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ABSOLUTE FINITE ELEMENT COORDINATES IN THE DYNAMICS OF LARGE FLEXIBLE STRUCTURES

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Abstract. *In the paper a novel method of finite elements in absolute coordinates is presented. The flexible elements and their node coordinate systems are considered free objects in space which motion is restricted by the elastic forces of the adjacent flexible elements. So, no kinematic restrictions, respectively algebraic equations, are imposed to the dynamic equations and the dynamic model is presented as Ordinary Differential Equations. Incremental approach for definition of system configuration during its global motion is applied, which avoids singularity of the large rotations. The proposed numerical procedure defines the element nodes as moving coordinate systems that makes possible their motion coordinates to be compared to rigid bodies and the large rotations to be taken into account. External excitations as wave propagation and earthquakes are subjects of the investigations.*

Generalized Newton – Euler dynamic equations for rigid and flexible bodies are applied for precise definition of the velocity dependent inertia forces in the dynamic equations. The seismic excitations are presented as reonomic constraints. Examples of large spatial flexible deflections of complex flexible structure verify the effectiveness of the method proposed.

1 INTRODUCTION

Multibody system methodology is a recently developing field in the theoretical and applied mechanics. Multibody dynamics as independent branch of mechanics was agreed to and established in 1977 at an IUTAM Symposium held in Munich, Germany and chaired by Magnus [1]. The first steps of the new discipline started with the dynamic analysis of rigid and flexible body systems striving to solve complicated nonlinear problems that the analytical methods cannon cover. In particular, elastic or flexible multibody systems, respectively, contact and impact problems where key issues for researchers worldwide. Multibody dynamics depends on computational dynamics and is closely related to control design and vibration theory.

Recently the multibody system methodology experienced huge progress including real time simulation, contact and impact problems, extension to electronics and mechatronics, dynamic strength analysis, optimization of design and control devices, integration codes in particular for differential algebraic systems, challenging applications in biomechanics, robotics and vehicle dynamics, manufacturing science and molecular dynamics and etc. Multibody dynamics is an excellent foundation for multivariable vibration analysis and sophisticated control design. The natural relations between the motion behavior of the mechanical systems with the sensors and the control devices make the mechatronic systems a basic subject for the Multibody system methodology. The precise analysis, simulation and planning of the system motion and design could be done deriving the dynamic equations including the control inputs and the information coming from the sensors. Obviously, the higher level of the mechatronic devices is the intelligent multibody systems which include all the characteristics of the mechatronics, as well as, the precise dynamics simulation and motion planning and control [2, 3].

Example of such mechatronic (intelligent) multibody systems are autonomous vehicles, space structures, structures imposed on external and seismic excitations, large flexible structures and wind generators. Solution of these tasks requires up-to-date methods for dynamics analysis and simulation, novel methods for numerical solution of ODE and DAE, real time simulation, passive, semi-passive and active control algorithms taking into account the control system and the feedback information.

The methods of the finite element theory (FET) [4] have been constantly developing to successfully solve the up-to-date tasks in structural statics. The problem of deriving the dynamics equations of flexible bodies using finite element discretization is really topical. Many books and papers [5, 6] discuss the problems that could arise using the finite element methodology and commercially available software (NASTRAN, ANSYS, ABAQUS) for simulation of nonlinear phenomena. It was pointed out that the FET approach cannot effectively simulate nonlinear effects of high velocity global motion in space and large flexible deformations.

The finite element theory proposed the theoretical basis for profound numerical analysis of complex flexible structures. The methodology for discretization of the continuum and the theory for polynomial approximation of the form of the deformations and on this basis computation and distribution of the mass and stiffness properties of the materials of flexible elements to discrete objects (nodes), actually, have been used afterward for development of various methods for static and dynamic analysis, even in the multibody system dynamics (MDS) simulation of rigid and flexible bodies. Attempts had been made for direct application of FET for dynamics simulation of systems that undergo global motion in space superimposed by large flexible deflections. In the recent decades the scientists successfully developed up-to-date methods for dynamics analysis of rigid and flexible mechanical systems setting the basis of multibody system methodology. Actually, the scientists use the theoretical rules of the FET for deriving the basic properties of the discretized elastic bodies but, on the other hand, they strive to escape from the motion parameters of the FET in order to catch the nonlinear effects of the large displacements and deformations. It was proven [7 – 9] that using the FET approach the dynamic equations are free of centrifugal and Coriolis accelerations and the corresponding inertia forces.

Khan and Anderson [10] proposed the most recent classification of the methods for MBS dynamics. Tthey applied the Floating Frame of Reference (FFR) and Absolute Nodal Coordinate Formulation (ANCF) approaches for realization of their numerical procedure.

The FFR approach was developed [7, 11, 12] and has been successfully applied for dynamics simulation of large spatial motion and rotation of flexible multibody systems. This approach is also used for application of the modal coordinates and significant decreasing of the flexible coordinates.

Recently a novel method of ANCF has been developed [13, 14]. It is based on the theory of the curvilinear coordinates. The slopes of the nodes in the tangential line or plane of the space curve or shell are applied as coordinates. The method proposes important advantages mainly in large rotation simulation and mass matrices formulation. Review on the ANCF method for large deformation dynamics simulation of flexible multibody systems was presented by Gerstmayr et al. [15]. The paper provided a comprehensive review of the advantages and further developments of the ANCF method.

As it was said above the dynamic equations derived on basis of the FET do not include the Coriolis and the centrifugal accelerations. Shabana [16] and Meijard [17] derived generalized Newton – Euler dynamic equations for the case of FFR formulation. The equations represent the inertia forces as function of the velocities and accelerations of the global and flexible coordinates. Zahariev [18] derived generalized Newto-Euler dynamics equations for the rigid and flexible bodies for which the kinetic energy is quadratic form of the velocities. The equations are applicable for the flexible elements of the FET. The inertia forces are with respect to the quasi velocities and accelerations and are invariant to the kind of the coordinates.

