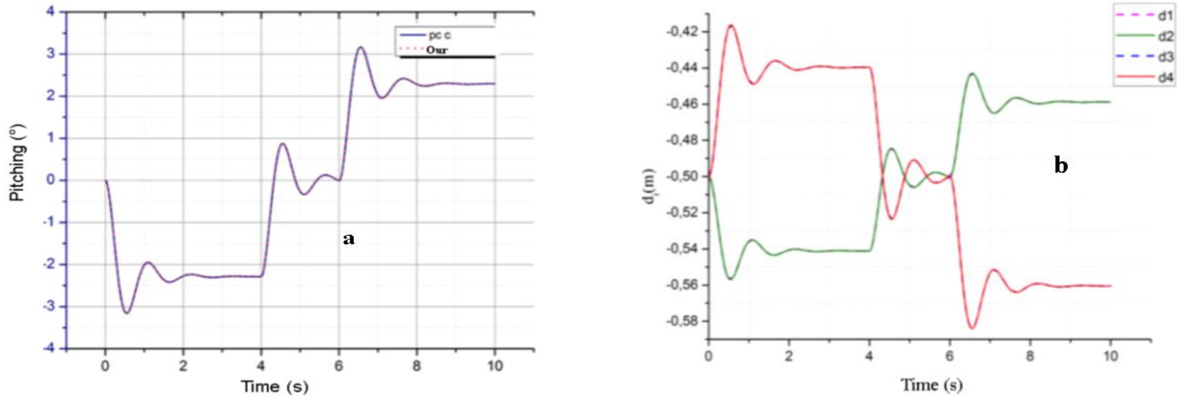


Figure 4: (a) Pitching angle and (b) elongation of spring of suspension

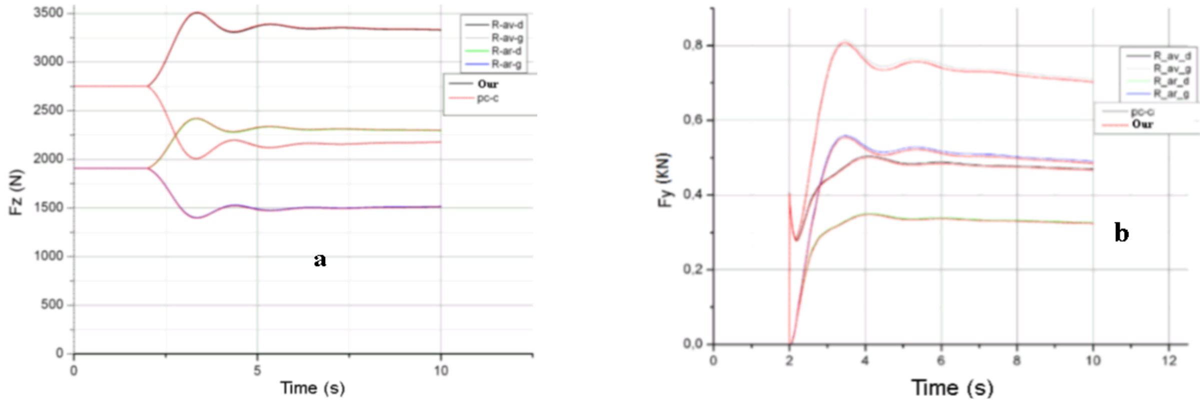


Figure 5: (a) Vertical and (b) longitudinal forces wheels

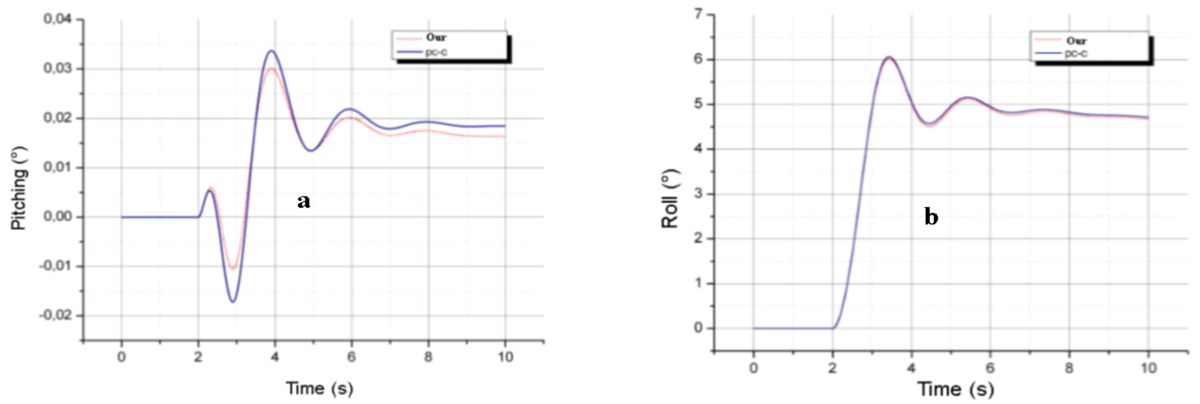


Figure 6: (a) Pitching angle and (b) roll angle

In the results for these two scenarios, we have already met with non-regularities of certain parameters (Figure 3b and 5b). In the first scenario, the irregular dynamic is caused by discontinuities of longitudinal acceleration, while for the second scenario; is approved by the sudden change in curvature radius.

4 APPLICATION TO COLLISION

Vehicle dynamic in normal operating mode remains predominant in the literature because it is usually sufficient to study a wide operating range, for this; we found a very little literature treating the irregular dynamic of the vehicle. However, the modeling of non-regular dynamic behavior of the vehicle will explain the phenomena which are introduced into the phase of an accident (the investigation of road accidents), and the operation of these models in vehicle construction or in mission planning an intervention autonomous vehicles.

To find a solution to our problems, we will be simply made to realize, first, an analytical study of the constraints equations and motions equations and, secondly, the resolution process. The approach should ensure convergence and allow the user to obtain the desired response at the instants for any case.

