ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

# MODELLING OF LONG WAVES (SEICHES) IN CASCAIS BAY

**Vera Bras**[1], **Antonio Trigo-Teixeira**[2]

[1] Instituto Superior Tecnico
Lisbon University
AV. Rovisco Pais, 1 1049-001 Lisboa
vera.bras@tecnico.ulisboa.pt

[2] CERIS – Instituto Superior Tecnico
Lisbon University
AV. Rovisco Pais, 1 1049-001 Lisboa
trigo.teixeira@tecnico.ulisboa.pt

**Keywords:** Seiches, harbour resonance, periodogram, pass-band filter, numerical modelling, CGWAVE.

***Abstract.*** *This study is focused on the analysis and characterization of long waves (seiches) in Cascais Bay-Portugal, where the phenomenon is known to occur. For this purpose, time series records of water surface elevation (WSE) from the tidal gauge were analyzed in time and frequency domains. The periodicities of the long waves were found, firstly with a plotting of time series, where two waves appear immediately: the astronomic tide (T=12h 25min) and a long wave with period of approximately (T=12min), and secondly with the application of the spectral FFT analysis to obtain the periodogram. Before the application of the FFT, the WSE elevation time series was first filtered for the astronomic signal using harmonic analysis and then a pass-band of second order filter was used to removal the noise. The frequency analysis allows identification of hidden periodicities and conclude that most of the spectral energy was concentrated on a single frequency. Finally, some statistics were calculated, which indicated the presence of long waves in the tidal record for most of the period of analysis, without apparent relationship with the season of the year: maritime summer (May-Oct) or maritime winter (Nov-Apr).*

*On a second phase the setup of a 2D finite element model to solve the mild-slope equation (MSE) was done to study the phenomenon of long wave propagation both into the bay and in a small craft harbour (Cascais Marina). The model used was CGWAVE* [1] *and it was forced with long waves of different frequencies on the boundary to investigate the appearance of seiches both in the bay and in the harbour. The model permits the identification of resonant periods and the location of nodal lines (where vertical WSE oscillation is nil and water velocity is at a maximum) and ventral lines (where just the opposite occurs). The hydrodynamic drag on boats moored on nodal lines is a cause of great trouble on the Marina operation. It might be concluded that seiches occur, not only in the marina basin, but in all water body of the bay, so it is a natural phenomenon on the area.*

## 1 INTRODUCTION

The present work focused on the modelling of long waves (seiches) in Cascais Bay- Portugal, which is a frequent local phenomenon. It is believed that the word “seiche” derives from the Latin word “*siccus*”, which means dry or expose. So, long waves (seiches) are “standing waves” as perceived by the periodic oscillations of the surface of an the enclosed basin or semi-enclosed body of water (lake, gulf, inland sea, bay, harbour) with periods from a few minutes to a few hours caused it by such phenomena as atmospheric pressure changes, winds, tidal currents and earthquakes. Usually, it is considered as a transient event because after some time subsequently to the end of the geophysical event that causes it, it eventually dissipates. However, in some places, it is also observed continuously with small amplitudes. [2], [3]

A natural or an artificial basin may offer good conditions of shelter for the open sea, particularly for wind waves (short waves), however both the geometry of the basin and their internal reflections can efficiently amplify the incident long waves, even for those with small amplitudes. So, this phenomenon of resonant oscillations is called seiches, harbour oscillations or harbour surging and is related to specific periods of the incident waves on the structure. For a basin, if its period of the oscillation is close to that the period of the incident long wave (or other possible excitation), an extreme seiche could result. Extreme seiches induce large oscillations and strong currents that might cause damage to ships and breaking moorings lines. From what was exposed, the study of long waves in coastal zones is very important. [4]–[7]

On the literature, various authors were dedicated to study of the resonant properties of harbour basins, which would be divided in two main groups: simple geometry (Miles and Munk 1961; Lee 1992; Ippen and Goda 1963; etc.) and complex geometry (Knapp and Vanoni 1945; Wilson 1960; Lee 1969; etc.). The first group provides theoretical expressions for approximated calculation of the natural period oscillation for closed and semi-enclosed basins. For the second, the use of theoretical formulas cause discrepant values of the reality because the non-linearity of the basin’s shape, so physical or numerical models had to be used in these cases. Numerical models are a powerful tool for studying long waves in basins with complex geometry (real basins) and for more accurate results, when the period of the waves is higher than 400 *s* they can replace the physical models. [8], [9]

Nevertheless it was important to refer that, theoretically, in semi-enclosed basins, like bays, it’s expected a nodal line across of this entrance, because the waters within the basin must communicate with waters outside. [10]

Recently, the implementation of modern’s digital tidal gauge in the coast allowed a faster recording rate (range of seconds), which is a huge advantage in the use of time series data to study long waves. However, these measurements are mostly used for analyzing extreme events such as tsunamis. [11]

This study had two main objectives. The first one was to analyze the time records of water surface elevation (WSE), from the digital tidal gauge of Cascais, for the characterization of long waves (period and amplitude). These periodicities were found by making the analysis on the time and frequency domains and, finally, the results of both methods were compared.