In [19] a method of Finite Elements in Relative Coordinates (FERC) based on the FET was proposed for dynamic simulation of large flexible structures. Using the generalized Newton-Euler dynamic equations nonlinear effects and velocity dependent terms are successfully simulated. But using this approach for complex structures with many mutually connected adjacent flexible bodies one could experience a lot of difficulties that mainly consists in its program system realization and complex pre-processor procedures.

A method of Finite Elements in Absolute Coordinates (FEAC) is discussed in the paper. It was recently presented in [18] and further developed for different applications. FEAC are proposed here for simulation of structures subject to seismic excitations. Acceleration and force methods are used in practice for dynamics simulation of the earthquake structure response. Acceleration approach consists in registration of the motion of a structure foundation by accelerographs and normally consists in three orthogonal components of the ground acceleration. The velocity and displacement of the ground are obtained integrating the data of the accelerograms. The statistical data for the specific regions are than applied as input data for the dynamic simulation. In [20] the accelerations of the foundation are reonomic constraints for the dynamic equations which are transformed with respect to the parameters that consists of structure displacement and the forces, applied to the foundation.

Investigation of the forces loading the basement is a natural way for simulation of the structure response because of earthquakes [21]. This task is implemented analyzing the soil-structure interaction. Computation of the forces loading the foundation as a function of the relative motion ground – basement is the first stage for the numerical simulation process using the multibody system methodology.

The problems related to the subjects of the large scale flexible structures are discussed in [22]. Problem oriented articles devoted to the structures subject to seismic excitation were collected in [23]. The up-to-date methods and devices for prevention of structure damages and collapse are based manly of passive, semi-active and active structure basement suspension. Mainly spring, dampers and rubber isolators are used for passive suppression of the structure deflections. Shock absorbers or dampers are added to the structure to dissipate the seismic shocks. Active Tuned Mass Dampers use a computer controlled counter moving weight to actively move against the building sway. Active large scale springs and dampers together with computer controlled motion of large masses ensure calculated action against the building deflections. The article of Yao [24] seems to be the cornerstone in the structural control in civil engineering. An early review article of active control in structural engineering was published by Soong [25]. Latest surveys in 2003 [26] and another recent one in 2012 [27] emphasize on the semi-active and active control of structures subject to earthquake excitations and discuss theoretical backgrounds of different control schemes.

Dynamics simulation is extremely important for the design process of slender and large scale flexible systems as skyscrapers, bridges, wind power generators and many others. The devices for deviation suppression including passive, semi-active or active, although being designed on the basis of the eigenvalue analysis, should be simulated and analyzed for long time operational time and different kind of external excitations. This process includes development of reliable dynamic model and deriving precise dynamic equations. For efficient dynamic and motion simulation of complex structures that implement spatial motion and experience external excitations an important thing is to provide numerical procedure that ensures long lasting operational time of simulation and convergence of the numerical integration. Keeping in mind that such structures are with tremendously high nodes and degree of freedom with many complicated relations and geometrical constraints the following requirements to the program systems for dynamic simulation are stated:

a) Simple and straight forward algorithm for pre-processor development of the input data compliant with the dynamic model.

b) An approach to presentation of the rotational parameters keeping in mind that the systems could implement large rotations and translations.

c) Efficient dynamic model for simulation of rigid and flexible systems taking into account nonlinear effects as geometrical stiffening, velocity dependent terms and external excitations;

d) Reliable methods for node, respectively, degree of freedom reduction.

e) Efficient program procedures for numerical integration of Ordinary Differential Equations (ODE) and Differential Algebraic Equations with proven convergence.

The efficient simulation procedure based on a precise dynamic model and efficient numerical solution significantly improves the design process making him more reliable and inexpensive. In the paper the main attention is paid to first three items (a, b, c). The numerical procedure proposed here defines the element nodes as moving coordinate systems that makes possible their motion coordinates to be compared to rigid bodies, respectively, the large rotations to be taken into account. In the paper a novel method of finite elements in absolute coordinates of modified finite elements is presented. External excitations as wave propagation and earthquakes are subjects of the investigations. The flexible elements and their node coordinate systems are considered free objects in space which motion is restricted by the elastic forces of the adjacent flexible elements. So, no kinematic restrictions, respectively algebraic equations, are imposed to the dynamic equations and the dynamic model is presented as ODE. Incremental approach for definition of system configuration during its global motion is applied, which avoids singularity of the large rotations.

Generalized Newton – Euler dynamic equations for rigid and flexible bodies are applied for precise definition of the velocity dependent inertia forces. Wave propagations and seismic excitation are presented as reonomic constraints. Examples of large spatial and flexible deflections of complex rigid and flexible body structure verify the effectiveness of the method proposed.

2 KINEMATICS OF LARGE SPATIAL ROTATIONS OF FLEXIBLE ELEMENTS

2.1 Relative node position of a flexible element

To understand the difference between the classical FET and the method of the finite elements in absolute nodal coordinates using multibody system methodology some explanations about the kinematics of flexible nodes will be presented here. Since the basic relations and deductions as discretization of the flexible elements, the forms of the deformations, as well as, the methods for computation of the mass and stiffness matrices are well known for the readers [4, 28] no special attention will be paid to that procedures.

In Figure 1 a non-isoparametric space beam element (with index *i*) is presented. This finite element is relatively simple for explanation but, at the same time, presents the most common six degree of freedom (dof) space deflections of the nodes. In the figure the beam and small translations and rotations, respectively $\boldsymbol{\Delta}_{i,k} = [\Delta x_{i,k} \;\; \Delta y_{i,k} \;\; \Delta z_{i,k}]^{\backslash}$, $\boldsymbol{\theta}_{i,k} = [\theta x_{i,k} \;\; \theta y_{i,k} \;\; \theta z_{i,k}]^{\backslash}$, $i = 1{,}2$ of its nodes (indices *i*,1 and *i*,2 – the first and second nodes of element *i*) with respect to the initial not deformable element coordinate system $\underline{X}_i \underline{Y}_i \underline{Z}_i$ are depicted [4].

With the superscript "\" a matrix transpose is denoted. The underlined notations denote initial configurations. With bold characters matrices are pointed out. For scalars and objects, for example points, bodies, coordinate systems, italic notations are used.

