ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

INFLUENCE OF MICRO CRACKS ON EFFECTIVE MATERIAL PROPERTIES IN FIBER REINFORCED SMART COMPOSITE MATERIALS

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Keywords: piezoelectric fiber reinforced composites, micro cracks, effective material properties, Galerkin BEM

Abstract. *In this paper, the analysis of boundary value problems with fiber reinforced piezoelectric representative volume elements (RVEs) containing micro cracks of arbitrary shape is presented. Interface cracks between fiber and matrix as well as cracks inside the matrix and fibers are investigated. For this purpose, the resulting boundary value problem is formulated as boundary integral equations (BIEs). The symmetric Galerkin method (SGBEM) is applied for the spatial discretization of the boundary to solve the BIEs numerically. Numerical examples will be presented and discussed to show the efficiency of the present SGBEM, the influences of the fiber variation, the micro cracks and the crack-face boundary condition on the effective material properties.*

1 INTRODUCTION

Smart composite materials offer certain important performance advantages over conventionally used metals. Due to the capability of converting electric energy into mechanical energy and vice versa, piezoelectric materials play an important role in smart structures. In recent years a new class of piezoelectric composites has been developed by combining passive elastic materials as matrix and piezoelectric ceramics or polymers as active fibers. These smart materials can be optimized by taking advantages of the most beneficial properties of each constituent and to satisfy the high-performance requirements according to different in-service conditions. Piezoelectric ceramics are very brittle with low fracture toughness and micro as well as macro cracks may be induced during the manufacturing and under the in-service condition [2]. Beside cracks inside the homogeneous matrix and fibers, interface cracks play an important role for the design and safety of real structures. The investigation of effective piezoelectric properties of fiber reinforced material containing micro-cracks is of special scientific significance and engineering importance. Due to the high mathematical complexity, the analytical solution of coupled multi-field problems is only possible to very simple geometrical configurations and loading conditions. Therefore, the solution of general boundary value problems requires advanced numerical methods. By applying a homogenization approach the representative volume element (RVE) is formulated and the Galerkin BEM is developed for a numerical evaluation of the effective material properties.

2 PROBLEM FORMULATION

Let us consider a linear piezoelectric fiber matrix composite with regularly distributed fibers. Micro-cracks of arbitrary shape can exist in the interface between fiber and matrix as well as inside the matrix and fibers. Without body forces, free electrical charges and by applying the generalized notation the constitutive equations are defined by

$$\sigma_{iJ}(\mathbf{x}) = C^{\zeta}_{iJKl}\varepsilon_{Kl}(\mathbf{x}) = C^{\zeta}_{iJKl}u_{K,l}(\mathbf{x}), \tag{1}$$

where u_I, σ_{iJ}, ε_{iJ} and C_{iJKl} are the generalized displacements, stresses, straines and elasticity tensor

$$u_I = \begin{cases} u_i, & I = i, \quad \text{(displacements)} \\ \varphi, & I = 4, \quad \text{(electric potential)}, \end{cases} \tag{2}$$

$$\sigma_{iJ} = \begin{cases} \sigma_{ij}, & J = j, \quad \text{(stresses)} \\ D_i, & J = 4, \quad \text{(electric displacements)}, \end{cases} \tag{3}$$

$$\varepsilon_{iJ} = \begin{cases} \varepsilon_{ij}, & J = j, \quad \text{(strains)} \\ E_i, & J = 4, \quad \text{(electric field)}, \end{cases} \tag{4}$$

$$C_{iJKl} = \begin{cases} c_{ijkl}, & J = j;\, K = k, \quad \text{(elasticity tensor)} \\ e_{lij}, & J = j;\, K = 4, \quad \text{(piezoelectric tensor)} \\ e_{ikl}, & J = 4;\, K = k, \quad \text{(piezoelectric tensor)} \\ -\kappa_{il}, & J = K = 4, \quad \text{(electric permittivity tensor)}. \end{cases} \tag{5}$$

