

ANALYSIS OF NON-LINEAR ENVIRONMENTAL LOAD COMBINATIONS BY EXTENDED CONTOUR-LINE ALGORITHMS

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Abstract. *For both ships and offshore structures, load effects caused by a combination of wind, current and waves need to be accounted for during the design process. It will easily become a very comprehensive task to consider all possible combinations of such load effects, and efficient methods to deal with this issue is generally in demand. The present paper deals with application of extended contour-line methods for this purpose. General methods for identification of load combinations in relation to continuous stochastic processes are first highlighted. The particular case of multiple FBC-processes with given amplitude distribution is subsequently addressed. (The FBC-process is named with reference to the work by Ferry-Borges&Castanheta (1971)). The case where all process components have identical basic time intervals is first considered. The relationship between the FBC-process and the so-called environmental contour methods (based on Inverse FORM techniques) which are presently being applied for various design purposes is elaborated. FBC-processes with widely different basic time intervals are next investigated. The present paper illustrates how a FORM search can be applied along the limit state surface in order to identify the relevant "load combination point" for such cases. This requires that a particular linear or non-linear combination of the load effects is specified. Such a combination will typically be based on a particular mechanical limit state.*

The motivation for the present work is to highlight and extend the methodology related to load combination rules to be applied for engineering design of marine structures which are subjected to stochastic environmental processes with multiple components.

Development of the relevant tools based on application of "translation processes" (which are transformed Gaussian processes) are outlined. Examples are given which represent cases with both two and three simultaneous loading components. For both examples, uniform as well as non-uniform time intervals for the process components are considered.

1 INTRODUCTION

In the present paper, general methods for identification of load combinations in relation to continuous stochastic processes are first highlighted. The particular case of multiple FBC-processes with known probability amplitude distribution is subsequently addressed. (The FBC-process is named with reference to the work by FerryBorges&Castanheta [1]). The case where all process components have identical basic time intervals is first considered. The relationship between the FBC-process and the so-called environmental contour methods which are presently applied for various design purposes is elaborated.

FBC-processes with widely different basic time intervals are next investigated. A methodology is outlined which enables to establish the environmental design contour also for this case.

Relationships between environmental parameters and structural load effects are frequently available once the particular characteristics of a specific structure to be installed are defined. This is most straightforward for the static type of response, while dynamic response (e.g. due to stochastic loading) generally requires significantly more effort. In some cases, simplified dynamic response analyses (e.g. based on regular wave excitation models) can further be performed in order to identify gross features of the dynamic response.

The present paper illustrates how a FORM search can be applied along the limit state surface in order to identify the relevant “load combination point” for such cases. This requires that a particular type of linear or non-linear combination of the load effects is also specified. This combination will typically be based on a particular mechanical limit state function which is relevant for the whole structure or one of its components.

The motivation for the present work is to highlight and further extend the methodology behind load combination rules which are to be applied for engineering design of structures which are subjected to stochastic environmental processes with multiple components. Examples of application to the combination of wind, wave and current loading are considered. Such load combinations are relevant e.g. for long-span marine bridges which are planned for crossing of the widest Norwegian fjords as part of the so-called “Ferry-free E39 Project”. One of the existing bridges with floating pontoons, i.e. the Norhordaland Bridge, is shown in Figure 1. An example of a future bridge concept, i.e. the submerged floating tunnel is shown in Figure 2. For the latter, first- and second-order wave loads as well as current loading (including the possibility of Vortex-Induced-Vibrations) are the most relevant ones.



Fig. 1. The Norhordaland Bridge with floating pontoons (located at the West Coast of Norway).

