

GLOBAL OPTIMIZATION OF THE SHAPE OF AN AEROSPACE VEHICLE, VIA ITERATIVE OPTIMUM-OPTIMORUM STRATEGY

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Abstract. *The determination of the global optimized (GO) shape of a flying configuration (FC), leads to an enlarged variational problem with free boundaries. An own developed evolutionary iterative optimum-optimorum (OO) strategy was developed in order to solve this problem. The GO shape of the FC is chosen among a class of elitary FCs. A lower limit hypersurface of the drag functional as function of the similarity parameters of the planforms of elitary FCs of the class is used and the elitary FC, which corresponds to the minimum of this hypersurface is, in the same time, the GO shape of the class. The iterative OO strategy uses, in its first step of iteration, analytical start solutions for the determination of the inviscid GO shape of FC, as surrogate model. The OO strategy and own developed software were used for the determination of inviscid GO shapes of three surrogate models, namely of ADELA (a delta wing alone) and of FADET I and FADET II, (two fully-integrated wing-fuselage configurations). These surrogate GO models were optimized with respect of inviscid minimum drag, respectively, at cruising Mach numbers $M_\infty = 2, 2.2, 3.0$. Further, the friction drag coefficients of the surrogate models are computed by using own developed hybrid solutions for the Navier-Stokes PDEs and are checked for the structure point of view. The iterative OO strategy uses, up its second step of iteration, the total drag as new functional and the analytical start solutions are replaced with Navier-Stokes solutions. A GO shape of a space vehicle model, in form of CATAMARAN II, which is optimized with respect of minimum drag, at Mach number $M_\infty = 3.0$, is proposed. It can be usefull for the design of performant sub-orbital space vehicles and for UAVs, It flies shock free, without sonic boom interference, has a reinforced structure and presents a high value of L/D .*

1 INTRODUCTION

The exploration of the space has taken a tremendous development. The new scientific research and the high technical performances make some dream of humans being a reality. At the same time more business, scientists and tourist travelers are interested to have faster intercontinental aircraft and to observe the earth during aerospace voyages. New cosmodromes are constructed and suborbital flights are planned by Virgin company, in order to respond to this increasing travel interest and to realize cheaper travels by increasing the number of passengers and by building more economic and ecologic acceptable supersonic transport aircraft (STA) and low earth orbit (LEO) space vehicles. The main aim of the design of GO shapes of supersonic FCs is to use them to increase the aerodynamic performances of aircraft and space vehicles.

The classic optimization of the shape of a FC consists in the determination of its surface of FC with fixed planform in order to reach a minimum drag at cruise. Such classic optimized shapes of FCs are here called **elitary FCs**. The shape of FC is GO if its camber, twist and thickness distributions and **also** the similarity parameters of its planform are **simultaneously** optimized in order to reach a minimum drag, at a chosen cruising Mach number. The determination of the GO shape of FC leads to an enlarged variational problem with free boundaries, which needs a special mathematical treatment. The own developed optimum-optimum (OO) and iterative optimum-optimum (IOO) theories are special strategies for the determination of the GO shape of the FC, inside of a class of elitary FCs, which satisfy some common properties, as in [1-4].

2 DETERMINATION OF THE INVISCID GLOBAL OPTIMIZED SHAPE OF A SURROGATE MODEL

Let us firstly consider an integrated wing-fuselage FC with arbitrary camber, twist and thickness distributions, which is flying at the cruising Mach number M_∞ . Dimensionless co-ordinates are used for the computation of the distributions of velocity's components:

$$\tilde{x}_1 = \frac{x_1}{h_1}, \quad \tilde{x}_2 = \frac{x_2}{\ell_1}, \quad \tilde{x}_3 = \frac{x_3}{h_1}, \quad \left(\tilde{y} = \frac{\tilde{x}_2}{\tilde{x}_1} \right) \quad (1)$$

Further, integrated wing-fuselage FCs are considered, namely, for which the mean surface is continuous and the thickness distributions on the wing and on the fuselage are different and along the junction lines wing-fuselage they have the same tangent planes. The downwashes on the thin and thick-symmetrical components of the integrated FCs, w and w^* , w'^* (on the wing and on the fuselage of FCs) are supposed to be expressed in form of superposition of homogeneous polynoms with arbitrary coefficients, namely:

$$w \equiv \tilde{w} = \sum_{m=1}^N \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}_{m-k-1,k} |\tilde{y}|^k, \quad w^* \equiv \tilde{w}^* = \sum_{m=1}^N \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}_{m-k-1,k}^* |\tilde{y}|^k, \quad w'^* \equiv \bar{w}^* = \sum_{m=1}^N \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \bar{w}_{m-k-1,k}^* |\tilde{y}|^k. \quad (2a-c)$$