Lower case Latin indices take the values 1 and 2 (elastic), while capital Latin indices take the values 1, 2 (elastic) and 4 (electric). A comma after a quantity designates spatial derivatives and unless otherwise stated, the conventional summation rule over repeated indices is implied. The boundary conditions

$$u_I(\mathbf{x}) = \bar{u}_I(\mathbf{x}), \quad \mathbf{x} \in \Gamma_u, \tag{6}$$

$$t_I(\mathbf{x},t) = \bar{t}_I(\mathbf{x},t), \quad \mathbf{x} \in \Gamma_t, \tag{7}$$

the continuity and equilibrium conditions on the interface

$$u_I^I(\mathbf{x}) = u_I^{II}(\mathbf{x}), \quad \mathbf{x} \in \Gamma_{if}, \tag{8}$$

$$t_I^I(\mathbf{x}) = -t_I^{II}(\mathbf{x}), \quad \mathbf{x} \in \Gamma_{if}, \tag{9}$$

are satisfied, with t_I being the generalized tractions

$$t_I(\mathbf{x}) = \sigma_{jI}(\mathbf{x})e_j(\mathbf{x}). \tag{10}$$

In the Eqs. (6)-(10), Γ_{if} is the interface between the homogeneous domains Ω^ζ $(\zeta = 1, 2, ..., N)$, Γ_t is the external boundary where the generalized tractions are prescribed, Γ_u is the external boundary where the generalized displacements are given and e_j is the outward unit normal vector. In the present work the crack is considered either as electrical impermeable

$$D_i(\mathbf{x} \in \Gamma_{c^+}) = D_i(\mathbf{x} \in \Gamma_{c^-}) = 0 \tag{11}$$

or as electrical permeable

$$D_i(\mathbf{x} \in \Gamma_{c^+}) = D_i(\mathbf{x} \in \Gamma_{c^-}), \quad \varphi(\mathbf{x} \in \Gamma_{c^+}) - \varphi(\mathbf{x} \in \Gamma_{c^-}) = 0. \tag{12}$$

Γ_{c^+} and Γ_{c^-} denote the upper and lower crack-face. The generalized crack-opening-displacements (CODs) are defined by

$$\Delta u_I(\mathbf{x}) = u_I(\mathbf{x} \in \Gamma_{c^+}) - u_I(\mathbf{x} \in \Gamma_{c^-}). \tag{13}$$

3 BOUNDARY INTEGRAL EQUATIONS AND FUNDAMENTAL SOLUTIONS

To solve the corresponding initial boundary value problem with a BEM it is formulated as boundary integral equations (BIEs). In the sense of a weighted residual formulation, the Galerkin BIEs are defined by

$$\begin{aligned}\int\limits_{\Gamma} \psi(\mathbf{x})u_J(\mathbf{x})\mathrm{d}\Gamma_x &= \int\limits_{\Gamma} \psi(\mathbf{x}) \int\limits_{\Gamma_b} \Big[u_{IJ}^G(\mathbf{x},\mathbf{y})\, t_I(\mathbf{y}) - t_{IJ}^G(\mathbf{x},\mathbf{y})\, u_I(\mathbf{y})\Big]\mathrm{d}\Gamma_y \mathrm{d}\Gamma_x \\ &+ \int\limits_{\Gamma} \psi(\mathbf{x}) \int\limits_{\Gamma_{c^+}} t_{IJ}^G(\mathbf{x},\mathbf{y})\, \Delta u_I(\mathbf{y})\mathrm{d}\Gamma_y \mathrm{d}\Gamma_x,\end{aligned} \tag{14}$$

where $\psi(\mathbf{x})$ is a weighting function, Γ_b is the external boundary, $\Gamma = \Gamma_b + \Gamma_{if} + \Gamma_{c^+}$ and $u_{IJ}^G(\mathbf{x},\mathbf{y})$ are the generalized displacement fundamental solutions. The generalized traction fundamental solutions $t_{IJ}^G(\mathbf{x},\mathbf{y})$ are obtained by substitution of the displacement fundamental solutions into the constitutive equation (1) and Eq. (10) as

$$t_{IJ}^G(\mathbf{x},\mathbf{y}) = C_{qIKr}e_q(\mathbf{y})u_{KJ,r}^G(\mathbf{x},\mathbf{y}). \tag{15}$$