Simulate a collision type of accident is to replicate the response of the vehicle involved in the accident, before, during and after the collision. This simulation requires the development and implementation of three models:

i) The collision detection model.
ii) The collision force calculate model.
iii) The model of the dynamic behavior of objects (vehicle, pedestrians ...) colliding.

In this work, we opted for a representation, not polyhydric, very simple based on modeling of vehicle and obstacles in the form of rectangular objects. The algorithm that we used for collision detection is based on the calculation of the distance between the rectangular objects. The distance between parallelepipeds is the smallest between the edges of the facets with respect to each other. The distance in the configuration illustrated in Figure 7, is given by:

$$L_{i/j} = \left({}^{j}R_{i} \cdot {}^{i}P_{2} - {}^{j}P_{5} \right) \cdot z_{i} \tag{14}$$

Either $L_{i/j}$ the smallest distance between the two parallelepipeds, whichever is negative, the objects collide. Otherwise, the objects are separated.

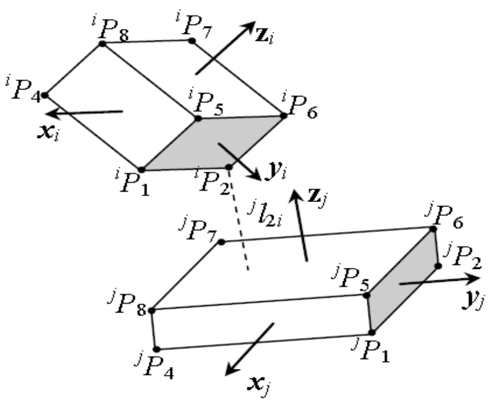


Figure 7: Calculating the distance between two parallelepipeds.