The coefficients of the downwashes and the similarity parameter $\nu = B \ell$ ($\ell = \ell_1 / h_1$, $B = \sqrt{M_\infty^2 - 1}$), of the planform of the wing are the free parameters of the optimization and ℓ , ℓ_1 , h_1 are the dimensionless span, the half-span and the depth of the planform of the delta wing. The quotient of the similarity parameters of the wing and of the fuselage, which depend on the purpose of the FC, is supposed constant. If the principle of minimal singularities (which fulfill the jumps of the velocity's components) and the hydrodynamic analogy of Carafoli are used, the following expressions for the axial disturbances of the thin and of the thick-

symmetrical components of the integrated wing-fuselage FC with subsonic leading edges are obtained, as in [1]:

$$u \equiv \ell \tilde{u} = \ell \sum_{n=1}^N \tilde{x}_1^{n-1} \left\{ \sum_{q=0}^{E\left(\frac{n}{2}\right)} \frac{\tilde{A}_{n,2q} \tilde{y}^{2q}}{\sqrt{1-\tilde{y}^2}} + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \tilde{C}_{n,2q} \tilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\tilde{y}^2}} \right\}, \quad (3a)$$

$$\begin{aligned} u^* \equiv \ell \tilde{u}^* = \ell \sum_{n=1}^N \tilde{x}_1^{n-1} & \left\{ \sum_{q=0}^{n-1} \tilde{H}_{nq}^* \tilde{y}^q \left(\cosh^{-1} M_1 + (-1)^q \cosh^{-1} M_2 \right) \right. \\ & + \sum_{q=0}^{n-1} \tilde{G}_{nq}^* \tilde{y}^q \left(\cosh^{-1} S_1 + (-1)^q \cosh^{-1} S_2 \right) \\ & + \left. \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \tilde{C}_{n,2q}^* \tilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\nu^2 \tilde{y}^2}} + \sum_{q=0}^{E\left(\frac{n-2}{2}\right)} \tilde{D}_{n,2q}^* \tilde{y}^{2q} \sqrt{1-\nu^2 \tilde{y}^2} \right\}, \quad (3b) \end{aligned}$$

$$\left(M_{1,2} = \sqrt{\frac{(1+\nu)(1 \mp \nu \tilde{y})}{2\nu(1 \mp \tilde{y})}} \right), \quad S_{1,2} = \sqrt{\frac{(1+\bar{\nu})(1 \mp \bar{\nu} \tilde{y})}{2(\bar{\nu} \mp \nu \tilde{y})}},$$

The lift and the pitching moment coefficients of the integrated wing-fuselage FCs, computed with the hyperbolic potential theory, are the following:

$$C_\ell \equiv 8\ell \int_{\tilde{\partial}\tilde{A}_1\tilde{C}_1} \tilde{u} \tilde{x}_1 d\tilde{x}_1 d\tilde{y}, \quad C_m \equiv -8\ell \int_{\tilde{\partial}\tilde{A}_1\tilde{C}_1} \tilde{u} \tilde{x}_1^2 d\tilde{x}_1 d\tilde{y}. \quad (4a,b)$$

The inviscid drag coefficients of the thin, thick-symmetrical and thick, lifting FCs with retracted flaps are quadratic forms with respect of the downwashes coefficients :

$$\begin{aligned} C_d & \equiv \ell \tilde{C}_d = 8\ell \int_{\tilde{\partial}\tilde{A}_1\tilde{C}} \tilde{u} \tilde{w} \tilde{x}_1 d\tilde{x}_1 d\tilde{y}, \\ C_d^* & \equiv \ell \tilde{C}_d^* = 8\ell \left[\int_{\tilde{\partial}\tilde{C}_1\tilde{C}} \tilde{u}^* \tilde{w}^* \tilde{x}_1 d\tilde{x}_1 d\tilde{y} + \int_{\tilde{\partial}\tilde{A}_1\tilde{C}_1} \tilde{u}^* \tilde{w}^* \tilde{x}_1 d\tilde{x}_1 d\tilde{y} \right], \\ C_d^{(i)} & \equiv \ell \tilde{C}_d^{(i)} = \ell \left(\tilde{C}_d + \tilde{C}_d^* \right). \quad (5a-c) \end{aligned}$$

The hyperbolic integrated solutions for the axial disturbance velocity given in (3a,b) are used as start solutions for the determination of the inviscid GO shape of the wing-fuselage FC, which is of minimum drag at cruising Mach number. The free parameters of the optimization are the coefficients of the downwashes w , w^* and w^{**} and also the similarity parameters of the planforms of the wing and of the fuselage. Further it is supposed that the quotient of these similarity parameters, which is determined for the purpose of the FC, is considered constant.