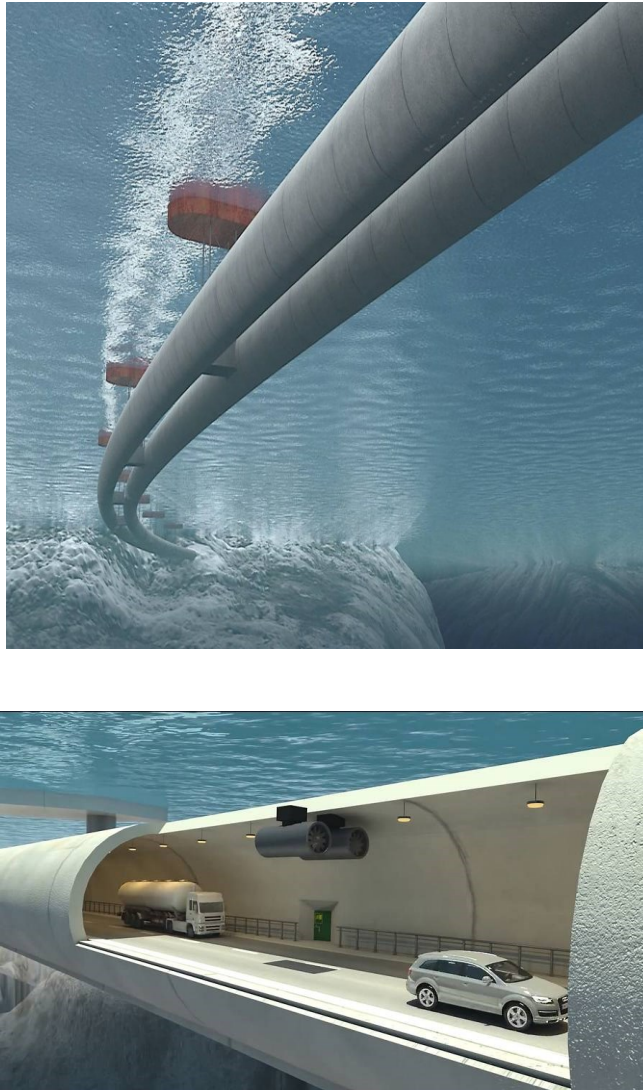


Fig. 2 Example of submerged tunnel concept (Source: Norwegian Public Road Authorities).

2 LOAD COMINATIONS FOR CONTINUOUS PROCESSES

A distinction should be made between combination of loads versus combination of load effects. Clearly, the combination of external loads with given magnitudes will in general imply different relative magnitudes between the associated load effects. In codified design, combination of different types of loading are typically specified in terms of return periods for the different environmental processes. As an example, for offshore structures the dominant load component is specified to have a return period of 100 years, while the secondary component is frequently specified to have a return period of 10 years (when the process components are assumed to be uncorrelated).

A further distinction should be made between situations where the relationship between the load-effects and environmental parameters are known and cases where these relationships have not yet been obtained. Even if the load effects are known, a further differentiation can be made between whether the capacity surface (mechanical limit state function) which corresponds to failure of a given structural component is available or not.

If a limit state function is specified, a load-effect combination needs to be analysed which frequently involve dynamic effects. In the present paper, focus is on continuous-time processes. For cases where the process components are also continuous-valued and in addition the limit state function is known, the combined load effect is frequently analyzed by means of the so-called up-crossing rate (or more generally the out-crossing rate for multidimensional formulations). For linear combinations, upper bound expressions can be derived which involve the up-crossing rate for each of the component processes, see e.g. Madsen et. al. [2] and Melchers [3].

Simplified methods for definition of relevant “point values” which are assumed to cover the most critical load combinations have also been introduced. One of these is the celebrated Turkstra rule which selects the expected extreme value of one component, which is then combined with the expected instantaneous values of the others, see Turkstra [4]. A sequence of such combinations (which is equal to the number of components) is then required.

A second type of simplified method is the so-called “Square-root-of-sum-of-squares” rule (SRSS-rule). The expected extreme values for all the components are then squared and added together, and the square-root of the resulting sum is then computed.

The simplified load combination methods do not explicitly take into account the particular distribution functions which apply for the involved components (except for computation of the associated expected values).

For process components which are discrete instead of continuous-valued the up-crossing rate can still be applied. However, for this case the analysis can be made somewhat simpler by utilization of the step-wise behavior of the sample functions. This is achieved by means of the FBC-process representation which was mentioned above. The particular type of distribution functions and the characteristic time interval for each process component can then be taken into account in a proper way.

In general the time interval will be different for the different process components. In some cases the lengths of the time scales are widely different as for example in connection with the joint representation of wind and snow parameters.