The second main objective was to evaluate the resonance phenomenon inside of the marina, using the CGWAVE model for this hydrodynamic modelling, because the complex shape of the marina. Therefore it is possible to match the information relatively to the periods of seiches and say if the range of periods, that were observed, correspond to the resonance periods of the basin, using the numerical modelling.

## 2 ANALYSIS OF THE LONG WAVES (SEICHES)

### 2.1 Introduction

In the ocean there are different types of long waves, generated by different phenomena and with different periods and wavelengths. When waves have wavelengths much higher than the depth water, they are called long waves and have periods between 5 *min* and 24 *h* (tides). These waves have a frequency band with relatively low spectral energy, which is between the frequency band of tides and wind waves, both with higher energetics bands. [5]

The measurements of long waves are not direct; the time series from the tidal gauge can be described by:

$$\eta(t) = \bar{\eta}(t) + T(t) + R(t) \qquad (1)$$

Where, $\eta(t)$ is the observed water surface elevation which varies with time; $\bar{\eta}$(t) is the mean sea level which changes slowly with time; $T(t)$ is the astronomical tidal component and $R(t)$ is the residual component (meteorological component and noise).

So, the meteorological tide originated by the meteorological phenomena, such as atmospheric pressure variation and wind's effect, which resulted on water oscillations of just a few centimeters and it was responsible for the difference between the predicted and observed levels. Therefore, the long wave signal has to be isolated from the records observed of WSE in the tidal gauge. The procedures of this treatment of the times series and the following characterization of the seiches, in time and frequency domains, are described below.

### 2.2 Data processing

The upgrade of the analogic coastal tide gauge into the digital one allowed a faster sampling rate, which permitted a better identification of long waves in the records because more sampling points (*N*) provided more information and, consequently, more accuracy.

The tidal gauge of Cascais is operated by the IGEO (Portuguese Geographic Institute) and the digital time series records of WSE are available online in their website since 2007. These are discrete time series because the observations are taken on equally spaced intervals, in this case with sampling rate of $\Delta t = 5\ s$ and it results in 17280 lines of information per day. When the daily records had failures (less than 17280 lines) they were not used. For the present work 17 months were analyzed: $1^{st}$ semester of 2015 and all year of 2014, except February (because it had gaps).

### 2.3 Time series graphs

Usually, the first step in the analysis of time series is to plot the data over time to obtain some characteristics like trend, variation in the time, periodicities and other fluctuations. Regarding this step, two types of waves were immediately apparent: the astronomic tide with a period of (T=12*h* 25*min*) and a long wave with a period of approximately (T=12 *min*).

The astronomic tide is generated by the combined effect of gravitational force that the Sun and Moon exerts on the Earth's and in the Cascais bay is of the mesotidal type (tidal range of 4.0 *m* in Spring Tides) and is semi-diurnal.

### 2.4 Filtering of the time series for the astronomic signal

When time series can be predicted exactly from past observations, because the successive observations are dependent, they are called deterministic, otherwise they are called stochastic. [12]

The deterministic component of the time series records of WSE corresponds to the astronomic tide and the stochastic to the meteorological tide. The first one can be predicted by a harmonic analysis, based on the movement of the Sun and the Moon, and in this step two methods were used: tide tables from IH (Portuguese Hydrograph Institute) and Wx-Tide software.

The astronomic component was determined based on these two methods, which had the same trends but different WSE. This difference can be explained with the global sea level rising, about 1.3 *mm* per year. While the tidal height from the tables does not take this factor in count, but refers that it should be added about 0.10 *m* to the predicted heights, the tidal height from the software had updated harmonic constants and, consequently, higher WSE.

The Wx-Tide just allows the minimum predicting rate of $\Delta t$= 60 *s* and, to assure that this parameter in this process is equal to the one from the tidal gauge ($\Delta t$=5 *s*), it was decided to consider the method of the tidal tables. However, an adjustment was made of about 0.10 *m*, as recommended. In the Figure 1 the results of the previous methodology for an extreme event of seiche are presented.