The second step of processing a collision problem is to calculate the collision forces. The methods used to assess these forces can be classified into three categories: the constraints methods, pulse-based methods and penalties methods. We chose a penalty method for the following reasons: *i)* penalties methods are implemented in several simulation software and *ii)* simplicity implementation. In this method, the collision force applied by the object *i* on the *j* item to the collision point, is given by:

$$F_{i/j} = -\left(k_{i/j} \cdot l_{i/j} + b_{i/j} \cdot \dot{l}_{i/j} \right) \left(\vec{n}_{i/j} - \mu_{i/j} \frac{\vec{v}_{i/j}}{\left\| \vec{v}_{i/j} \right\|} \right) and\ F_{i/j} = -F_{j/i} \tag{15}$$

With $\vec{n}_{i/j}$ the common normal to the contact surfaces; $k_{i/j}$ and $b_{i/j}$ are the coefficients of stiffness and damping of the two objects at the point of collision; $\vec{v}_{i/j}$ the relative sliding velocity at the contact point; $\dot{l}_{i/j} = l_{i/j}/dt$: is the velocity of interpenetration. The value of this speed is calculated by a numerical derivation of $l_{i/j}$.

$$l_{i/j}\left(t+dt\right)=\left(l_{i/j}\left(t+dt\right)-l_{i/j}\left(t\right)\right)/dt \tag{16}$$

4.1 Simulation algorithm of vehicle's collision

The simulation process starts with the initial conditions (position, orientation and vehicle velocity). During its movement, the vehicle is subjected to the force of gravity, the wheels/ground interaction efforts and inertia forces. The collision forces are not taken into account if there is a collision between the vehicle and its environment. It is therefore necessary to detect whether there is a collision or not. In a collision, it is necessary to calculate the impact point, the normal to the surfaces in this point and the interpenetration distance, and evaluate the collision forces. Then, these forces are taken into account in the vehicle dynamics. Once the full dynamic is calculated, the overall configuration system is updated.

4.2 Simulation of collision vehicle/fixed obstacle

To simulate the non-regular dynamic behavior of a vehicle collision phase. The vehicle in rectilinear motion with constant speed, it collides with a fixed obstacle (a wall) considered a perfectly rigid body. The angle α is the orientation between the vehicle speed and the normal to the wall (Figure 8), we considered two cases of collision:

Case 1: a frontal collision for α = 180 °

Case 2: an oblique collision for α = 170 °.

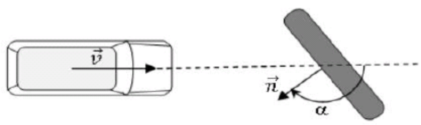
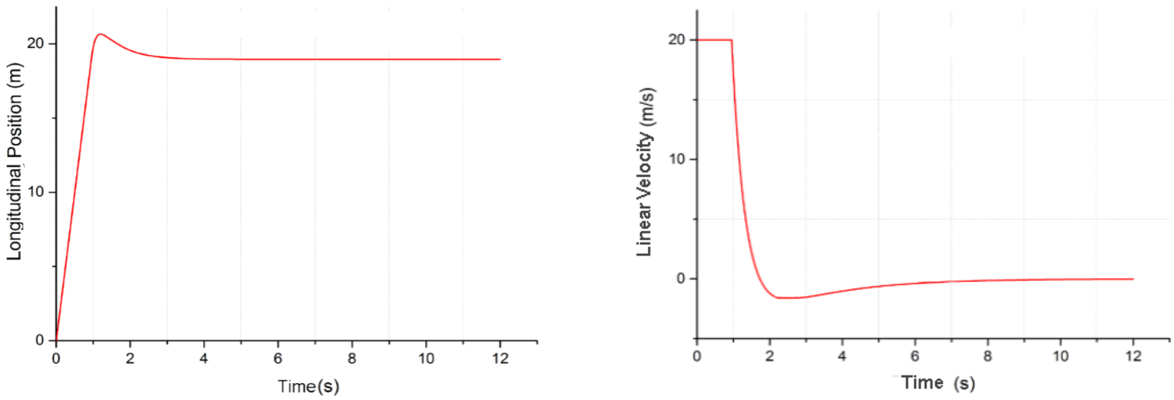
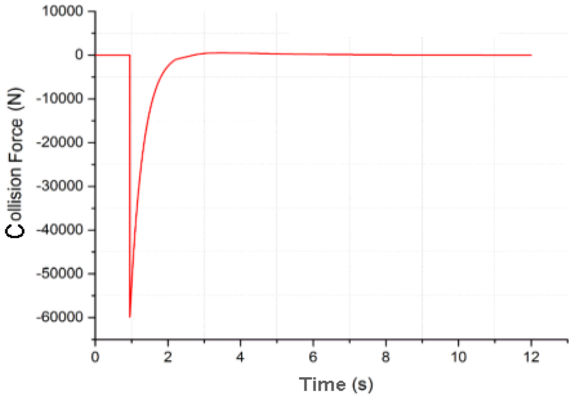


Figure 8: Collision of a vehicle with the wall.