3 CONTOUR METHODS FOR PROCESS COMPONENTS WITH IDENTICAL BASIC TIME INTERVALS

3.1 General

For cases where the limit state function is not specified, a range of environmental conditions that are relevant for analysis can still be identified. This is based on consideration of the multi-dimensional joint probability density and distribution functions which define the long-term statistical properties of the process components. Iso-probability surfaces can then be computed which correspond to a specified exceedance probability (or equivalently a specified return period) for the components.

In the following, a brief review of the much applied contour methods for identification of such relevant design events is first given. The connection with the FBC process is also highlighted. Subsequently, load effect combinations for cases with known limit state functions are considered. Identification of the associated “load combination point” for such cases is addressed.

Having obtained the design contour and the associated critical point, calculation of so-called “long-term” load effects can be avoided. Such calculations would require that the load effect is evaluated for each combination of the environmental parameters. An integration (or more typically a weighted summation) across the entire variation range for each parameter is

then required. This can be quite laborious, especially for a large number of environmental parameters.

3.2 Design contours

Environmental processes such as wind and wave characteristics (e.g. mean wind velocity and significant wave height) are generally of a non-stationary character. A simplified representation is typically applied where these processes are modeled according to the step-wise representation as mentioned above. The “step-levels” of the basic components are generally non-Gaussian distributed. However, they can still be represented as being transformations of processes which have Gaussian distributed step levels. Such transformed processes are frequently referred to as “translation processes”.

The transformation between these basic processes and the auxiliary normalized Gaussian processes is in that case provided by the Rosenblatt transformation, see e.g. Madsen et. al. [2], Melchers [3]. For two process components this transformation is expressed as:

$$\begin{aligned}\Phi(u_1(t)) &= F_{x_1}(x_1(t)) \\ \Phi(u_2(t)) &= F_{x_2|x_1}(x_2(t)|x_1(t))\end{aligned}\quad (1)$$

where the second line involves the conditional distribution function of x_2 given x_1 .

For the case of uncorrelated basic components only the diagonal terms will be non-zero, and the elements of the Jacobian matrix simplify into the following expressions:

$$\partial u_i / \partial x_i = J_{ij}(x_i) = f_i(x_i) / \phi(u_i(x_i)) \quad (2)$$

In the case of correlated basic components more complex expression apply although they can in principle be evaluated in a straightforward way.

Other possible types of transformations also exist, somewhat depending on the type of statistical information which is available. As an example, the Nataf transformation can be applied if only the marginal distributions and the pairwise correlation coefficients are known, see Nataf [5], Der Kiureghian and Liu [6].

Having performed the transformation into normalized components, the corresponding cumulative distribution for the distance from the origin to a specific point will be independent of the direction in the transformed space. This is due to the isotropic properties of the transformed processes. Hence, only the length of the radius vector will be of significance. This implies that the iso-probability levels correspond to concentric circles. The probability of exceeding a given value of the radius (R) in any direction is hence given by the following expression

$$p_f(R) = 1 - \Phi(R) = \Phi(-R) \quad (3)$$

This probability of exceedance can also be interpreted in terms of a specific return period in the following manner: Designating the number of events (i.e. number of repetitions of the basic time interval) which corresponds to the given return period by N , the probability of exceeding the corresponding radius value is expressed as:

$$p_f(R) = \Phi(-R) = 1 - (1/N) \quad (4)$$

Examples of 2D and 3D contours corresponding to given return periods are given in Sections 5 and 6 of the present paper.

3.3 Load effects and identification of “Load Combination Point”

For each point along the design contour (in the basic parameter space) the corresponding load effect can be computed once a sufficient number of structural properties are given. Static load effects can (at least in principle) be expressed directly as functions of the environmental parameters. For stochastic dynamic load effects such relationships can usually only be established for the parameters of the probability distributions of the response processes. However, by specifying a given fractile of the response distribution, functional relationships which are similar to the static case can be obtained.

Introducing a limit state function for a specific structural member, a linear or non-linear combination of the load effects will result. The associated limit state surface can also be transformed into the space of the normalized Gaussian processes. The most critical combination of the associated load parameters will then correspond to the point on the limit state surface which has the minimum distance to the origin. This point can e.g. be identified based on FORM/SORM algorithms, see e.g. Madsen et. al. [2], Melchers [3].