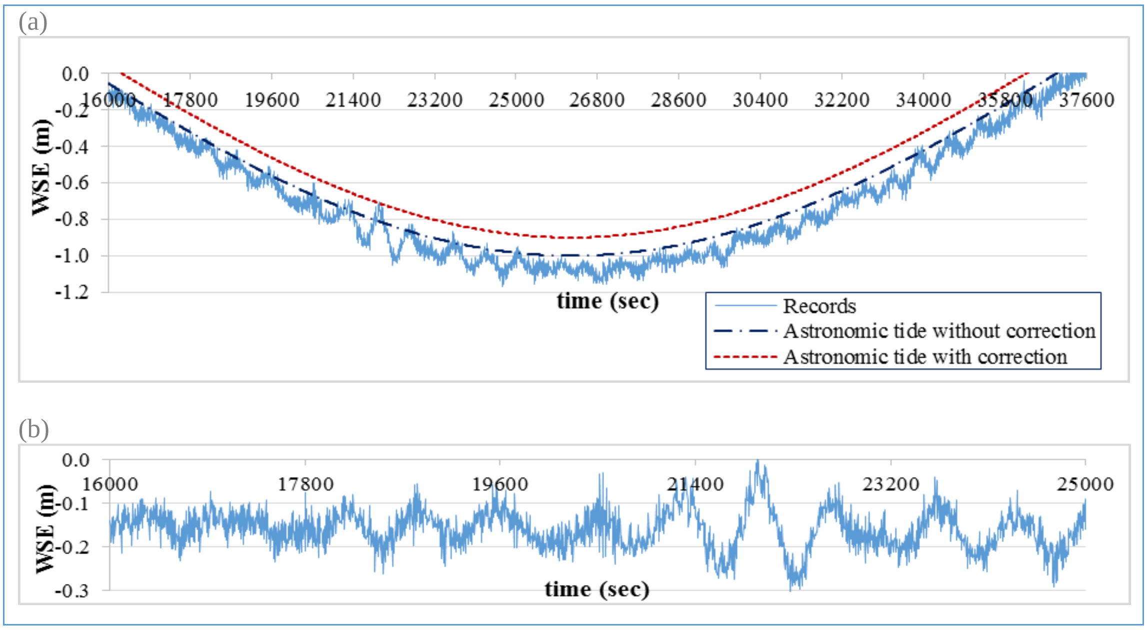


Figure 1: (a) Time series records of WSE (6*h*) from the Cascais tide gauge and astronomic trend; (b) astronomic tide removal for the time series – exert (2*h* 30*min*).

By observing the Figure 1, it is possible to see that the astronomic tide trend was removed by the difference between the records of WSE and the predicted tide by the harmonic analysis. The oscillations of the seiches had, usually, small amplitudes when compared with the tide oscillations. Thus, the long wave oscillations had a periodical trend which can be approximately described for a sinusoid, given by:

$$\eta(t) = \overline{\eta} + Asen(\omega t + \phi) \quad (2)$$

Where, $\eta(t)$ is the water surface elevation at time $t$; $\overline{\eta}$ the mean level of water surface elevation; $A$ the amplitude of the oscillation; $\omega$ the angular velocity, which can be obtained by $2\pi/T$, where $T$ is the period and, finally, $\varnothing$ is the phase lag relative to some defined time zero.

### 2.5 Pass-band filter of second order

In fact, the noise is a random signal and consequently it can be eliminated in time domain. In this phase a band-pass of second order filter (Butterworth filter) was applied to remove the noise in frequency domain. A “butter” function was used from the “Signal Processing Toolbox” - MATLAB software.

It was decided that the digital Buttherworth filter had a normalized cutoff frequency between 1/400 *Hz* and 1/900 *Hz*, which corresponds to the interval of frequencies of the seiches. This means that all signals out of this range are eliminated.

### 2.6 Statistical analysis

By direct observation, when regularity in the distances between peaks is observed, this period of time was counted, as well as the number of intervals between peaks. The period of this long wave is calculated by the ratio between this interval of time and the number of peaks. Half of the distance in vertical between two consecutives peaks gives the amplitude of this oscillation. [13]

In the Figure 2 some results of the statistical analysis can be observed.

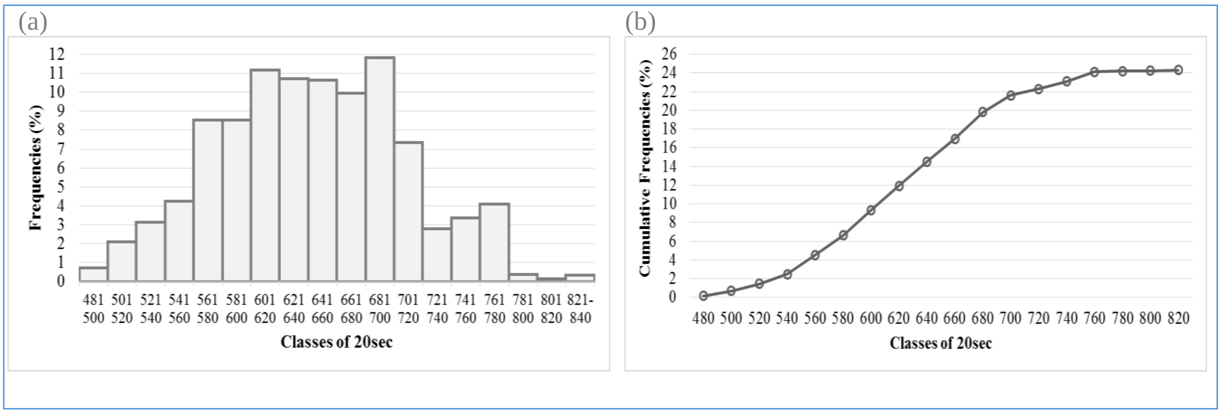


Figure 2: (a) Frequency Histogram in relation to the total time of observation of the seicha phenomenon (2420*h*) (b) cumulative frequency curve in relation to the total time of analysis (9960*h*).