The traction BIEs are derived by substituting Eq. (14) into Eqs. (1) and (10) as

$$\begin{aligned}\int\limits_{\Gamma} \psi(\mathbf{x})t_J(\mathbf{x})\mathrm{d}\Gamma_x &= \int\limits_{\Gamma} \psi(\mathbf{x}) \int\limits_{\Gamma_b} \Big[v_{IJ}^G(\mathbf{x},\mathbf{y})\, t_I(\mathbf{y}) - w_{IJ}^G(\mathbf{x},\mathbf{y})\, u_I(\mathbf{y})\Big]\mathrm{d}\Gamma_y \mathrm{d}\Gamma_x \\ &+ \int\limits_{\Gamma} \psi(\mathbf{x}) \int\limits_{\Gamma_{c^+}} w_{IJ}^G(\mathbf{x},\mathbf{y})\, \Delta u_I(\mathbf{y})\mathrm{d}\Gamma_y \mathrm{d}\Gamma_x,\end{aligned} \tag{16}$$

with the traction and the higher-order traction fundamental solutions

$$v_{IJ}^{G}(\mathbf{x},\mathbf{y}) = -C_{pIKs}e_p(\mathbf{x})u_{KJ,s}^{G}(\mathbf{x},\mathbf{y}), \tag{17}$$

$$w_{IJ}^{G}(\mathbf{x},\mathbf{y}) = C_{pIKs}e_p(\mathbf{x})C_{qJLr}e_q(\mathbf{y})u_{KL,sr}^{G}(\mathbf{x},\mathbf{y}). \tag{18}$$

The fundamental solutions for homogeneous, generally anisotropic and linear piezoelectric solids are applied [5].

4 SOLUTION ALGORITHM

For the spatial discretization, the external boundary Γ_b, the interface Γ_{if} and the crack-face Γ_{c^+} of the piezoelectric solid are discretized by quadratic elements

$$\Gamma = \Gamma_b + \Gamma_{if} + \Gamma_{c^+} = \sum_{e=1}^{E}\Gamma_e, \tag{19}$$

with $E = E_b + E_{if} + E_{c^+}$. The strongly singular and hypersingular boundary integrals are computed directly with special analytical and numerical techniques [1]. The symmetry properties of the fundamental solutions are utilized to enhance the efficiency of the present Galerkin-BEM.

After spatial discretizations the BIEs (14) and (16) lead to the following systems of linear algebraic equations for each subdomain Ω^ζ $(\zeta = 1, 2, ..., N)$

$$\mathbf{C}_\zeta\mathbf{u}_\zeta = \mathbf{U}_\zeta^S\mathbf{t}_\zeta - \mathbf{T}_\zeta^S\mathbf{u}_\zeta + \mathbf{T}_\zeta^S\Delta\mathbf{u}_\zeta, \tag{20}$$

$$\mathbf{D}_\zeta\mathbf{t}_\zeta = \mathbf{V}_\zeta^S\mathbf{t}_\zeta - \mathbf{W}_\zeta^S\mathbf{u}_\zeta + \mathbf{W}_\zeta^S\Delta\mathbf{u}_\zeta. \tag{21}$$

By invoking the boundary conditions (6),(7), the interface conditions (8) and (9) and the crack-face conditions (11), (12) all equations can be summarized to the system

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{22}$$

where **A** is the system matrix, **x** is the vector of the unknown boundary data and the vector **b** contains the prescribed boundary data