It should be said that in FET these deflections are with respect to the element reference frame in its initial position (not deformed position) and no rotation of the element coordinate system is taken into account. Of course, coordinate system transformation with respect to the absolute reference frame $X_0 Y_0 Z_0$ of mass and stiffness matrices is considered but for the structure initial configuration only. As a result of the node deflections the element coordinate system $X_i Y_i Z_i$, respectively the node *i*,1 coordinate system $X_{i,1} Y_{i,1} Z_{i,1}$, moves to a new position defined by the radius – vector $\boldsymbol{\rho}_{i,1}$ that could be easily estimated adding the initial

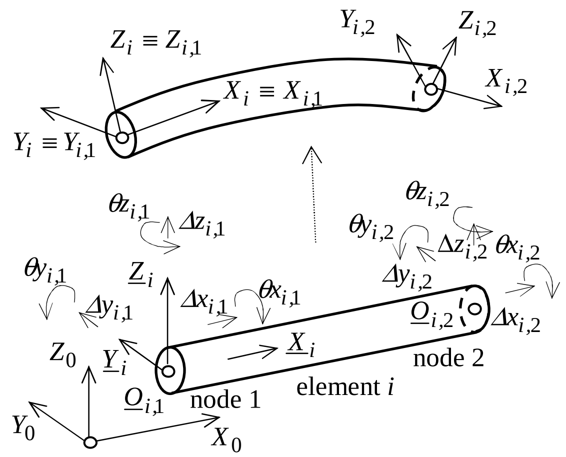


Figure 1: Small flexible node deflections of space non-isoparametric beam finite element

position of element coordinate system origin, radius – vector $\underline{\boldsymbol{\rho}}_i \equiv \underline{\boldsymbol{\rho}}_{i,1}$, and the node i,1 deflections $\boldsymbol{\Delta}_{i,1}$. The same holds for the radius –vector $\boldsymbol{\rho}_{i,2}$, taking into account the initial position of node i,2 coordinate system origin - radius – vector $\underline{\boldsymbol{\rho}}_{i,2}$, and the node i,2 deflections $\boldsymbol{\Delta}_{i,2}$. The node deflections of the i-*th* beam element are set in a 12×1 matrix – column $\boldsymbol{\Theta}_i = \left[\boldsymbol{\Theta}_{i,1}^{\prime} \;\; \boldsymbol{\Theta}_{i,2}^{\prime}\right]^{\prime} = \left[\boldsymbol{\Delta}_{i,1}^{\prime} \;\; \boldsymbol{\theta}_{i,1}^{\prime} \;\; \boldsymbol{\Delta}_{i,2}^{\prime} \;\; \boldsymbol{\theta}_{i,2}^{\prime}\right]^{\prime}$.

In Figure 2 the element i is not drawn but only the coordinate systems i,1 and i,2 of its nodes are shown in their initial and final configurations. The nodes are actually free objects, respectively coordinate systems that implement free motion in space restricted by the elastic forces only. Six possible deflections of the nodes are assumed (translations and rotations). The elastic forces $\mathbf{F}_{i,1}$ and torques $\mathbf{L}_{i,1}$ in the first node, as well as, $\mathbf{F}_{i,2}$, $\mathbf{L}_{i,2}$ in the second node, which are 3 × 1 matrix – vectors, are set in a 12×1 matrix – column $\boldsymbol{\Xi}_i = \left[\mathbf{F}_{i,1}^{\prime} \;\; \mathbf{L}_{i,1}^{\prime} \;\; \mathbf{F}_{i,2}^{\prime} \;\; \mathbf{L}_{i,2}^{\prime}\right]^{\prime}$ arranged in the same order as the corresponding elements of $\boldsymbol{\Theta}_i$.

Actually, the position of the element coordinate system $X_i Y_i Z_i$ in the deformed configuration coincides with the coordinate system $X_{i,1} Y_{i,1} Z_{i,1}$ of node i,1. So, the deflections of the flexible element could be considered as free motion of the element coordinate system and the flexible deflections of the second node with respect to it. These are the flexible deflections relative to the element coordinate system.

In Figure 3 the beam element in the configuration as this of Figure 1 is depicted including the length L_i of the beam (along axis X) and the small deflections $\boldsymbol{\Delta}_{i,2\rangle i}$ of node i,2 relative to the beam coordinate system i. In the right subscript $\boldsymbol{\Delta}_{i,2\rangle i}$ the symbol "$\rangle$" serves as an arrow pointing out that the coordinate system with index i,2 is with respect to coordinate system i.

The right symbol of the subscript could be omitted if it points the absolute reference frame with index "0" – zero. Obviously, the beam configuration is quite the same and the fact that the flexible deflections are read relative to the moving element coordinate system does not change the regulations for FET discretization and for computation of the mass and stiffness matrices. The position of node i,2 coordinate system relative to the coordinate system of the moving beam is presented by 4×4 homogeneous transformation matrix $\mathbf{T}_{i,2\rangle i}$, i.e.:

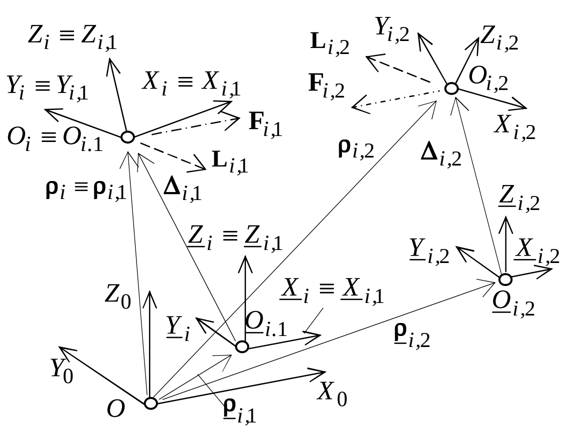


Figure 2: Presentation of the nodes as moving coordinate systems

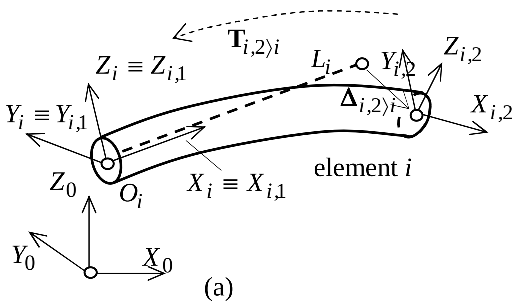


Figure 3: Finite beam element in relative coordinates

$$\mathbf{T}_{i,2\rangle i} = \begin{bmatrix} {}^{3.3}\boldsymbol{\tau}_{i,2\rangle i} & {}^{3,1}\boldsymbol{\rho}_{i,2\rangle i} \\ {}^{1,3}\mathbf{0} & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & -\theta z_{i,2\rangle i} & \theta y_{i,2\rangle i} & L_i + \Delta x_{i,2\rangle i} \\ \theta z_{i,2\rangle i} & 0 & -\theta x_{i,2\rangle i} & \Delta y_{i,2\rangle i} \\ -\theta y_{i,2\rangle i} & \theta x_{i,2\rangle i} & 0 & zy_{i,2\rangle i} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -q_{i_6} & q_{i_5} & L + q_{i_1} \\ q_{i_6} & 0 & -q_{i_4} & q_{i_2} \\ -q_{i_5} & q_{i_4} & 0 & q_{i_3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

which for small rotational deflections does not include trigonometric functions. With this assumption that the homogeneous transformation matrix is linear for the relative coordinates $q_{i_j}, j = 1,2,...6$ of element i that include the small translational and rotational flexible deflections. ${}^{3,3}\boldsymbol{\tau}_{i,2\rangle i}$ is 3×3 rotational matrix; the left superscripts point out (used if needed) the matrix dimensions (rows, columns).

Since the beam coordinate system is moving the flexible deflections of node i,1 relative to the element coordinate system are zero and the matrix vector of the beam relative coordinates $\mathbf{q}_i$ is $\mathbf{\Theta}_{i\rangle i} = \left[\mathbf{\Delta}^{\backslash}_{i,2\rangle i} \quad \mathbf{\theta}^{\backslash}_{i,2\rangle i}\right]^{\backslash} = \mathbf{q}_i = \left[q_{i_1} \quad q_{i_2} \quad \dots \quad q_{i_6}\right]^{\backslash}$ (see Equation 1), as well as, the beam mass and stiffness matrices are constant.

2.2 Finite elements in absolute coordinates

In Figure 4 the coordinate systems of the beam element and its nodes are depicted without the beam shape and configuration. As we said above the beam coordinate system and that of node i,1 coincide and their relative position does not change during the beam motion. That is why we shall regard the motion of node coordinate systems taking into account which one is of the beam. Loading the nodes with the inertia, elastic and external forces, just like free rigid bodies they will implement, for a small increment of time, small increments of the motion parameters. Six are the minimal number of the coordinates that define the position as of a rigid body, so of a node. When a flexible element implements global motion in space overlapped by flexible deflections even small increments of the node coordinates with respect to their initial position do not present the real deformation of the flexible element.

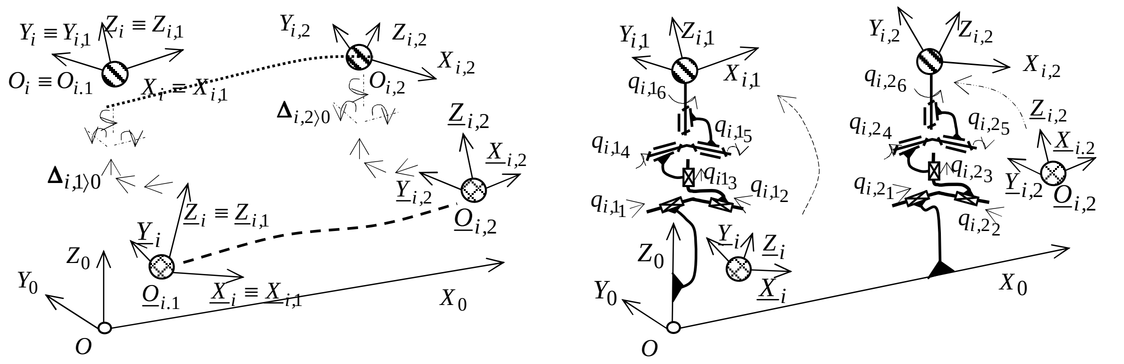
The solution of the problem how to use the FET in case of global motion and large flexible deflections consists in:

a) definition of proper node reference frame coordinates for the case of small increments of time, respectively, for small increments of that coordinates;

b) estimation of the magnitudes of the elastic deformations.

The solution of problem (a) consists in selection of node reference frame coordinates compatible with the coordinates used in FET. The solution of problem (b) consists in separation of the flexible deflections from the global motion of the node coordinate systems.

In Figure 4, similar to Figure 2, the coordinate systems of nodes *i*,1 and *i*,2, $X_{i,k}Y_{i,k}Z_{i,k}$; $i=1,2$, are located in their final configuration. The initial positions of the node coordinate systems are presented by the same but underlined notations. FET claims for the small deflections to be read with respect to the system initial configuration.



(a) possible small global deflections (b) virtual joint presentation of the deflections

Figure 4: Nodes as free coordinate systems and corresponding motion coordinates

In Figure 4 (a) these deflections are presented by arrows and in Figure 4 (b) they are presented by virtual translational and rotational joints. These presentations are one and the same but give proper comparison to the free moving objects, constraint kinematics, joint coordinates, as well as, reveal the generality of the methodology proposed in the paper.