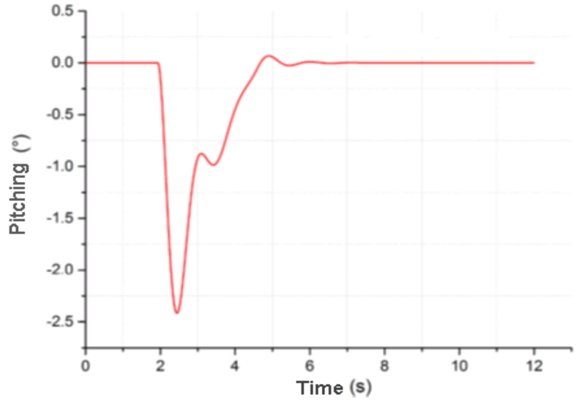


a) Longitudinal position

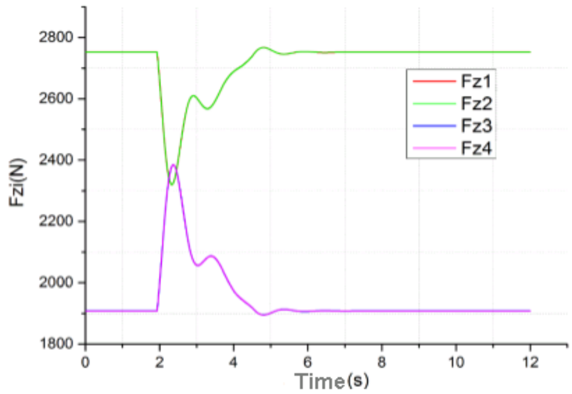
b) Linear velocity



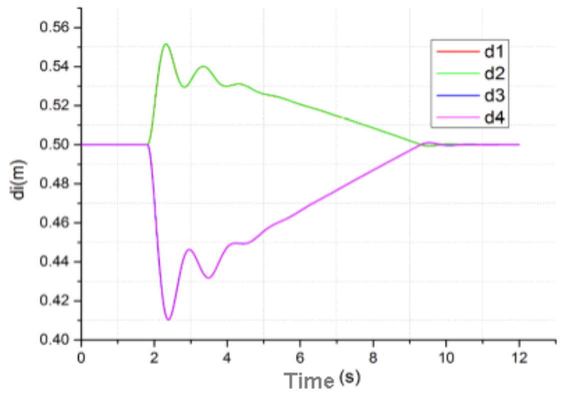
c) Collision forces



d) Pitching angle

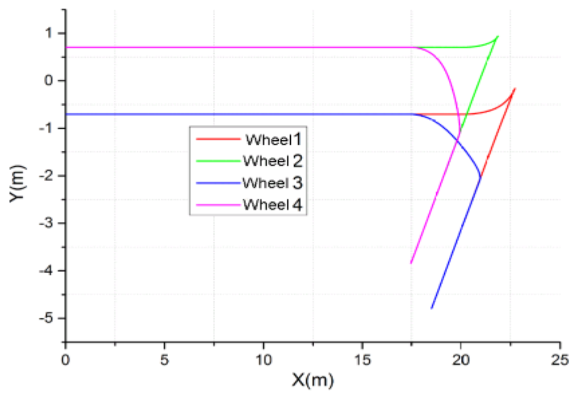


e) Vertical forces

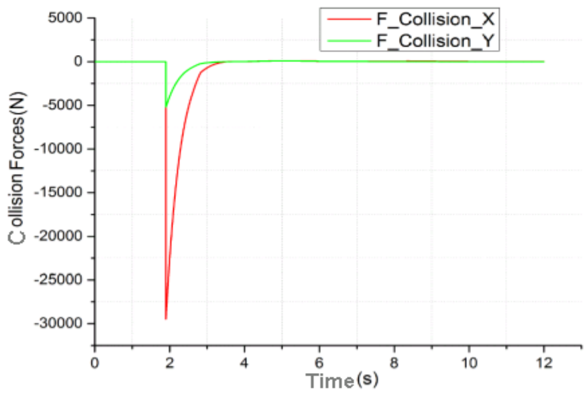


f) Elongation of springs

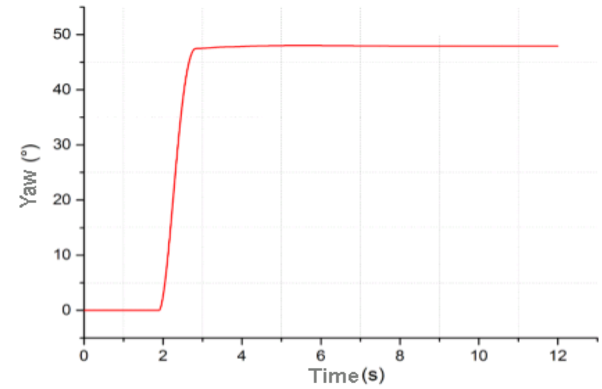