The constraints of the inviscid GO shape's design are: the given lift, pitching moment and the Kutta condition along the subsonic leading edges of the thin FC component (in order to cancel the induced drag at cruise and to suppress the transversal contournement of the flow around the leading edges, in order to increase the lift) and the given relative volumes of the wing and of the fuselage zone, the cancellation of thickness along the leading edges and the new introduced integration conditions along the junction lines between the wing and fuselage zone of the thick-symmetrical FC component (in order to avoid the detachment of the flow along these lines). According to the optimum-optimorum strategy, the GO shape of the FC is searched among the elitary FCs with the same area of their planforms which belong to the same class of FCs. The class is defined by the common properties of the elitary FCs, which belong to this class. The similarity parameter ν of the planform of FC is sequentially varied and a lower limit-line of the inviscid drag functional of elitary FCs, as function of this similarity parameter ν , is obtained. For FCs with subsonic leading edges, is: $0 < \nu < 1$. The position of the minimum of this limit-line gives the optimal value of the similarity parameter $\nu = \nu_{opt}$ and the corresponding elitary FC is, at the same time, the GO FC of the class.

The author has used the OO strategy for the determination of the GO shapes of three models, namely, ADELA (a wing alone) and FADET I and FADET II (two fully-integrated wing-fuselage FCs), which are of minimum drag at, respectively, cruising Mach numbers $M_\infty = 2; 2.2; 3$.

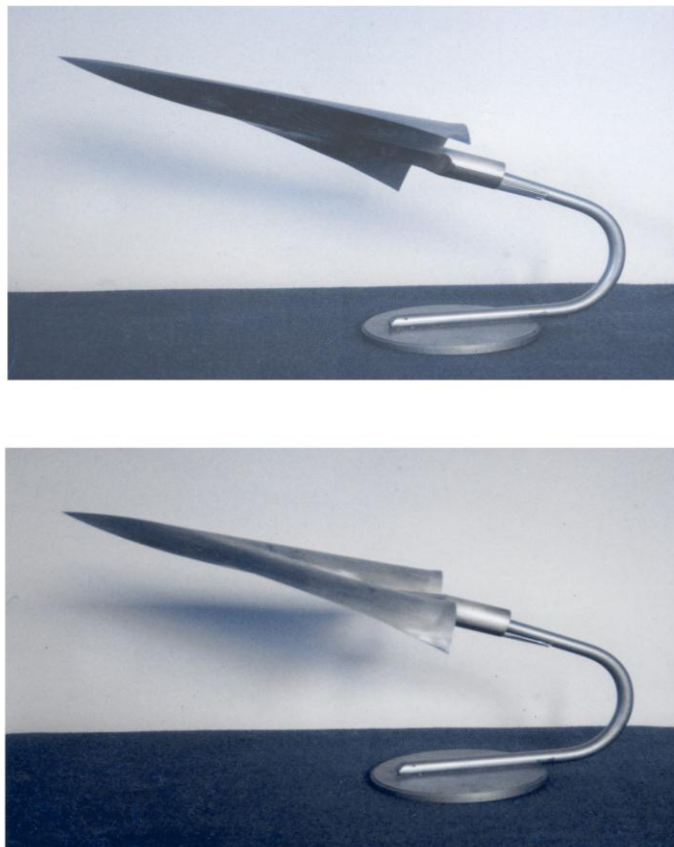


Fig. 1a,b The Global Optimized Model FADET II

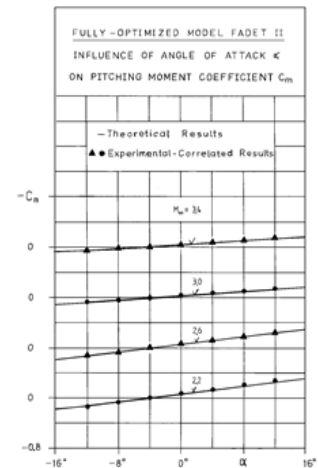
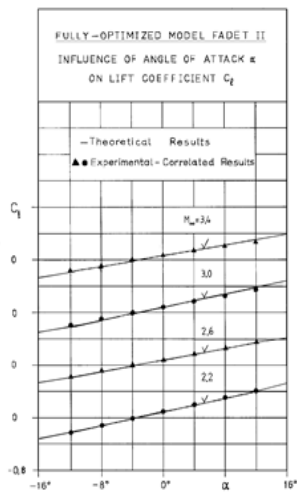