Having obtained the point on the limit state surface which is closest to the origin, a scaling can be performed in the direction of the origin. This scaling is performed such that the resulting new point is located on the environmental contour surface (i.e. the contour which corresponds to the given return period). This scaled point then represents the relevant “load combination point”.

Comparison of this load combination point can then be made with the points which correspond to other procedures such as the Turkstra and SRSS rules which do not utilize the distribution functions explicitly.

4 ENVIRONMENTAL CONTOURS AND LOAD COMBINATIONS FOR PROCESSES WITH NON-UNIFORM BASIC TIME INTERVALS

4.1 General

Two main options for analysis of the case with non-uniform time intervals for the basic components are considered in the following:

Option (i) Redefinition of the cumulative distribution functions for all the components except the one with the longest time interval. Introducing the notation $n_i = (\Delta T / \Delta T_i)$ for the ratio between the longest time interval and the interval length for component number i , the modified distribution function then reads:

$$F_{X_{i,\Delta T}}(x) = \{F_{X_i}(x_i)\}^{n_i} \quad (5)$$

where $F_{X_i}(x_i)$ is the cumulative distribution function for the short time interval, while $F_{X_{i,\Delta T}}(x)$ is the corresponding cumulative distribution corresponding to the longest time interval (which now becomes the common interval for all the components). The transformation into the normalized components can be performed in the same way as before.

Option (ii) Direct transformation into normalized components and subsequently accounting for the differences in reference time intervals. This procedure is based on re-scaling of the components with the shortest time intervals. Some further details of the direct transformation approach are outlined below.

4.2 Direct transformation

Also for the case on non-uniform time intervals, the same type of transformation as for FBC processes with identical basic time intervals can clearly be applied for each component. The “radius value” of normalized component number i which corresponds to the given return period is denoted by R_i . This quantity is obtained by solving Equation (4) when inserting the number of load interval repetitions which corresponds to that particular component.

To simplify the description, the two-dimensional case is considered as an example. Denoting the direction angle in the normalized plane by θ , the two components are now expressed by decomposing the respective component values to the U_1 and U_2 axis. This gives on component form:

$$\begin{aligned} u_1(\theta) &= R_1 \cos(\theta) \\ u_2(\theta) &= R_2 \sin(\theta) \end{aligned} \quad (6)$$

This can also be expressed in terms of the equation for a corresponding ellipse as

$$\left(\frac{u_1}{R_1}\right)^2 + \left(\frac{u_2}{R_2}\right)^2 = 1 \quad (7)$$

Examples of such an extreme contour ellipse (ECE) in the “normalized” plane with $N_1=100$ and $N_2=10000$ is shown in Figure 1(a). The corresponding result for $N_1=100$ and $N_2=1000$ is shown in Figure 1(b). As observed, there is a marked downwards shift of the maximum point along the vertical axis for case (b) as compared to case (a). The points which correspond to the Turkstra and SRSS rules are also shown in the figure.