The homogeneous transformation matrices $\mathbf{T}_{i,1\rangle 0}$, $\mathbf{T}_{i,2\rangle 0}$ of the absolute position of the node coordinate systems *i*,1 and *i*,2, respectively, are expressed by the matrices $\underline{\mathbf{T}}_{i,1\rangle 0}$ and $\underline{\mathbf{T}}_{i,2\rangle 0}$ of their initial configuration and the transformation matrix because of the small node deflections.

$$\mathbf{T}_{i,k\rangle 0}=\left[\begin{array}{c|c}\boldsymbol{\tau\Delta}_{i,k\rangle 0}\cdot\underline{\boldsymbol{\tau}}_{i,k} & \underline{\boldsymbol{\rho}}_{i,k}+\boldsymbol{\Delta}_{i,k}\\ \hline {}_{1,3}\mathbf{0} & 1\end{array}\right]=\begin{bmatrix} & & & \underline{\rho x}_{i,k}+q_{i,k_1}\\ & \boldsymbol{\tau\Delta}_{i,k\rangle 0}\cdot\underline{\boldsymbol{\tau}}_{i,k} & & \underline{\rho y}_{i,k}+q_{i,k_2}\\ & & & \underline{\rho z}_{i,k}+q_{i,k_3}\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$\boldsymbol{\Delta}'_{i,k\rangle 0}=\left[q_{i,k_1}\quad q_{i,k_{21}}\quad q_{i,k_3}\right]'\;; \tag{2}$$

$$\boldsymbol{\tau\Delta}_{i,k\rangle 0}=\begin{bmatrix}0 & -\theta z_{i,k\rangle 0} & \theta y_{i,k\rangle 0}\\ \theta z_{i,k\rangle 0} & 0 & -\theta x_{i,k\rangle 0}\\ -\theta y_{i,k\rangle 0} & \theta x_{i,k\rangle 0} & 0\end{bmatrix}=\begin{bmatrix}0 & -q_{i,k_6} & q_{i,k_5}\\ q_{i,k_6} & 0 & -q_{i,k_4}\\ -q_{i,k_5} & q_{i,k_4} & 0\end{bmatrix}\quad k=1,2,$$

where $\mathbf{q}_{i,k}=\left[\boldsymbol{\Delta}'_{i,k\rangle 0}\quad \boldsymbol{\theta}'_{i,k\rangle 0}\right]'=\left[q_{i,k_1}\quad q_{i,k_2}\quad \dots\quad q_{i,k_6}\right]'$, $k=1,2$ are the deflections of node *k* from element *i* with respect to the absolute reference frame (index 0), which are also the coordinates of the free moving nodes. To apply Equation (2) the deflections are to be small although they do not present the flexible deformations only. The deflections q_{i,k_n} , $k = 1,2; n = 1,6$ include also the global motion of the nodes. To define the flexible deformations of the element *i* the relative position of node *i*,2 with respect to node *i*,1 that coincides with the element coordinate system *i*, should be defined, i.e.:

$$\mathbf{T}_{i,2\rangle i,1} = \mathbf{T}_{i,1\rangle 0}^{-1} \cdot \mathbf{T}_{i,2\rangle 0} = \left[\underline{\mathbf{T}}_{i,1\rangle 0} \cdot \mathbf{T\Delta}_{i,1}\right]^{-1} \cdot \underline{\mathbf{T}}_{i,2\rangle 0} \cdot \mathbf{T\Delta}_{i,2} = \\ = \mathbf{T\Delta}_{i,1}^{-1} \underline{\mathbf{T}}_{i,1\rangle 0}^{-1} \cdot \underline{\mathbf{T}}_{i,2\rangle 0} \cdot \mathbf{T\Delta}_{i,2} \tag{3}$$

The matrix elements $\mathbf{T}_{i,2\rangle i,1} = \mathbf{T}_{i,2\rangle i}$ are actually the flexible deformations (rotations and translations) and, since they are to be small, the rotations could be presented without trigonometric functions, i.e.:

$$\mathbf{T}_{i,2\rangle i} = \begin{bmatrix} 0 & -\theta z_{i,2\rangle i} & \theta y_{i,2\rangle i} & L_i + \Delta x_{i,2\rangle i} \\ \theta z_{i,2\rangle i} & 0 & -\theta x_{i,2\rangle i} & \Delta y_{i,2\rangle i} \\ -\theta y_{i,2\rangle i} & \theta x_{i,2\rangle i} & 0 & zy_{i,2\rangle i} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Taking into account that the relative deformations of node 1 are zero, the flexible deformations of the element *i* relative to its own reference frame are as follows $\mathbf{\Theta}_{i\rangle i} = \left[\mathbf{\Delta}^{\backslash}_{i,2\rangle i} \;\; \mathbf{\theta}^{\backslash}_{i,2\rangle i}\right]^{\backslash}$ and appear explicitly in matrix $\mathbf{T}_{i,2\rangle i}$ without trigonometric functions.

To realize the numerical algorithm with the proposed coordinates the integration process should be realized with small increments of time, respectively, with small increments of the nodal coordinates. For this purpose on every step of the time increment the new configuration of the system is the initial configuration for the next iteration and the absolute nodal coordinates start again with zero values. That process is not disadvantage since during the numerical integration of the dynamic equations the new positions of the coordinate systems are to be recalculated and the new mass and stiffness properties of the bodies are to be computed. Detailed statements of the procedures are presented in [29].

3 FORCES LOADING THE NODES OF FLEXIBLE ELEMENTS

The forces loading the nodes are internal elastic forces, external forces and, during the global motion, the inertia forces. Regarding static structures the internal elastic and external forces are in equilibrium. The external forces could be included in the dynamic equations using the principle of the virtual work and here we will not discuss this matter. The main subjects related to the dynamics in FET these are the internal elastic and inertia forces. The stiffness matrices play the major role in computation of the elastic forces, while the mass matrices are used for computation of the inertia force. Since the nodes of the flexible elements are considered coordinate systems implementing free motion in space they are kinematical independent, which means that the coordinates of a node do not depend on the coordinates of the adjacent ones. Unfortunately the static and dynamic equations cannot be solved separately for the nodes since deflections of a node give rise of elastic forces added to the adjacent nodes. The same holds for the velocities and accelerations and the resulting inertia forces.

