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# NEW BENCHMARK FOR THE LIFE ASSESSMENT OF A THIN-WALLED PIPE SUBJECTED TO STRESS ASSISTED CORROSION

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**Keywords:** Mechanochemical Corrosion, Thin-Walled Tube, High Pressure Vessel, Analytical Solution.

**Abstract.** *A thin-walled tube subjected to internal and external pressure under the conditions of double-sided mechanochemical corrosion is considered. The rates of corrosion are supposed to be linearly dependent on the maximum principal stress. Previous solutions for thin cylindrical shells obtained by other authors reflect only the effect of differential pressure (i.e., the difference between internal and external pressure) and do not depend on the internal and external pressure values themselves. The model proposed here allows to take into account the effect of hydrostatic pressure on the durability of the tube exposed to corrosive environment, but it is not so cumbersome as the accurate solution for the double-sided corrosion of a pressurized tube based on the solution of Lamé problem for a thick-walled cylinder. The problem is reduced to an ordinary differential equation of the first order, which is solved in a closed form. It has been observed that the computational results for the classical thin shell model and for the proposed here refined model are very close to each other when either internal or external pressure is equal to zero. But the difference between the mentioned results grows as the minimum of the pressure values increases.*

## 1 INTRODUCTION

Corrosion is a process of material destruction that occurs as a result of chemical interaction between a material and its environment. General corrosion is one of the most common types of corrosion, that damages the entire surface of the material at about the same rate, and as a consequence, results in a failure. If a structural element, as well as being in aggressive environment, is subjected to mechanical load, then the mechanochemical effect may occur when corrosion is enhanced by stress [1, 2]. In such situations for the assessment of the durability of structural elements, initial boundary value problems with evolving boundaries have to be considered. Most of them are solved by numerical methods. However, it is reasonable to have closed-form benchmarks for the verification of numerical techniques. A number of closed-form solutions have been found for the cases of uniform corrosion, e.g. by the authors of [3, 4, 5, 6, 7, 8, 9].

This paper presents an analytical solution for a thin-walled cylindrical tube subjected to internal and external pressure under the conditions of double-sided mechanochemical dissolution. The rates of corrosion are supposed to be linearly dependent on the maximum principal stress [1]. There is a lot of works devoted to thin-walled structures [3, 4, 5, 6]. However, previous solutions for thin shells obtained by other authors, do not depend on the internal and external pressures themselves, but depend only on their difference. Therefore, they do not reflect the effect of hydrostatic pressure (i.e. the minimum of the internal and external pressures) on lifetime of the vessel. At the same time, the known accurate solutions for double-sided corrosion of elastic tube under internal and external pressure based on the solution of Lamé problem are rather cumbersome [10, 11]. The model proposed here is not so complicated as these accurate solutions, but it allows to take into account the effect of hydrostatic pressure on the durability of thin shells under the mechanochemical corrosion conditions. The same questions for spherical shells were investigated in [12, 13, 14, 15]. The problem of calculation of the optimal initial thickness of a spherical vessel operating in mechanochemical corrosion conditions was solved in [16]. The results can be taken into account for the lifetime assessment of thin-walled structural elements subjected to the combined action of mechanical loads and an aggressive environment [17, 18, 19, 20, 21].

## 2 THE DESCRIPTION OF THE PROBLEM

Consider a linearly elastic long thin-walled cylindrical tube with the inner $r$ and outer $R$ radii subjected to internal $p_r$ and external $p_R$ pressure which are given constant. The article covers the following cases: an incompressible tube subjected to the plane-strain condition characterized by vanishing axial strain and a closed cylindrical vessel. The action of the ends of the cylinder is not taken into account. Cylinder is uniformly corroding from the inside and outside with the dissolution rates $v_r$ and $v_R$, respectively. Change of the tube's thickness is assumed to be quasi-static and uniform throughout the entire length: the inner radius increases while the outer radius decreases with time $t$. Let the tube's radii at the initial time $t_0$ be denoted by $r_0$ and $R_0$.

According to [1, 3], the rates of corrosion on the inner and outer surfaces are:

$$v_r = \frac{dr}{dt} = a_r + m_r\sigma_1(r), \qquad v_R = -\frac{dR}{dt} = a_R + m_R\sigma_1(R), \tag{1}$$

where $a_r,\ a_R,\ m_r,$ and $m_R$ are experimentally determined constants, which, in general, are different for tension and compression [1, 7]; $\sigma_1$ is the maximum principal stress at the corresponding surface of the tube; $\operatorname{sign} m = \operatorname{sign}\sigma_1$.