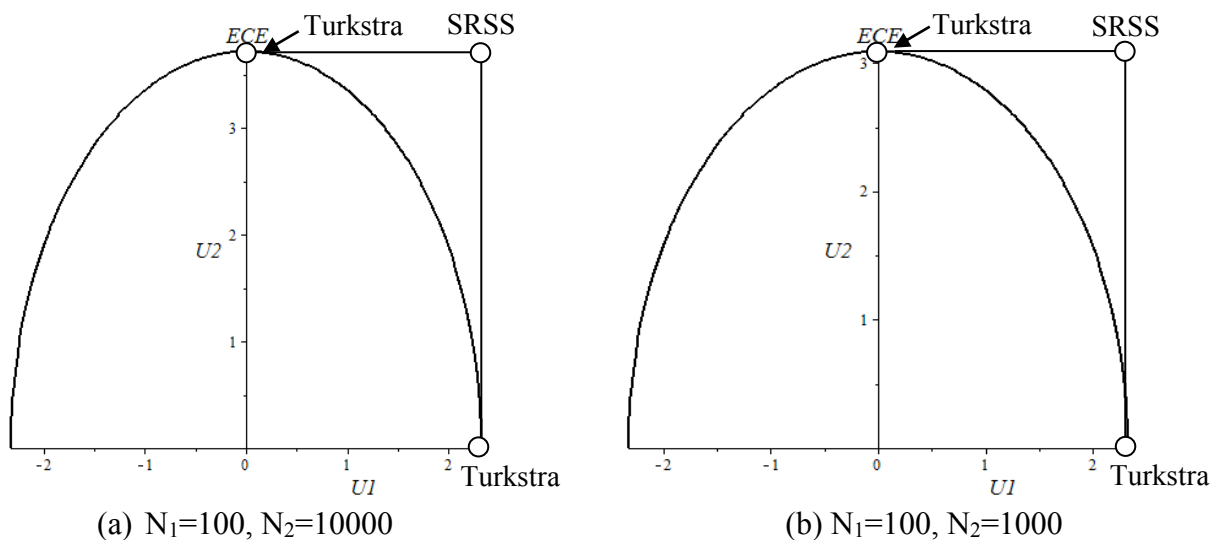


Figure 1. Comparison of contours in the “normalized” plane.

4.3 Search for load combination point

For option (1) the search for the “load combination point” is performed in the same manner as for the case with identical time intervals. For option (2), a re-scaling of the component axes corresponding to all but the longest basic time interval is performed in the normalized space. The scaling is expressed by:

$$u_{i,\text{mod}} = u_i \cdot \left(\frac{R}{R_i} \right) \quad (8)$$

where R is the “radius value” which corresponds to the given return period for the component with the longest time interval.

Having performed such a scaling, the search for the design point proceeds in the same way as for the first option.

5 EXAMPLE OF A TWO-DIMENSIONAL LOAD COMBINATION

5.1 General Description

“Environmental parameter processes” which correspond to mean wind and significant wave height are first considered. Both the mean wind velocity and the significant wave height are assumed to be characterized by their respective Weibull distributions. Both processes are first assumed to have the same basic time interval which is taken to be 3 hours. For a return period of 100 years, the values of R_1 and R_2 will then both have values of 4.5. The effect of applying different basic time intervals for the two components is subsequently investigated.

The two processes are presently assumed to be uncorrelated. Furthermore, *normalized processes are applied* for the two environmental parameters, which means that both of them have scale factors which are equal to unity. This implies that in order to obtain the physical magnitudes of the environmental parameters, they need to be multiplied by the respective Weibull scale parameters for each specific case.

The shape parameter of the distribution for the mean wind velocity is set to $\eta_u = 2.2$, while for the significant wave height a value of $\eta_w = 1.6$ is applied. This means that the cumulative distribution function for the process “step levels” in both cases is given by

$$F_X(x) = 1 - \exp[-(x)^\eta] \quad (9)$$

where η is the particular shape parameter that applies for each component, i.e. $\eta = \eta_u = 2.2$ or $\eta = \eta_w = 1.6$, respectively. As mentioned, the variable x represents the physical quantity divided by the corresponding Weibull scale parameter.

For each of the independent load components which are Weibull distributed, the transformation into standard Gaussian variables is then expressed as follows:

$$u_i(x) = \Phi^{-1} \left\{ 1 - \exp[-(x)^\eta] \right\} \quad (10)$$

where $\Phi^{-1}\{ \}$ is the inverse of the standard Gaussian distribution function.

Subsequently, the combined static load effect due to these two environmental processes is

considered and a limiting capacity value for this combined load effect is introduced. The relevant “load combination point” identified in the response plane by means of the FORM algorithm is described in the next section.

5.2 Contour and design point for identical basic time intervals

The two-dimensional contour which corresponds to the (dimensionless) wind-wave environmental processes with a return period of 100 years (i.e. $N = 292\,000$) is shown in both Figure 2 and Figure 3 below. In Figure 3, the contour corresponding to non-uniform time scales is also included for the purpose of comparison.