3.1 Elastic and inertia forces of flexible elements

In the paper special attention is paid to the elastic and inertia forces loading the nodes of flexible elements. As it could be seen above the relative flexible deformations of nodes with respect to the element coordinate system, $\mathbf{\Theta}_{i\rangle i}$ play significant role for FEAC. The relative flexible deformations for the first node, in which the moving coordinate system of the element is located, are equal zero. The matrix-vector $\mathbf{\Theta}_{i\rangle i}$ is compiled without the deformations in the first node and is used for computation of the elastic forces loading both element nodes, i.e.:

$$\mathbf{\Xi}_{i\rangle i} = -\mathbf{K}_{i\rangle i} \cdot \mathbf{\Theta}_{i\rangle i}\,, \tag{5}$$

where for a beam element $\mathbf{K}_{i\rangle i}$ is 12×6 matrix compiled from the last six columns of the conventional FET stiff matrix of element *i* relative to its own beam coordinate system. The first six elements of $\mathbf{\Xi}_{i\rangle i}$ are the forces and torques loading the coordinate system of node *i*,1 and the last six elements are the forces and torques loading node *i*,2.

Taking into account that the nodes are coordinate systems and summarizing the results for FEAC methodologies one concludes that the dynamics analysis of multibody systems of rigid and flexible bodies could be analyzed by a common methodology. As it was discussed above one of the major problems for application of the FET in dynamics of rigid and flexible mutibody systems it is definition of the node accelerations, respectively, the inertia forces. In [18] generalized Newton-Euler dynamic equations for rigid and flexible bodies discretized using the FET are derived. The inertia forces in the nodes of the flexible elements are expressed with respect to the quasi velocities and accelerations and are independent of the kind of coordinates. The notations for the quasi-velocities and accelerations of node *i,k* are $\dot{\mathbf{\Theta}}_{i,k} = \left[\mathbf{v}_{i,k}^{\backslash} \;\; \boldsymbol{\omega}_{i,k}^{\backslash}\right]^{\backslash}$, $\ddot{\mathbf{\Theta}}_{i,k} = \left[\mathbf{a}_{i,k}^{\backslash} \;\; \boldsymbol{\varepsilon}_{i,k}^{\backslash}\right]^{\backslash}$ where $\mathbf{v}_{i,k}^{\backslash}$ and $\mathbf{a}_{i,k}^{\backslash}$, respectively, $\boldsymbol{\omega}_{i,k}^{\backslash}$ and $\boldsymbol{\varepsilon}_{i,k}^{\backslash}$, are the node linear and angular velocities. For a flexible element *i* with two nodes (indices 1 and 2), and 12×12 mass matrix **M** the inertia forces and torques, 12×1 matrix vector $\mathbf{F}_i = \left[\mathbf{P}_{i,1}^{\backslash} \;\; \mathbf{\Phi}_{i,1}^{\backslash} \;\; \mathbf{P}_{i,2}^{\backslash} \;\; \mathbf{\Phi}_{i,2}^{\backslash}\right]^{\backslash}$, is as follows:

$$\mathbf{F}_i = \mathbf{M}_i \cdot \begin{bmatrix} \ddot{\mathbf{\Theta}}_{i,1} \\ \ddot{\mathbf{\Theta}}_{i,2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}_{i,1}^{\times} & {}^{3,3}\mathbf{0} & {}^{3,3}\mathbf{0} & {}^{3,3}\mathbf{0} \\ \mathbf{v}_{i,1}^{\times} & \boldsymbol{\omega}_{1}^{\times} & {}^{3,3}\mathbf{0} & {}^{3,3}\mathbf{0} \\ {}^{3,3}\mathbf{0} & {}^{3,3}\mathbf{0} & \boldsymbol{\omega}_{i,2}^{\times} & {}^{3,3}\mathbf{0} \\ {}^{3,3}\mathbf{0} & {}^{3,3}\mathbf{0} & \mathbf{v}_{i,2}^{\times} & \boldsymbol{\omega}_{i,2}^{\times} \end{bmatrix} \cdot \mathbf{M}_i \cdot \begin{bmatrix} \dot{\mathbf{\Theta}}_{i,1} \\ \dot{\mathbf{\Theta}}_{i,2} \end{bmatrix} - \mathbf{M}_i \cdot \begin{bmatrix} \boldsymbol{\omega}_{i,1}^{\times} \cdot \mathbf{v}_{i,1} \\ {}^{3,1}\mathbf{0} \\ \boldsymbol{\omega}_{i,2}^{\times} \cdot \mathbf{v}_{i,2} \\ {}^{3,1}\mathbf{0} \end{bmatrix}, \tag{6}$$

where the notation “×” means skew-symmetric matrix. In Equations (8) all vectors and matrices are to be with respect to one and the same coordinate system. For elements with more than two nodes, for examples plates and shells, their elements are to be arranged in a similar order. The forces of elements in the common nodes are to be summarized.

The general Newton – Euler dynamic equations are applicable as for rigid, so for flexible systems and provide the basis for effective general recursive algorithms and program systems for multibody system dynamics analysis. The elements of the homogeneous transformation matrices are explicitly defined with respect to the deflections and computational expenses are low. The numerical procedure uses constant relative mass and stiffness matrices for definition of the elastic and inertia forces. Here we do not discuss the methodology for application of the principle of virtual work for reduction of forces to the system coordinates and deriving the dynamic equations since that is well known procedure. Possible kinematic constraints are to be expressed in the same way as in the case of rigid bodies.

4 DYNAMICS OF STRUCTURES SUBJECT TO SEISMIC EXCITATIONS

The methodology based on the FEAC has been successfully applied for large structures with many flexible elements and nodes that compose closed chains with many dependent coordinates. The approach is successfully applied for simulation of complex structures subject to external disturbances like seismic excitations.

Two methods are used for simulation of seismic excitation, ground – structure force interaction method and the basement acceleration method. The first one uses data from simulation of ground structure interaction and the output data so obtained, the forces loading the basement, are input data for the dynamics analysis of the structure. This approach is the easiest for dynamics simulation of the structures and is fully compatible with the methods discussed above.

The acceleration based approach uses statistical data for the accelerograms for specific regions and the motion characteristics of the basement are input data for the dynamics simulation process. Actually, the basement acceleration and the integrated velocities and displacements are reonomic constraints for the dynamic equations. Further down an approach for solution of the dynamic equations subject to reonomic constraints and on application of FEAC is presented.