According to the Figure 2 it is possible to conclude that the seiches occur with a frequency of 24.30% and the most frequent periods are in the range of 600-700 *s* (10.0-11.7 *min*). The phenomenon occurs for periods between 480 *s* and 840*s* (8-14 *min*).

Since low pressures are associated to dangerous and extreme meteorological phenomenon, an attempt to correlate them with an occurrence of seiche and pressure variations was made. However, the results did not indicate any type of relation with this possible cause of the origin of seiches in Cascais-bay.

### 2.7 Spectral analysis - Periodogram with the FFT algorithm

The Fourier analysis is an advantageous technique to represent time series in terms of the frequencies distribution. If certain conditions are satisfied, any function may be formulated as a sum of a series of sines and cosines of a spectral resolution ($\Delta f = 1/N\Delta t$). However, the spectral resolution depends only on the length of the time records, which means that short time series’ performance is not as good. [14]

The periodogram provides the graph representation of the PSD (Power Spectral Density) *versus* spectral resolution, with a FFT algorithm.

The results of this spectral analysis were presented in Figure 3, for an extreme event of seiche and for different spectral resolutions.

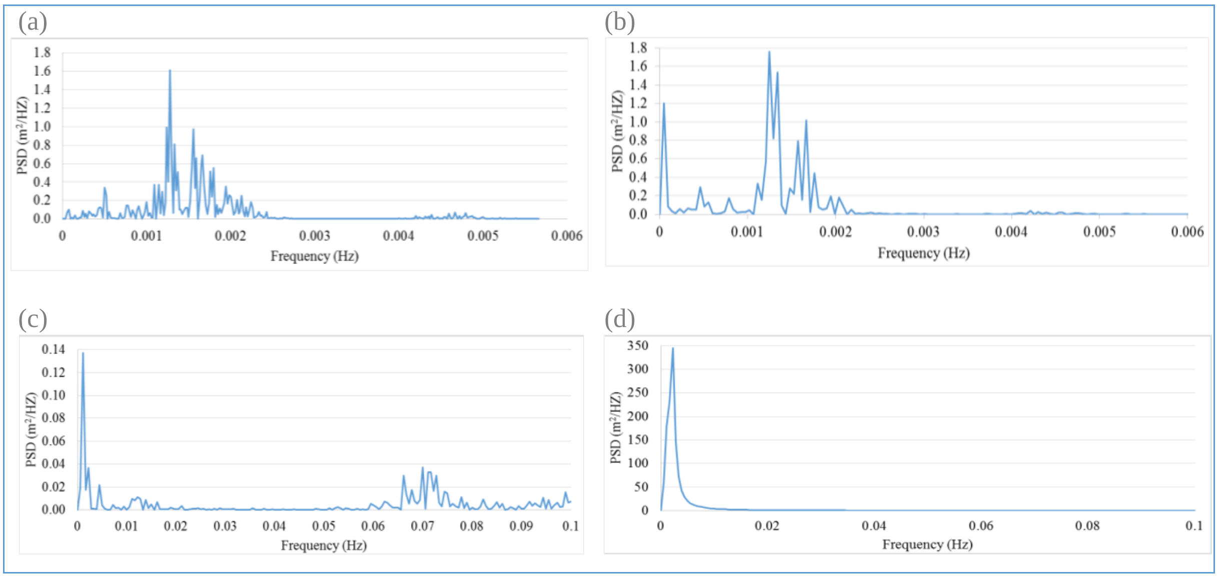


Figure 3: Identification of seicha phenomenon used the periodogram method. (a) computed to time of seiche (15*h*), with $T_{peak} = 783$ *s*; (b) computed to 6*h*, with $T_{peak} = 800$ *s*; (c) computed to 30 *min*, with $T_{peak} = 900$ *s* (d) application of the pass-band filter (time series of 30 *min*), with $T_{peak} = 485$ *s*.

In first instance, it was observed that, for a greater number of samplings, the resolution is finer, and a correspondent spectral plot is noisier and irregular. So, a more acceptable performance is obtained with 30 *min* of times series, which provides 360 time observations but only 180 (*N/2*) were used in the FFT algorithm. Thus, after the application of the pass-band filter the noise signal (highs frequencies) was completely eliminated.