Fig. 2a,b The Agreement of the Theoretical Determined Lift and Pitching Moment Coefficients of the Global Optimized Model FADET II with the Experimental Results

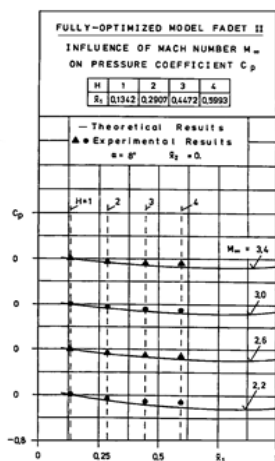
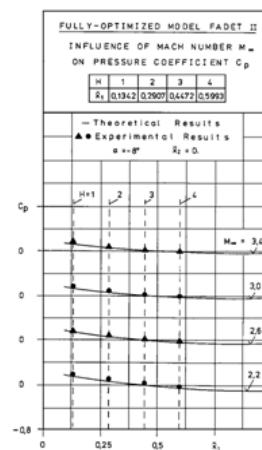
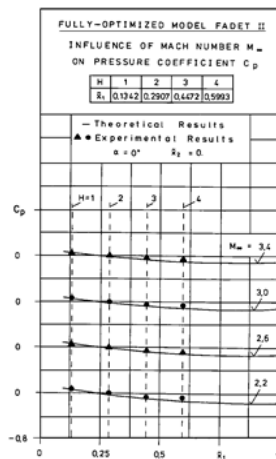


Fig. 3a-c Comparison of Theoretical and Experimental Determined Pressure Coefficients on the Central Longitudinal Cut of the Global Optimized Model FADET II, at the Angles of Attack $\alpha = -8^\circ, 0^\circ, 8^\circ$

In the (Figs. 1a,b) are presented two views of the GO shape of the fully-integrated Model FADET II. It was measured in the trisonic wind tunnel of DLR-Cologne, in the frame of research projects of the author, sponsored by the DFG. The comparisons of theoretical and experimental-correlated values of the lift and pitching moment coefficients of all these models were in very good agreements with the experimental results. In the (Fig. 2a,b) are presented these agreements for the lift and pitching moment coefficients of the model FADETt II with subsonic leading edges at moderate angles of attack, as exemplification.

In the (Fig3a-c) are presented the agreements between the theoretical predicted and the measured pressure coefficients along the central longitudinal cut of the model FADET II at moderate angles of attack $\alpha = -8^\circ, 0^\circ, 8^\circ$.

3 HYBRID SOLUTIONS FOR THE THREE-DIMENSIONAL COMPRESSIBLE NAVIER-STOKES LAYER

The new developed, hybrid, meshless solutions for the boundary value problems of the PDEs of the Navier-Stokes layer, proposed here, use the hyperbolic potential solutions of the flow on the same FC twice, namely: at the NSL's edge (instead of parallel flow used by Prandtl in his boundary layer theory) and in the structure of the velocity's components, which are expressed inside the NSL, as products between the corresponding potential velocity's components with polynoms with arbitrary coefficients, versus a spectral variable. These coefficients are used to satisfy the NSL's PDEs, in an arbitrary chosen number of points. Let us firstly introduce a spectral variable:

$$\eta = \frac{x_3 - Z(x_1, x_2)}{\delta(x_1, x_2)} \quad (0 \leq \eta \leq 1) \quad (6)$$

The proposed forms for the hybrid numerical solutions of the velocity's components are, as in [1-4], the following:

$$u_\delta = u_e \sum_{i=1}^N u_i \eta^i, \quad v_\delta = v_e \sum_{i=1}^N v_i \eta^i, \quad w_\delta = w_e \sum_{i=1}^N w_i \eta^i. \quad (7a-c)$$

The here introduced logarithmic density function $R = \ln \rho$ and the absolute temperature T are the following:

$$R = R_w + (R_e - R_w) \sum_{i=1}^N r_i \eta^i, \quad T = T_w + (T_e - T_w) \sum_{i=1}^N t_i \eta^i. \quad (8a,b)$$