Using the principle of the virtual work the forces are reduced to the system coordinates to derive the dynamic equations

$$\mathbf{M} \cdot \ddot{\mathbf{q}} + \mathbf{B}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{S} \tag{7}$$

where $\mathbf{S}$ is $n \times 1$ matrix-vector of the generalized forces, $\mathbf{B}(\mathbf{q}, \dot{\mathbf{q}})$ is velocity depend term.

Earthquake shaking affects forced motion of the structure basement. For a region using statistical data the accelerograms for the space displacements are as follows:

$$\ddot{q}_i = \ddot{q}_i(t), i = 1,2,3 \tag{8}$$

These are reonomic constraints that depend on time. For a multibody system with dof n which motion is described by ODE, Equation (7), subject to m reonomic constraints, Equation (8), the dynamic equations are presented as follows:

$$\mathbf{M} \cdot [\ddot{q}_1 \ \cdots \ \ddot{q}_{k+1} \ \cdots \ \ddot{q}_{k+m} \ \cdots \ \ddot{q}_n]^{\backslash} + \mathbf{B}(\mathbf{q}, \dot{\mathbf{q}}) = [S_1 \ \cdots \ S_{k+1} \ \cdots \ S_{k+m} \ \cdots \ S_n]^{\backslash} \tag{9}$$

In Equation (9) the coordinates $\ddot{q}_i = \ddot{q}_i(t), i = k+1, k+2, .., k+m$ are known, while the generalized forces $S_i = S_i(t), i = k+1, k+2..., k+m$ are unknown. The dynamic equations, Equation (9), are transformed as follows:

$$\underline{\mathbf{M}} \cdot [\ddot{q}_1 \ \cdots \ S_{k+1} \ \cdots \ S_{k+m} \ \cdots \ \ddot{q}_n]^{\backslash} + \mathbf{B}(\mathbf{q}, \dot{\mathbf{q}}) = -\underline{\underline{\mathbf{M}}} \cdot [\ddot{q}_{k+1} \ \cdots \ \ddot{q}_{k+m}]^{\backslash} + \underline{\mathbf{S}} \tag{10}$$

where the matrices $\underline{\mathbf{S}}$, $\underline{\underline{\mathbf{M}}}$, $\underline{\mathbf{M}}$ are compiled according to the indices of the coordinates.

The well know beam of Kane et al. [30] is an example of the dynamic analysis subject to a reonomic constraint. It is a horizontal cantilever beam that rotates around a vertical axis. The rotation is prescribed by its angular velocity and this example is a test for many researchers analyzing their methods. Here the same example is used to present FEAC method in case of reonomic constraints differentiating the velocity function to obtain the angular acceleration as inpt data. In Figure 5 the results of the simulation are presented. The angular velocity of the beam input shaft (coordinate q_1) are presented in the Figure 5 (a). The mass and inertia characteristics of the beam are: (all measures are in SI UNIT): modulus of elasticity $E = 7 \cdot 10^{10}$; shear modulus $G = 2.8 \cdot 10^{10}$; length L=10.; density $\rho = 3000$; cross section area $S = 4 \cdot 10^{-4}$; cross-section moments of inertia $I_x = I_z = \frac{1}{2} I_c = 2.10^{-7}$. The beam discretization is of three elements. In Figure 5 (b) the experimental results are presented while in Figure 4 (c) the results of the simulation using FEAC are depicted.

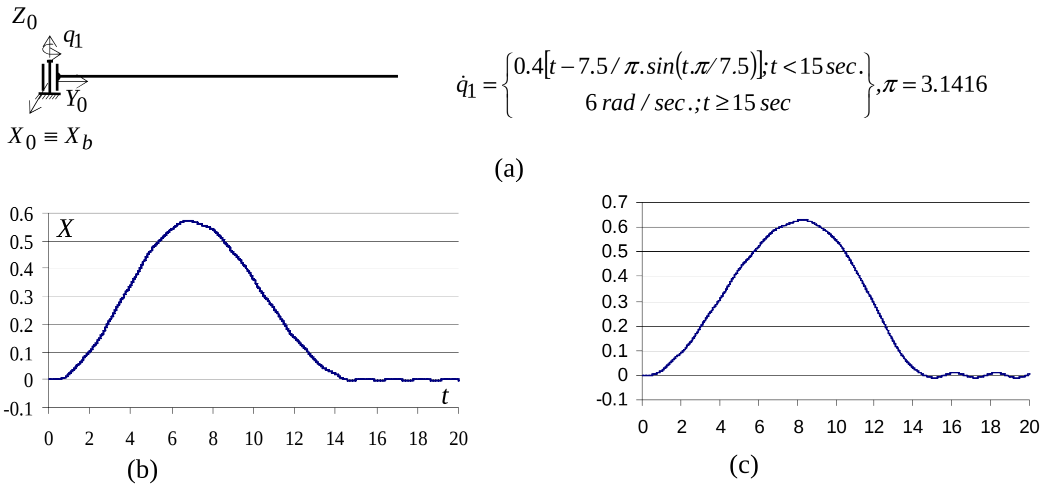


Figure 5: Kane's flexible beam [30]:
(a) the beam and driving motion; (b) experimental results; (c) results of the FEAC.

5 EXAMPLE

In Figure 6 an example of space flexible structure compiled of rigidly connected beams that build many closed contours is presented. The flexible beams are consequently numbered, as well as the nodes are also numbered by numbers in ellipses. The structure is mounted on a basement imposed on external excitation – one dimensional motion q_1 in the horizontal plane and rotational excitation q_2 along the vertical axes. Because of the external excitations the basement implements two dimensional motion which functions of the accelerations are also shown in the figure. In the function of $\ddot{q}_1$ **G** is the earth acceleration. The structure will be simulated using the method of FEAC proposed in the paper.