5 COMPUTATION OF EFFECTIVE MATERIAL PROPERTIES

To compute the effective material properties of a fiber reinforced piezoelectric composite a homogenization procedure is applied. A uniform distribution of piezoelectric fibers is assumed in the present case and the corresponding representative volume element is formulated. The relation between the generalized average stresses and the boundary tractions is expressed as

$$\langle\sigma_{iJ}\rangle = \frac{1}{V}\int_\Omega \sigma_{iJ}\mathrm{d}\Omega = \begin{cases} \dfrac{1}{2V}\displaystyle\int_\Gamma (t_i x_j + t_j x_i)\,\mathrm{d}\Gamma, & i,j = 1,2 \\ \dfrac{1}{V}\displaystyle\int_\Gamma t_J x_i\,\mathrm{d}\Gamma, & i = 1,2; J = 4, \end{cases} \tag{23}$$

where Ω is the domain of the RVE and Γ is the whole boundary of the RVE containing the external boundary Γ_b, the crack-faces Γ_c, and the interfaces Γ_{if}. In the same sense the volume integral of the generalized average straines can be transformed to the boundary by

$$\langle \varepsilon_{iJ} \rangle = \frac{1}{V} \int_{\Omega} \varepsilon_{iJ} \mathrm{d}\Omega = \begin{cases} \dfrac{1}{2V} \displaystyle\int_{\Gamma} (u_i e_j + u_j e_i) \, \mathrm{d}\Gamma, & i, j = 1, 2 \\ \dfrac{1}{V} \displaystyle\int_{\Gamma} u_J e_i \, \mathrm{d}\Gamma, & i = 1, 2; J = 4. \end{cases} \tag{24}$$

Taking into account the constitutive relationships

$$\langle \sigma_{iJ} \rangle = C^{eff}_{iJKl} \langle \varepsilon_{Kl} \rangle \tag{25}$$

the effective material coefficients C^{eff}_{iJKl} can be computed using the generalized average stresses (23) and the generalized average strains obtained from the numerical solution of properly selected boundary value problems in the RVE sample [3, 4].

6 NUMERICAL RESULTS

As a numerical example, we consider a square RVE of length l with a central circular fiber of radius r. Between the fiber and the matrix a interface crack of the arc length 2α exist.

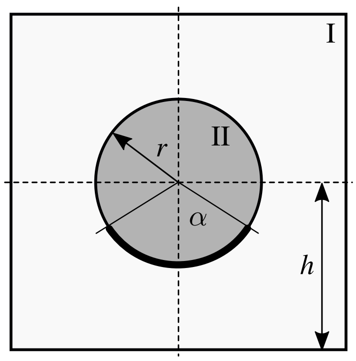

Figure 1: A square RVE with a interface crack between the central circular fiber and the matrix

As non-piezoelectric matrix (domain I) Epoxy is chosen, which has the material parameters

$$\begin{aligned} &C_{11} = 8.0\text{GPa}, \quad C_{12} = 4.4\text{GPa}, \quad C_{22} = 8.0\text{GPa}, \quad C_{66} = 1.8\text{GPa}, \\ &\kappa_{11} = 0.0372\text{C/(GVm)}, \quad \kappa_{22} = 0.0372\text{C/(GVm)}, \end{aligned} \tag{26}$$

while for the fiber (domain II) PZT7 with the following material constants

$$\begin{aligned} &C_{11} = 154.8\text{GPa}, \quad C_{12} = 82.7\text{GPa}, \quad C_{22} = 131.4\text{GPa}, \quad C_{66} = 25.7\text{GPa}, \\ &e_{21} = -2.121\text{C/m}^2, \quad e_{22} = 9.522\text{C/m}^2, \quad e_{16} = 9.349\text{C/m}^2, \\ &\kappa_{11} = 4.065\text{C/(GVm)}, \quad \kappa_{22} = 2.079\text{C/(GVm)} \end{aligned} \tag{27}$$

is applied. For the numerical computations the external boundary of the RVE is discretized by using a uniform mesh with 16 quadratic elements and the circular interface including the crack is divided into 20 elements. The crack is assumed as impermeable for the electric field. The normalized effective material properties for a volume fraction of 20% of the piezoelectric fiber and different crack angles α are shown in Fig. 2.