Figure 9**:** Results obtained of frontal collision vehicle/wall.



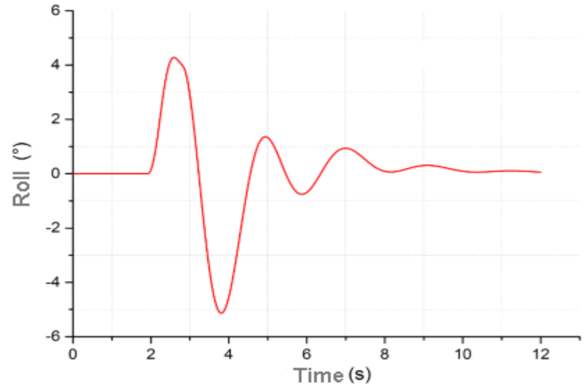
a) Vehicle trajectory



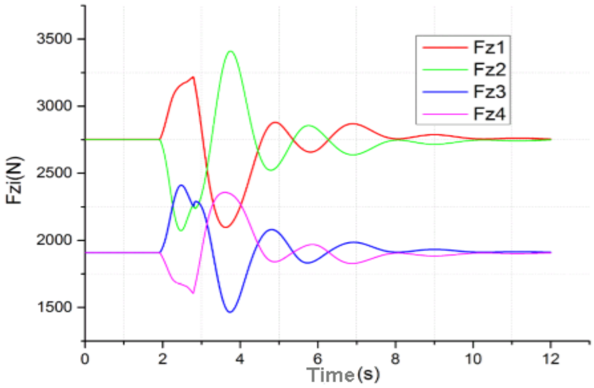
b) Collision forces



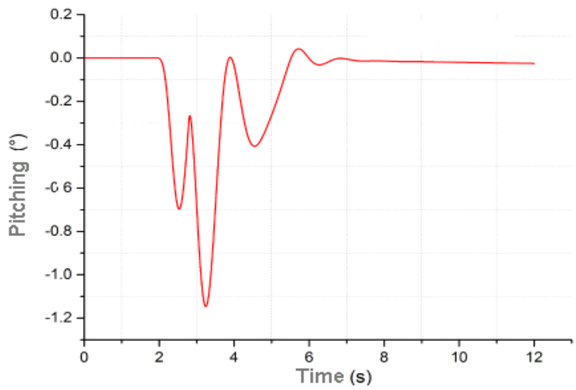
c) Yaw angle



d) Roll angle



e) Vertical forces



f) Pitching angle

Figure 10: Results obtained of an oblique collision vehicle/wall.