In Figure 7 the nodes of the beams are shown as a moving coordinate system. The nodes connected to the basement are assumed connected to a rigid body (the basement). The nodes are loaded by internal elastic forces $\mathbf{F}_i, 1,2,...,16$ and torques $\mathbf{L}_i, 1,2,...,16$ caused as a result of the flexible deflections. The nodes are loaded by inertia forces and torques (not shown on the figure) exaggerated because of forced motion of the basement and the resulting space motion of the whole structure. The nodes 1 – 4 are connected to the basement and are considered part of the basement as a rigid body, i.e.: their motion characteristics coincide with that of the basement q_1 and q_2.

The size, the mass and the stiffness properties of the vertical beams are the same as these of the beam of Figure 5. The lengths of the horizontal beams are 0.7 of the lengths of the vertical beams.

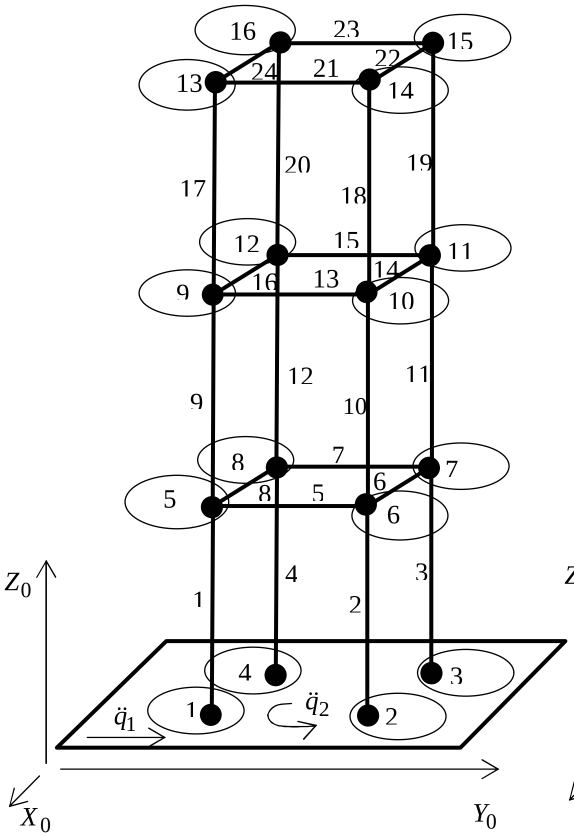


Figure 6: Flexible space beam structure. Elements and nodes

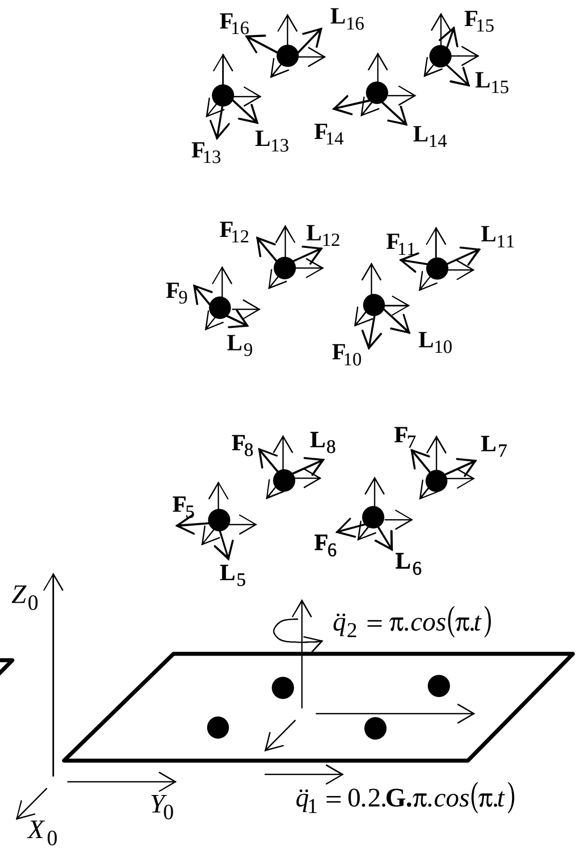


Figure 7: The dynamic model of the structure as free nodes – coordinate system

The dynamic model of Figure 7 is derived using the method proposed in the paper and the incremental approach for numerical integration of the dynamic equations [29, 31]. In Figure 8 the configuration of the structure for three instants of time are depicted.

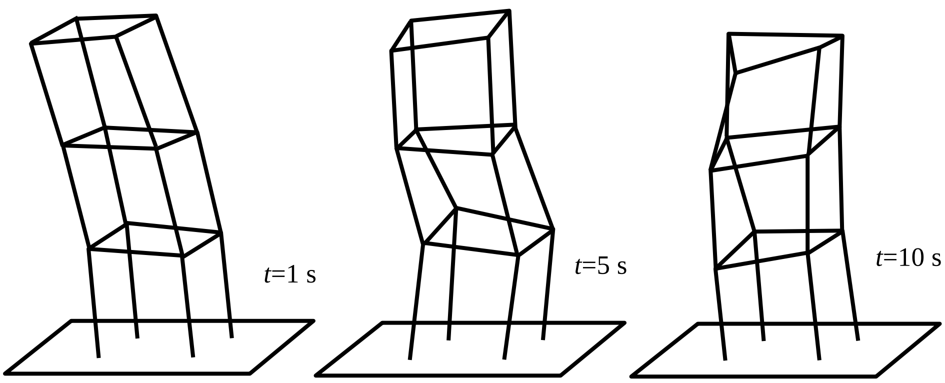


Figure 8: Configuration of the structure for three instants of time

6 CONCLUSIONS

- A method of Finite Elements in Absolute Coordinates is applied for dynamic simulation of large flexible structures subject to external excitations.
- The method is applicable for structures with many flexible elements and degrees of freedom. It is realized using simple and straight forward procedure with no kinematic constraints, as well as, algebraic constraint to the dynamic equations.
- Since the kinematic constraints are substituted by internal elastic forces of the elements large amplitudes of these forces are observed that could decrease the precision of the numerical algorithm and increase the time of simulation. This effect could be observed in the test example of Figure 5.
- The next step in application of the method will consist in development of a numerical procedure for solution of ordinary differential equations that truncate the high amplitudes of the elastic forces.

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