The pressure p is computed by using the physical equation of perfect gas and, for the viscosity μ , an exponential law is used

$$p = R_g \rho T = R_g e^R T, \quad \mu = \mu_\infty \left(\frac{T}{T_\infty} \right)^{n_1} \quad (9a,b)$$

The free coefficients u_i, v_i, w_i, r_i and t_i are used to satisfy the NSL's PDEs in some chosen points. If the hybrid forms for the velocity's components (7a-c) are introduced in the continuity's PDE and the collocation method is used, the coefficients r_i are determined only as func-

tions of the coefficients of the velocity's components, by solving a linear algebraic system and the coefficients t_i satisfy the PDE of absolute temperature and are also obtained only as functions of the coefficients of the velocity's components by solving of a transcendental algebraic system. A splitting of the NSL's PDEs is obtained and the physical entities are expressed only as function of the spectral coefficients of the velocities components and can be easily updated in an iterative process. A speed up of computation time is obtained. The coefficients of velocity's components are determined by using the impulse PDEs, which are iteratively solved, as in [1-4].

The hybrid solutions for the NSL presented here are reinforced numerical solutions, which present important analytical properties, namely: they have correct last behaviors, they have correct jumps due to the singularities located only along the singular lines (like the junction lines wing-fuselage and the subsonic leading edges of the wing of the FC) obtained according to the principle of minimal singularities which fulfil the jumps and the singularities are balanced, they are accurate because the partial derivatives of velocity's components can be exactly computed, they are split due to the use of the logarithmic density function and therefore they produce a speed up of the computation time, they fulfil automatically the non-slip condition on the FCs surface, they are matched with the outer potential flow and for moderate perturbations, they are reduced to the potential solutions at the NSL's edge and they do not need interface. Additionally, for hyperbolic PDEs the boundary condition on its characteristic surface is automatically fulfilled.

The hybrid solutions of the NSL's PDEs are useful for the computation of the friction drag coefficient of the FC. The skin friction coefficient at the wall takes the form:

$$\tau_{x_1}^{(w)} \equiv \tau_{x_1} \Big|_{\eta=0} = \mu_f \frac{\partial u_\delta}{\partial \eta} \Big|_{\eta=0} = \mu_f u_1 u_e \quad . \quad (10)$$

The friction drag coefficient and the total drag of the FC, with arbitrary camber, twist and thickness distributions are:

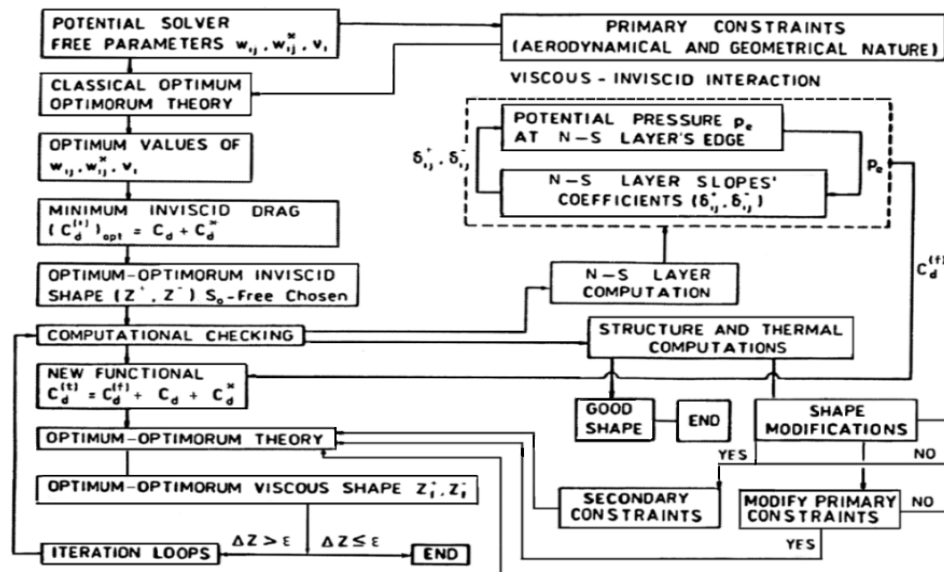
$$C_d^{(f)} = 8 \nu_f u_1 \int_{\tilde{\partial A_1} \tilde{c}} u_e x_1 dx_1 dy \quad , \quad C_d^{(t)} = C_d^{(f)} + C_d^{(i)} \quad . \quad (11a,b)$$

These hybrid NSL's solutions are also used for the viscous design of the GO shape of FC.