Application of the present contour in connection with identification of the load combination “point” based on a particular limit state function is next considered. The static response of the structure due to the mean wind, $r_{s,u}$ is assumed to be given by an expression of the following type:

$$r_{s,u} = C_u \cdot U^2 \quad (11)$$

where C_u is a coefficient which depends on the geometry of the part of the structure which is subjected to the wind, in addition to the stiffness and material properties of the structure itself. U is the *normalized wind velocity* with a probability distribution function of the type given in Equation (10) with a shape factor of 2.2. In the present example the coefficient C_u has a value of $(1/(3.6 \cdot 3.6))$.

Similarly, the static response of the structure due to the (second order) action of the waves is represented by the following expression:

$$r_{s,w} = C_w \cdot W^2 \quad (12)$$

where the proportionality factor for this load effect is equal to $C_w = (1/(5.5 \cdot 5.5))$. The design limit now corresponds to the normalized stress being equal to 1.0 which gives

$$r_{s,u} + r_{s,w} = 1 \quad \Rightarrow \quad C_u U^2 + C_w \cdot W^2 = 1 \quad (13)$$

or

$$g(U, W) = 1 - (C_u U^2 + C_w \cdot W^2) \quad (14)$$

where $g(U, W)$ designates the limit state function. The corresponding limit state corresponds to this function being equal to zero. This constitutes an ellipse in the (non-dimensional) wind-wave-parameter plane.

The associated 100-year contour and the limit state surface are shown in Figure 2. The point with the minimum distance to the origin in the normalized plane is identified by the FORM algorithm which was referred to above. The point obtained by scaling this point to the design contour represents the “load combination point”.

In the standard Gaussian plane, the point on the failure surface has coordinates (4.5, 1.5) which corresponds to (4.9, 1.6) in the non-dimensional wave-wind plane. The corresponding point on the contour has the following coordinates: (4.6, 1.5). The physical values will depend on the Weibull scale parameters. As an example, consider a case for which the scale parameter for the wave height is 5 m and 10 m/s for the wind velocity which gives combination point coordinates of (23 m, 15 m/s).

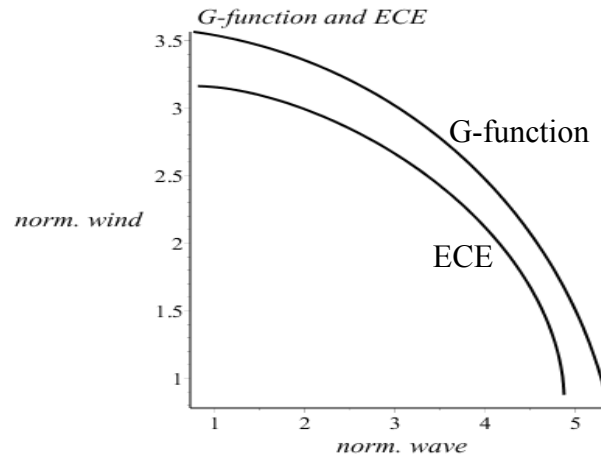


Figure 2. Contour and limit state surface for the case with uniform time intervals for both wind and wave components ($N = 292\,000$).

In the standard Gaussian plane, the load “combination point” has coordinates (4.3,1.4). By dividing these coordinates with the 100-year return value (i.e. $R_1=R_2=R=4.5$) for both components, we obtain the following ratios $(4.3/4.5, 1.4/4.5) = (0.95, 0.3)$. The corresponding return periods for these down-scaled environmental parameters can then also be determined.

5.3 Contour and design point for non-uniform time intervals

The time interval for the wind process is next taken to correspond to 10 minutes, which is 1/18 of the time interval for the wave process (which is 3 hours).

For the wind process, the transition between different time intervals can be performed by application of proper conversion factors for the mean wind velocities (corresponding to different averaging times). Particular expressions for such a conversion are summarized e.g. in Ghiocel and Lungu [7]. As an example, the conversion factor from the 10-minute average value to the 1-hour average value is $(1/1.20) = 0.83$ for city areas, while it is $(1/1.05) = 0.95$ for the seacoast. The corresponding values for conversion from 10-minutes to 3 hours are respectively $(1/1.35) = 0.74$ and $(1/1.07) = 0.93$ for city areas and the seacoast. The latter value (i.e. $(1/1.07)$ for the seacoast) is applied in the present study.