Finally, for the 30 *min* time series the spectral energy is mostly concentrated on a single frequency (low frequency) and these periods are in agreement with the ones of the direct analysis. So, one of the advantages of this analysis is the possibility of being able to easily identify the frequency where the spectral energy was concentrated – frequencies of the peak.

## 3 NUMERICAL MODELLING

In this section, the setup of a numerical model was done to study the phenomenon of long wave propagation, both into the bay and in a small craft harbour (Cascais marina). The model used was CGWAVE, a linear finite element method based on the elliptic mild-slope wave equation (MSE), and can simultaneously simulate the effects of refraction, diffraction, reflections by the bathymetry and structures, dissipation due to friction and breaking and nonlinear amplitude dispersion. [1]

### 3.1 Set up of the model

The model setup uses a detailed LIDAR bathymetry acquired recently for the coastal area and land boundaries are discretized based on the type of boundary. The ocean boundary is a semicircular arc enclosing the area of interest. For the computational simulations, the domain was defined with a linear triangular finite element mesh with 10 nodes per wavelength.

Typically, the minimum radius of the semicircle arc should be about two or three times the wavelength (*L*), which for long waves is given by:

$$L = T\sqrt{gd} \tag{3}$$

Being, *T*, period; *g*, gravity acceleration and *d*, depth of the water. According to this, for a long wave with period of T=650 *s* and a constant depth of d=1 *m*, the minimum radius obtained, for this conditions, should be between 4070 *m* and 6105 *m*.

Beside this, the distance between the open boundary and the shoreline is also important. If they are too close, the bottom effects could influence the output results from wave height and, if they are too far, it is possible that the model does not obtain sufficient bathymetry information and the simulations do not run. After several attempts, the domain was defined according to the previous specifications, with a final value for the radius of 9300 *m*.

### 3.2 Friction coefficient

Typically the values for friction coefficient are related with the bottom irregularity, roughness of the bottom, structures that modify the flow field and other parameters. This effect, on the period of resonant oscillations of the basin, depend slightly on water depth, decreasing as depth increases. [10]

Some attempts were made to understand the influence of the friction coefficient and it was observed that this parameter is not relevant for the output results. Regarding this conclusion, and because taking the friction in consideration is very time consuming, the model was performed for the "no friction".

### 3.3 Reflection coefficient

According to literature, for long waves it is possible to affirm that the reflection coefficient is unitary for the land boundaries, which means that total reflection is admitted.[15] However, for different coastline materials the reflection coefficient had values between 0 and 1 as shown in the table below. [16]

| Coastline material | Vertical wall | Shore defense (rocks) | Beach |
|---|---|---|---|
| Reflection Coefficient (-) | 0.90 | 0.25 | 0.10 |

Table 1: Reflection coefficients to coastline materials

### 3.4 Definition of the study scenarios

The most dangerous stages of agitation for the numerical propagations of the incident waves were associated to the wave directions of 45º and 60º, because the predominant wave direction is south-west (45º). So, the model was forced for these five scenarios, for events with the long waves amplitude's of 0.5 *m* (1 *m* of height) in the open boundary and for periods between 350 *s* and 950 *s*, with increments in time of 50 *s*.

| Parameters | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 | Scenario 5 |
|---|---|---|---|---|---|
| Direction (º) | 60 | 45 | 45 | 45 | 45 |
| Reflection Coefficient (-) | 1.0 | 1.0 | variable | 0.5 | 0.9 |

Table 2: CGWAVE boundary conditions – different scenarios

## 4 RESULTS AND DISCUSSION OF THE NUMERICAL MODELING

The response curve provides the resonant periods, which are the periods that correspond to the amplification factor higher than 1. It is relevant to show that the amplification factor (A) is given by:

$$A = \frac{Hi}{Hr} \tag{4}$$

Where the *Hi* is the height of the wave in a certain point and the *Hr* is the height of the incident wave (*Hr*=1*)*. When *A*>1 the wave is amplified and harbour resonance occurs. Nevertheless, it is important to note that, in the real events of resonance, these amplified waves do not have necessarily heights of 1 *m* (or more), it is a value that will give agitation indexes, since the model is linear and the height of the wave in the open ocean is not known. The response curves for five positions inside of marina are shown.