The results obtained for the first case are illustrated in Figure 9. We note, in phase collision, a sharp decrease of the robot speed (Figure b). Then after the crash phase, a movement of the vehicle to the rear until the stoppage of vehicle (Figure 9a). This frontal collision, causes an inclination of the vehicle backward (change in the negative pitch angle (Figure 9d)), which causes an increase of the vertical forces applied to the rear wheel and a decrease in those front wheels (Figure 9f).

In the second case, the vehicle changes its direction after the collision (Figure 10a) and obviously, the longitudinal component of force is higher than that of the transversal component (Figure 10b). The vehicle is moving at an angle of 50 ° with respect to the yaw axis and then stabilizes at that angle (Figure 10c), and unlike the first case, we observe a change in the roll angle due to the orientation of the wall (Figure 10d).

Before the collision, the vertical forces applied on the same axle are identical. After that, we find that the vertical forces on the right side conversely increases the left side (Figure 10f), which shows that the point of application of the collision is in the driver's side.

4.3 Validation results of the frontal collision vehicle/wall

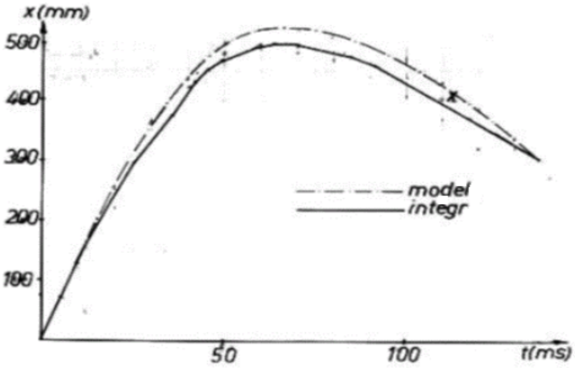
The validation of the collision model was performed by comparison with those given experimentally Crash-test [24]. For this, we have considered a vehicle of mass m = 2000 Kg in rectilinear motion with constant velocity v = 13.42 m/s. In this validation, we considered that the collision vehicle/wall is composed of two phases:

i) A deformation phase with stiffness $K_1 = 1.243\ 10^6\ N / m^2$.

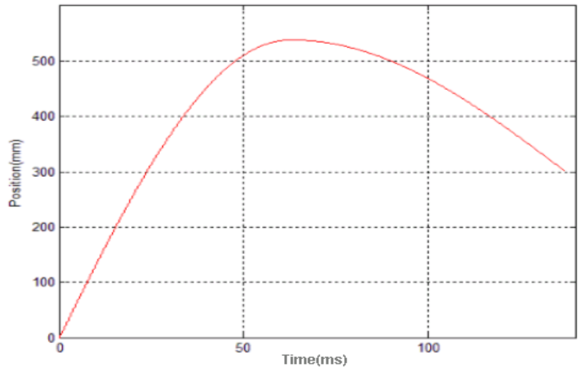
ii) A restitution phase with a stiffness $K_2 = 896.6\ 10^3\ N / m^2$.
The collision force applied by the wall on the vehicle, is defined by the following relationship:

$$F_{w/v} = \begin{cases} -\left(k_1 \cdot l_{w/v}\right) x_v & if\ \dot{l}_{w/v} > 0 \\ -k_2 \cdot \left(l_{w/v} - l_p\right) x_v & if\ \dot{l}_{w/v} \le 0 \end{cases} \tag{17}$$

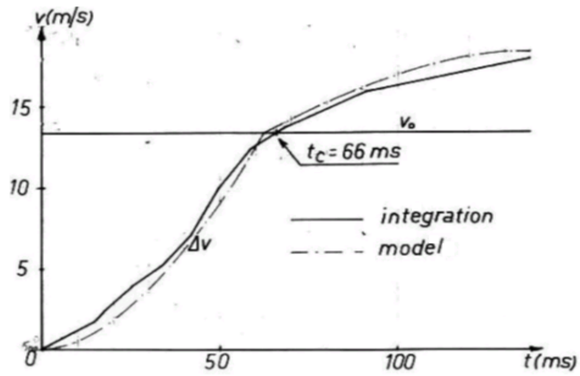
With $l_{w/v}$ the penetration distance of the vehicle in the wall; l_p : Distance from permanent deformation.