For the wind velocity there will hence be a two-fold effect related to application of shorter basic time intervals: (i) The probability distribution for the average wind velocity is shifted to higher values and (ii) The number of basic time intervals (i.e. number of repetitions) is significantly increased which also serves to shift the probability distribution upwards.

Clearly, the assumption of independence between the 18 repeated 10-minute average sequences is in general highly questionable. The presence of correlation would imply that instead of 18 independent repetitions a reduced “equivalent” number could be applied.

The contours which correspond to a 3hr versus 10 minute time interval for the wind process are compared in Figure 3. There is a very strong difference between the contour shapes for the two cases.

The design point in the non-dimensional wave wind plane now has coordinates (2.2, 3.3). The corresponding point in the transformed Gaussian plane has coordinates (1.9,3.7). Scaling this to the environmental contour, the load combination point in the wave-wind plane has coordinates (2.2,3.2) and (1.9,3.5) in the Gaussian plane. By dividing these values with the corresponding 100-year return values we obtain $(1.9/4.5, 3.5/5.08) = (0.4, 0.7)$ for the present case. This implies that the relative influence of the wind load has increased significantly as compared to the uniform case.

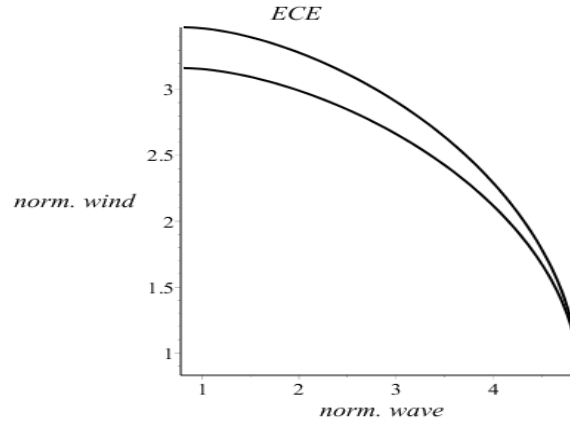


Figure 3. Comparison of contours for the case with (a) uniform time intervals ($N_1 = N_2 = 292\,000$) (b) non-uniform time intervals for the wave-wind process. ($N_1 = 292\,000$, $N_2 = 5\,256\,000$).

6 EXAMPLE OF A THREE-DIMENSIONAL LOAD COMBINATION

6.1 General

An example with three different environmental components is next considered. The two first components are the same as in the previous example, while the third component corresponds to the water current velocity.

Until now it has been common practice to use averaging periods of 10 minutes and longer (e.g. 30 minutes) when recording the current flow directly. However, much shorter intervals have also been considered as relevant, Yttervik [8].

It seems that studies on conversion factors between velocities for different averaging times are not available and will probably be very site dependent. In the present analysis a basic time interval length of 10 minutes is applied.

The cumulative probability distribution of the (dimensionless) mean current velocity is also assumed to be given by a Weibull distribution. The current velocity is normalized such that the scale factor is equal to unity. The corresponding shape factor is $\eta_c = 1.2$.

The static load effect due to the current is given by an expression which is similar to those for the static wind and wave loads:

$$r_{s,c} = C_c \cdot C^2 \quad (15)$$

The values of the three constants for the three-dimensional case are now set equal to $C_u = 1/(10 \cdot 10)$, $C_w = 1/(5.5 \cdot 5.5)$ and $C_c = 1/(6 \cdot 6)$. The total static load effect is expressed as the sum of all the three contributions, and the resulting limit state function then becomes:

$$g(U, W, C) = 1 - (C_u U^2 + C_w W^2 + C_c C^2) \quad (16)$$

which represents the surface of an ellipsoid in the three-dimensional wave-wind-current space (when the limit state function is equal to zero).

6.2 Contour and design point for identical basic time intervals

The contour surface which corresponds to non-dimensional wave, wind and current values is shown in Figure 4 for the case with $N_1=N_2=N_3=N=292\,000$ (which corresponds to a return period of 100 years).