3 THE ITERATIVE OPTIMUM-OPTIMORUM STRATEGY

The viscous iterative OO theory of the author is proposed for the viscous determination of the GO shape of the FC in order to present a total minimum drag at cruising Mach number. The viscous iterative OO strategy uses the inviscid hyperbolic potential solutions as start solutions and the inviscid GO shape of these FCs as surrogate models, **only** in its first step of iteration. An intermediate computational checking of this inviscid GO shape of the FC is made with own hybrid solvers, for the three-dimensional compressible NSL. The friction drag coefficient $C_d^{(f)}$ of the FC is computed and the inviscid GO shape is checked also for the structure point of view. A weak interaction aerodynamics-structure is proposed. Additional or modified constraints, introduced in order to control the camber, twist and thickness distributions of the GO shape, for structure reasons, are here proposed. In the second step of optimization, the predicted inviscid GO shape of the FC is corrected by including these additional constraints in

the variational problem and of the friction drag coefficient in the drag functional. The chart flow of the iterative OO strategy is given in the (Fig. 4).



4. The Iterative Optimum-Optimorum Strategy

4 THE PROPOSED AEROSPACE VEHICLE MODEL FOR LEO

The main aim of the design of new GO shapes of supersonic FCs is to use them to increase the aerodynamic performances of supersonic transport aircraft (STA) and of low earth orbit (LEO) space vehicles and to build new supersonic FCs, which are more economical and ecological acceptable.

The proposed GO shape of Catamaran II with respect of minimum drag, at cruising Mach number $M_\infty = 3$ is presented in (Fig. 5). It has twin fuselages, partially embedded in the thickness of the GO FCs.

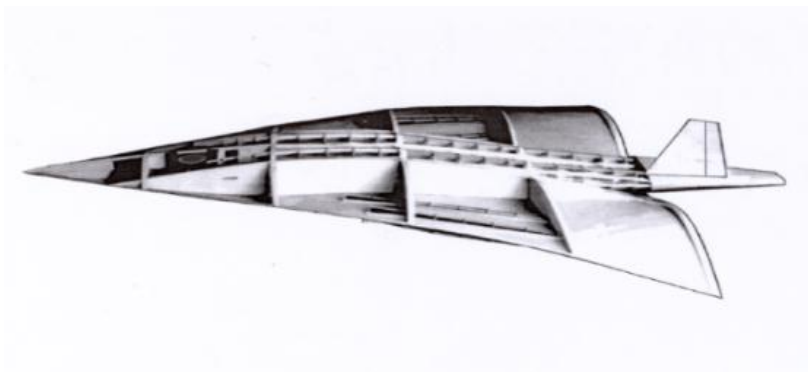


Fig. 5 GO Shape of Catamaran II

The following benefits are expected by using such GO shapes of FCs:

- Due to the global optimization with respect of minimum drag the GO shape are economical, chipper for passengers and produce less pollution;

Due to the premises and of judicious choose of constraints it fly shock free, has no sonic boom interference and has more stiffness.

5 CONCLUSIONS

The Catamaran II can be used also as sources of inspirations for the shapes of the future new generation of aerospace vehicles for sub-orbital flights and for UAVS.

The Catamaran II has some advantages, when it is compared with the FCs with one, central, non-integrated fuselage, carrying the same number of passengers, at the same cruising Mach number:

- it flies with a *shock-free surface*;
- it has *no sonic boom interference* because it flies with one *characteristic surface* (the classical FCs with one central non-integrated fuselage flies with two shock surfaces, one produced at the frontal part of the fuselage and the other at the roots of the wing and in their intersection zones, the sonic boom interference occurs);
- it has a *better structural stiffness and increased lateral stability*; because instead of one long fuselage there are two twin fuselages embedded in the wing, with half length;
- it needs *less trim* because the weight is better distributed and the pressure center and the center of gravity points are closer together;
- it has a *higher L/D*, due to global optimization, full integration, of flattened form and due to the fulfilling of the Kutta condition along its subsonic leading edges, which avoids the leading edges contournements, which cancels the induced drag, destroys the leading edge vortices and increases the lift, not only at cruise, but also for large ranges of Mach numbers and angles of attack.

The proposed GO shapes of supersonic FCs look like birds in gliding flight!

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