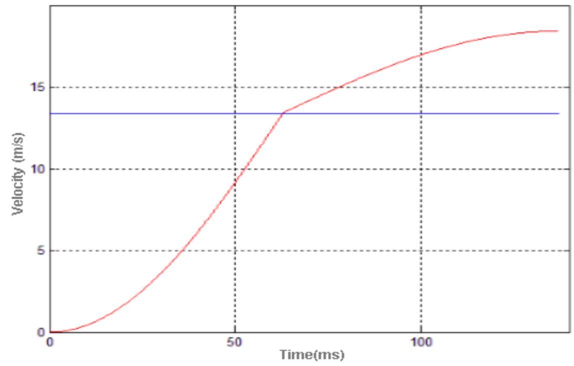
a) Penetration distance (Crash-test) **[24]**



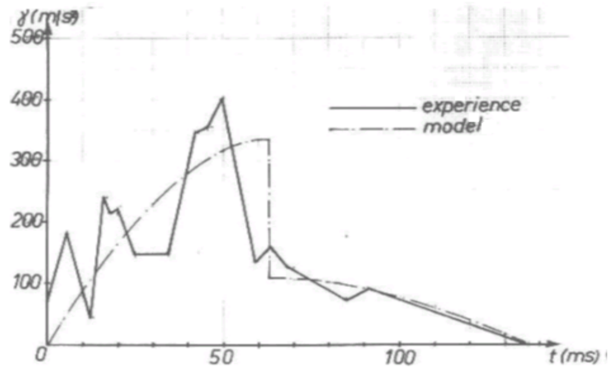
b) Penetration distance (our model)



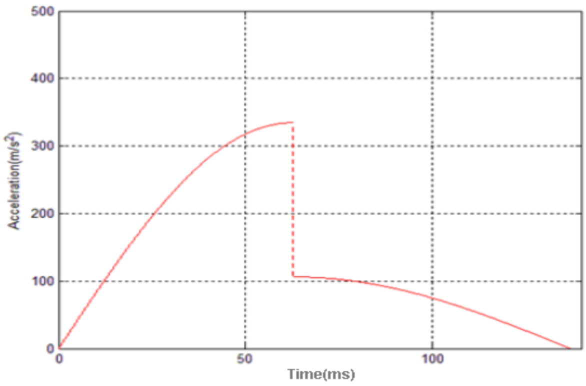
c) Penetration velocity (Crash-test) **[24]**



d) Penetration velocity (our model)



e) Penetration acceleration (Crash-test) **[24]**



f) Penetration acceleration (our model)

Figure 11: Validation results of the collision model vehicle/wall.

Our simulation results and those given by experimental measurement Crash test are presented in Figure 11. We find, from these results, that the curves of the displacement, velocity and acceleration of penetration are substantially the same compared with those of the experiment. We can say that the modeling of the head-on collision of a vehicle against a wall is correct.

5 CONCLUSION

In this work, we introduced the steps necessary for dynamic modeling of the vehicle. The first step relates to the mechanical system description. In this step, the vehicle is considered as a rigid body connected to the ground via the suspension system. The second step is devoted to the generation of system constraints equations to know the wheels/ground holding contact conditions and the Ackermann steering. The third step relates to the generation of the motions equations. While the final step is devoted to solving the system of differential and algebraic equations by the Euler explicit method. The validation results of the developed dynamic model was carried out by comparison with those given by the PC-Crash software. We show that the results are very satisfactory. Subsequently, we focused our study on the non-regular behavior of a vehicle during a collision with a fixed obstacle. We have proposed a method based on the evaluation of the distance to vehicle /obstacle interpenetration. The collision force is calculated, thereafter, via a penalty method. The results of this method are validated experimentally with those given by Crash-test. These results show the affinity of the developed model. Moreover, the choice of model validation was not trivial but rather conditioned by the availability of middle address the same problems.

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