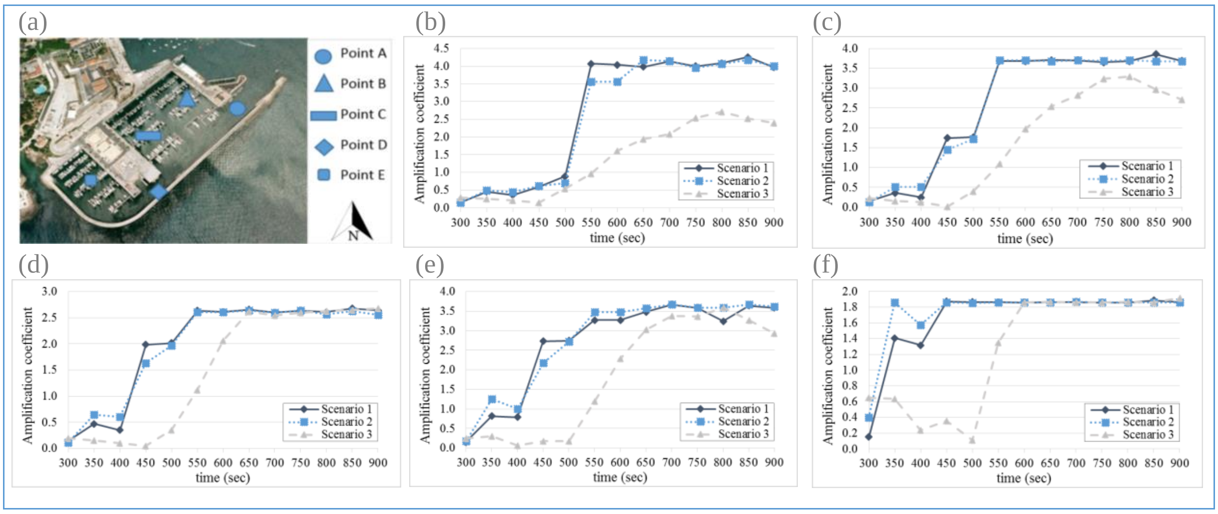


Figure 4: (a) Position of the points inside of the marina. Image from google earth, adapted. Response curves from scenarios 1, 2 and 3 in (b) Point A; (c) Point B; (d) Point C; (e) Point D; (f) Point E

Focusing in the Figure 4 it was perceived that it would be possible to decrease the interval between runs to provide a more precise curve of amplification factor but, for this problem, the analysis seems to be enough detailed.

For the range of periods that was observed in time series records of the WSE, the amplification factor is higher than 1 and consequently resonance really occurs inside of the marina. The trend of the response curve of the scenarios 1 and 2 are practically the same in both cases (maxima and minima are situated on the same range of periods), as well as their asymptotes.

The response curves of the scenario 3 also showed a similar trend in almost all positions (less for point A) that displayed a lower amplification coefficient. On the other hand, for this situation, the resonance happens on higher periods and this can be explained by a variable and smaller reflection coefficient on the boundaries.

Relatively to all of the scenarios analyzed, it is evident that, for starting periods around the 500 *s*, the resonance curve reaches an asymptote and these values of the amplification factor remain almost constant.

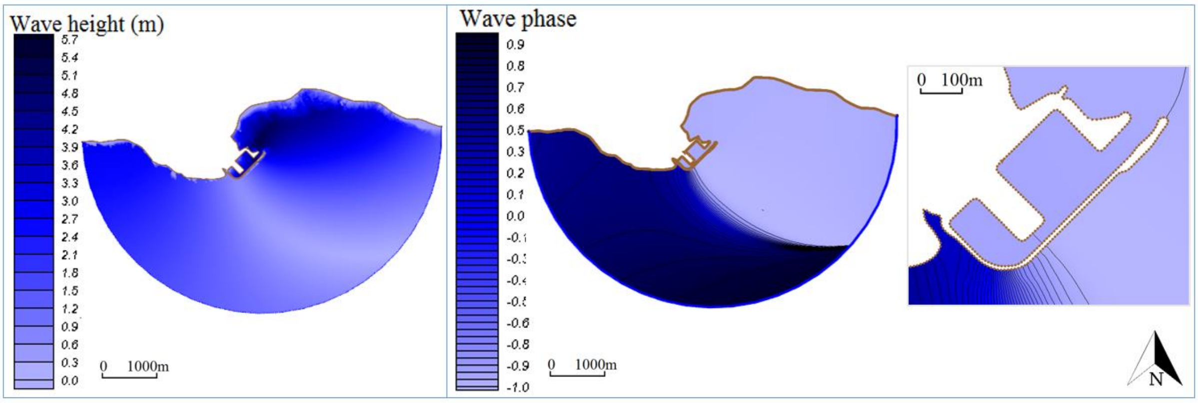


Figure 5: CGWAVE results – Cascais Bay and Marina. Scenario 1 for incident long wave of T=800 *s*.