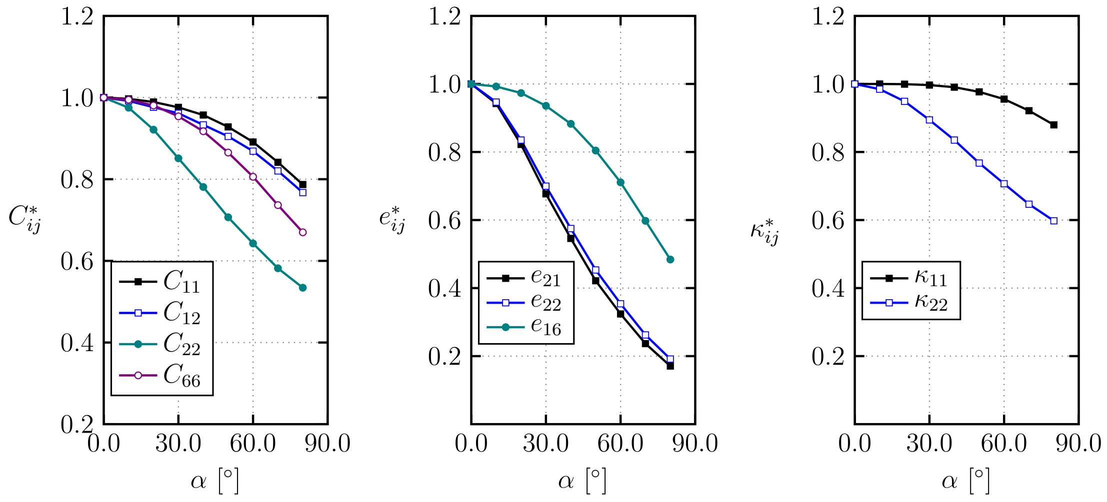

Figure 2: Effective material properties for different crack lengths

As depicted in Fig. 2 the interface crack has a significant influence on the effective material properties of the composite. The elastic constants decrease drastically with increasing crack angles α. For a crack of the arc length of $90°$ the elastic constants are only 55-80% of the uncracked composite. In the same sense the electric permittivities κ_{11} and κ_{22} reduce significantly with increasing interface crack lengths. It should be mentioned that this is the consequence of the applied electric impermeable crack-face boundary condition. This implies that the crack is free of electric charges and the electric field lines circumvent the crack. In contrast if the crack is considered as fully permeable for the electric field both crack-faces have equal electric potentials and therefore the crack is fully penetrated by the electric field. In other words the crack does not exist for the electric field and the application of the permeable crack modell would lead to identical electric permittivities for the RVE with and without a crack. As well expected the piezoelectric constants show a similar tendency and decrease with a increasing interface crack arc length.

7 CONCLUSIONS

The analysis of effective material properties for fiber reinforced piezoelectric representative volume elements (RVEs) containing micro cracks of arbitrary shape is presented in this paper. Both, interface cracks between fiber and matrix as well as cracks inside the matrix and fibers are investigated. In order to solve the resulting boundary value problem with the Galerkin BEM it is formulated as boundary integral equations (BIEs). Analytical and numerical techniques are used for a direct computation of the strongly singular and hypersingular boundary integrals. The developed BEM is general without limitations of the geometry, poling direction and material anisotropy. Numerical examples will be presented and discussed to show the efficiency of the present Galerkin BEM, the influences of the fiber variation, the micro cracks and the crack-face boundary condition on the effective material properties.

ACKNOWLEDGEMENT

This work is supported by the Slovak Academy of Sciences Project (SASPRO) 0106/01/01. The financial support is gratefully acknowledged.

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