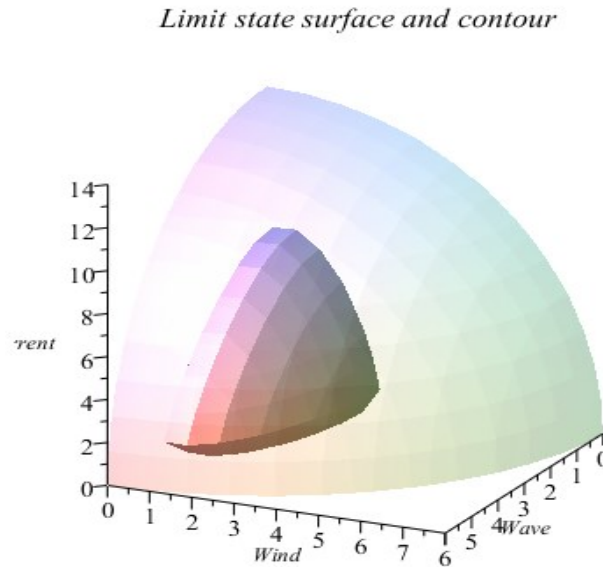


Figure 4. Contour and failure surface in the non-dimensional “wave-wind-current” space for the case with identical time intervals ($N_1=N_2=N_3=N=292\,000$, corresponding to a return period of 100 years).

The point on the failure surface which is identified to be closest to the origin based on application of the FORM algorithm is found to have coordinates (4.0, 1.0, 4.5) in the normalized wave-wind-current space. The physical values can subsequently be obtained by multiplying with the respective scale parameters. It is seen that the wave and the current are the dominant load parameters for the present combination.

6.3 Contour and design point for identical basic time intervals

As the next step, a different FBC-model is applied where the basic time interval for both the average wind and current velocity are 10 minutes, while that for the significant wave height is 3 hours. The contour surface for this case is shown in Figure 5. It is observed that it is widely different from the contour surface in Figure 4.

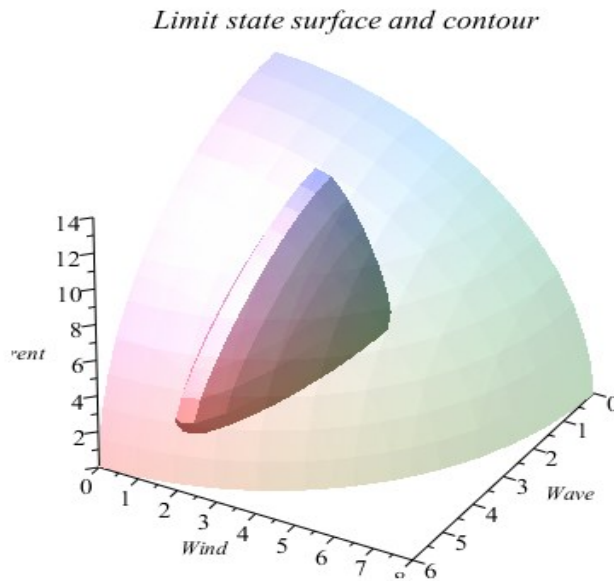


Figure 5. Contour and failure surface in the non-dimensional wind-wave-current space with non-uniform basic time intervals. ($N_1=292\,000$, $N_2=5\,256\,000$, $N_3=5\,256\,000$)

The coordinates of the “load combination point” in the normalized wave-wind-current space are now computed as (2.5, 3.4, 5.7). This implies that the current is the dominating load parameter also for this case, while the wind now is the second most parameter as opposed to the wave for the previous case.

7 CONCLUDING REMARKS

The role of contours in relation to calculation of design load effects and the associated proper load combinations was highlighted. Application examples were given, both for the case with identical basic time intervals and for the case with non-uniform intervals. A non-linear combination of load effects was applied to illustrate the implications of the analysis procedure.

For the mean wind velocity (and also for the mean current velocity) the issue of averaging period seems to have a strong influence on the resulting contour shape. Accordingly, relevant conversion formulas between different averaging periods for the environmental processes that are involved should be readily available for user of design codes in order to achieve a transparent formulation. The assumption of independence between values for reduced averaging periods also needs further clarification.

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