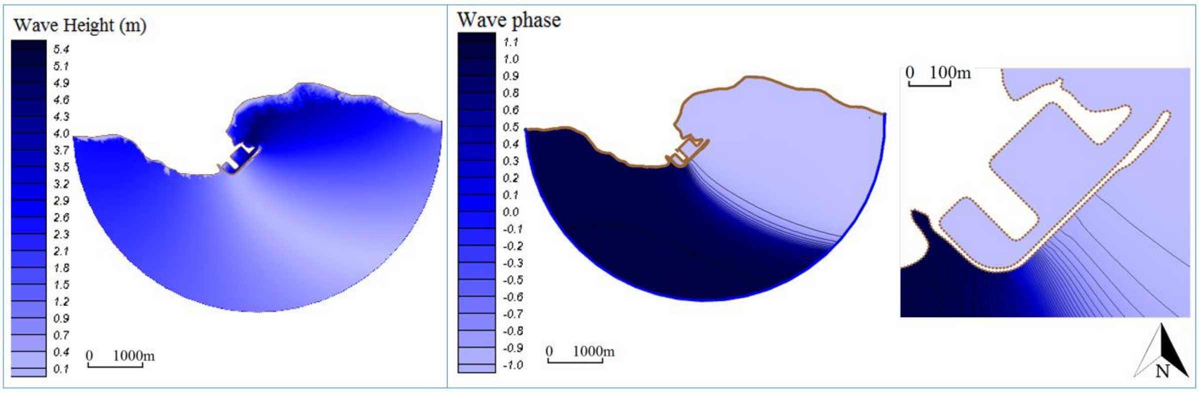


Figure 6: CGWAVE results – Cascais Bay and Marina. Scenario 2 for incident long wave of T=800 *s*.

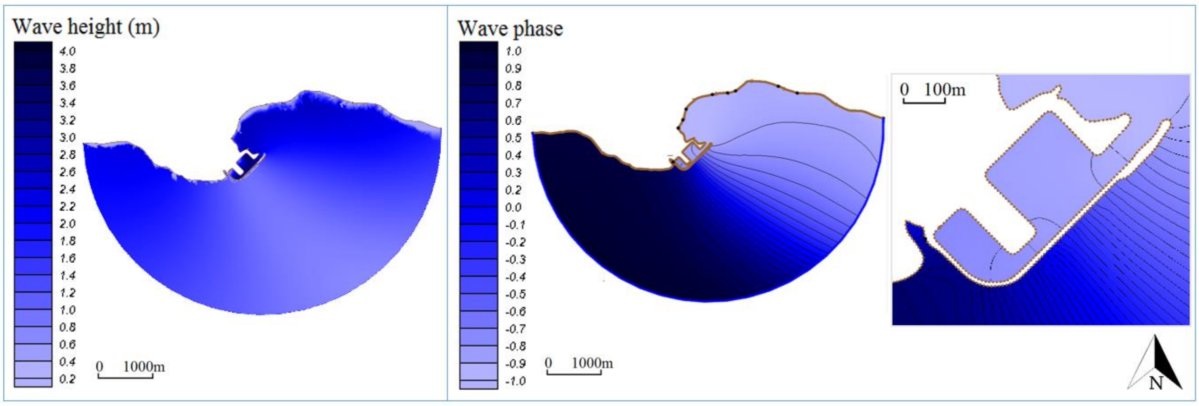


Figure 7: CGWAVE results – Cascais Bay and Marina. Scenario 3 for incident long wave of T=800 *s*.

The output results of the wave phase were used on the identification of the nodal lines (where vertical WSE oscillation is nil and the water velocity is at a maximum), which corresponds to the wave phase when it is equal to zero. In ventral lines the opposite occurs.

All scenarios are devoid of nodal lines inside the marina or in their mouth, and the single nodal line was approximately on the middle of the exterior of the marina, and it was regularly extended across the whole bay. For the resonant periods, the bay responds like a big reservoir, without great vertical oscillations of the water body or great waters velocities.

Therefore, for an instant, the water levels are practically the same in all places of the bay and a current allows, both the entrance and the exit, of the water from the bay to the inside of the marina, making the water level to fluctuate.

## 5 CONCLUSIONS

One of the main purposes of this work was analysis of the characteristics of the long waves in Cascais-bay. From the analysis and data processing of the time series records of WSE from the local tide gauge, it was concluded that the seiche event occurs with a frequency of 24.30% of the total observations time, for a range between 480 *s* and 840 *s* (8-14 *min*), with mean values of about 660 *s* (11 *min*). The amplitude of these oscillations are normally uniform, with ranges of a few centimeters however, in an exceptional event, the maximum observed amplitude was of 0.54 *m*.

From the statistical calculations it was observed that the presence of long waves are a frequent event (presents in 272 days) but it is very probable that these oscillations had been more frequent in Cascais-bay than the present study reports. This happens because the local tide gauge has gaps in the measurements, in extreme situations even for a month. From the pressure records from the local tide gauge, no evidence about any relation of occurrence of this phenomenon with the season of year was found. However, the amplitudes of the seiches are larger on the maritime winter (November to April).

A Matlab program was written for the acquisition of the heights of the astronomic tide by the harmonic analysis (tide tables), as well for the formulation of the periodogram method and the pass-band filter of second order (Butterworth filter). The spectral analysis allows a complementary and alternative option in relation to the direct method (time domain) for the measurement of the seiches periodicities.

Relatively to the periodogram, for exerts of 30 *min* of the time series, this peak period is in good agreement with the periods of seiches obtained by direct observation. So, these convergent results allows a confirmation of the primary periodicities of the long wave oscillations observed in Cascais-bay, as well as an extra care for hidden periodicities. Although different domains for the analysis of the period seiches were used, both methods are valid.

From the CGWAVE simulations of the long wave penetration in Cascais-Bay it was possible study two problems: the resonance phenomenon and the nodal lines position. About the resonance phenomenon, firstly the resonant periods are between values of around 400 *s* and 900 *s*. Secondly the bay response to the forced open ocean waves did not have significant variations with the incident angle. On the other hand, the reflection coefficient had some implications on the response curve, like the increase of the resonant periods on the basin response. The main advantage of this is that the highest periods are harder to occur in reality, so the occurrence of harbour resonance is less likely. So, these results were useful on showing that, even for long waves, the model is sensible to different reflection coefficients, especially for lower periods, which does not corroborate the theoretical hypothesis.

Another significant conclusion is that the periods of seiches that were obtained in the first phase of this work are in good agreement with the computed resonant periods of the bay (when $A>1$). This means that the periodicities of the observed oscillations really correspond to the amplification of the incident long waves from the open ocean. So, CGWAVE model gives good results for the present study.

Finally, according to the non-existence of nodal lines inside of the marina and in their mouth, it was possible to conclude that the basins oscillates in unison, so dangerous situations in moored positions and problems in the marina operation are not expected to occur.

In order to find more information about the physical processes that originates seiches in the Cascais bay, further studies are needed. An attempt to link the occurrence of seiches with the passing of atmospheric depressions did not show any correlation.

## REFERENCES


[1] V. G. Panchang and B. Xu, "CGWAVE: A Coastal Wave Transformation Model for Arbitrary Domains." Tech. Report. Dept. of Civil and Environmental Engineering, University of Maine, 1995.

[2] A. B. Rabinovich, "Seiches and Harbor Oscillations," *Handb. Coast. Ocean Eng. World Sci. Publ., Singapoure*, 2009.

[3] J. Park, J. Macmahan, W. V Sweet, and K. Kotun, "Continuous seiche in bays and harbors," *Ocean Sci. Discuss*, vol. 12, pp. 2361–2394, 2015.

[4] J. W. Miles, "Harbor Seiching," *Annu. Rev. Fluid Mech.*, vol. 6, pp. 17–35, 1974.

[5] W. H. Munk, "Origin and generation of waves." Coastal Engeering. La Jolla, California: Institute of Geophysics and Scrips Institution of Oceanography - University of California, 1950.

[6] A. R. Kabiri-Samani, "Natural Frequencies of Seiche in a Closed Trapezoidal Basin with Internal Barriers," *J. Civ. Eng. Res.*, vol. 3, no. 1, pp. 22–34, 2013.

[7] J.-J. Lee, "Wave induced oscillations in harbours of arbitrary shape," *J. Fluid Mech.* , vol. 45, no. 02, pp. 375–394, 1969.

[8] U.S.Army Corps of Engineers, *Coastal Engineering Manual, Parte II-7, Harbor Hydrodynamics*. Washington DC: U.S.Army Corps of Engineers , 2008.

[9] L. S. Lillycrop, "Barbers Point Harbor, Oahu, HI, Monitoring Study." U.S. Army Corps of Engineers, Washington DC, 1993.

[10] L. C. Breaker, Y.-H. Tseng, and X. Wang, "On the natural oscillations of Monterey Bay: Observations, modeling, and origins," *Prog. Oceanogr.*, vol. 86, pp. 380–395, 2010.

[11] L. Bressan and Tinti. S., "Statistical properties of coastal long waves analysed through sea-level time-gradient functions: exemplary analysis of the Siracusa, Italy, tide-gauge data," *Nat. Hazards Earth Syst. Sci. Discuss.*, vol. 3, no. 9, pp. 5247–5286, 2015.

[12] C. Chatfield, *The Analysis of Time Series: An Introduction*, 2nd ed. New York: Chapman and Hall, 1980.

[13] F. M. Abecasis, "Estudo do problema das ondulações de longo periodo na ampliação do porto de Luanda," *"Técnica" Rev. dos alunos do I.S.T.*, vol. 303, p. 483 a 492, 1960.

[14] R. E. Thomson and W. J. Emery, Eds., *Data Analysis Methods in Physical Oceanography* , Third . Elsevier B.V. , 2014.

[15] L. H. Holthuijsen, *Waves in Oceanic and Coastal Waters*. United States of America by Cambridge University Press, New York, 2007.

[16] E. F. Thomson, H. S. Chen, and L. L. Hadley, "Validation of Numerical Model for Wind Waves and Swell in Harbours," *J. Waterw. port, Coast. Ocean enginnering*, pp. 245–257, 1996.