Consider a cylindrical coordinate system $(\rho, \theta, z)$ with the $z$-axis coincide with the axis of the tube. In the previous solutions for thin-walled tubes obtained by other authors (for example [4]), $\sigma_1$ was chosen equal through the thickness of the shell:

$$\sigma_1(r) = \sigma_1(R) = \sigma_{\theta\theta} = \frac{\Delta p R_c}{h}, \tag{2}$$

where $R_c = (R_0 + r_0)/2$ is the radius of the middle surface of the cylindrical tube (which is supposed to be constant during the corrosion), $h = R - r$ is the thickness of the tube, and $\Delta p = p_r - p_R$.

It's obvious that the solution derived by the use of equation (2) does not depend on the pressure values $p_r$ and $p_R$, but depends only on the difference $\Delta p$. However, it is known that hydrostatic pressure $p = \min\{p_r, p_R\}$ can affect corrosion kinetics [1].

The purpose of this paper is to derive a new (more accurate, but not more complicated than previous) solution which allow to take into account the effect of hydrostatic pressure $p$ on the assessment of the durability of thin-walled tubes subjected to mechanochemical corrosion.

## 3 THE SOLUTION OF THE PROBLEM

### 3.1 Effective stress

According to the G. Lame's formulas for a thick-walled tube under internal and external pressure, maximum principal stress at every time $t$ is determined as

$$\sigma_{\theta\theta}(\rho) = \frac{p_r r^2 - p_R R^2}{R^2 - r^2} + \frac{(p_r - p_R)}{R^2 - r^2}\frac{r^2 R^2}{\rho^2}, \qquad r \le \rho \le R. \tag{3}$$

Solution to the problem of mechanochemical corrosion obtained by the use of formula (3) is rather cumbersome [11]. Let us simplify formula (3) taking into account that for a thin-walled shell we can assume that $h/R_c << 1$. Using other notations: $r = R_c - \delta$ and $R = R_c + \delta$ (where $\delta = h/2$), we can write

$$\begin{aligned}\sigma_{\theta\theta}(r) &= \frac{p_r(2R_c^2 + 2\delta^2) - 2p_R(R_c^2 + 2R_c\delta + \delta^2)}{4R_c\delta} \\ &= \frac{p_r\left(1 + \delta^2/R_c^2\right) - p_R\left(1 + 2\delta/R_c + \delta^2/R_c^2\right)}{2\delta/R_c},\end{aligned} \tag{4}$$

and

$$\begin{aligned}\sigma_{\theta\theta}(R) &= \frac{-p_R(2R_c^2 + 2\delta^2) + 2p_r(R_c^2 - 2R_c\delta + \delta^2)}{4R_c\delta} \\ &= \frac{-p_R\left(1 + \delta^2/R_c^2\right) + p_r\left(1 - 2\delta/R_c + \delta^2/R_c^2\right)}{2\delta/R_c}.\end{aligned} \tag{5}$$

Due to the fact that $\delta/R_c << 1$, the terms $\delta^2/R_c^2$ in equations (4) and (5) can be neglected compared to 1. However, we leave the terms $\delta/R_c$ which will allow us to take into account the effect of hydrostatic pressure $p = \min\{p_r, p_R\}$. As a result we obtain

$$\sigma_{\theta\theta}(r) = \frac{\Delta p R_c}{h} - p_R, \qquad \sigma_{\theta\theta}(R) = \frac{\Delta p R_c}{h} - p_r. \tag{6}$$

### 3.2 Basic differential equation

Let us assume that during the corrosion process the radius $R_c$ remains constant, while the thickness $h$ is changing with time.

Combining equations (1) and (6) gives the ordinary differential equation of the first order

$$\frac{dh}{dt} = -(a_r + a_R - m_R p_r - m_r p_R) - (m_r + m_R)\frac{\Delta p\, R_c}{h}. \tag{7}$$

The initial condition to be satisfied at $t = t_0$ is $h|_{t_0} = R_0 - r_0$.

### 3.3 Solution of the basic equation

The integral of the differential equation (7), satisfying the above initial conditions is

$$t = t_0 + \frac{m\,\Delta p\, R_c}{a^2}\ \ln\frac{m\,\Delta p\, R_c + ah}{m\,\Delta p\, R_c + ah_0} - \frac{h - h_0}{a}, \tag{8}$$

where $a = a_r + a_R - m_R p_r - m_r p_R$ and $m = m_r + m_R$.

Maximum principal stresses at the tube's surfaces can then be calculated by the use of equations (6) for every $(t, h)$.

### 3.4 The assessment of the lifetime

In the framework of the maximum normal stress criterion, since $|\sigma_1(r)| \geq |\sigma_1(R)|$, for the assessment of the durability of the tube we should track the stress $\sigma_1(r)$. Let us denote it by $\sigma$. The lifetime of the tube can be defined as the time $t^*$ at which $\sigma$ reaches a strength limit $\sigma^*$ : $\sigma(t^*) = \sigma^*$. Therefore, the lifetime can be determined by equation (8) with $h^* = \Delta p\, R_c/(\sigma^* + p_R)$ for $h$.

## 4 CALCULATION RESULTS

Let us compare calculation results obtained by the proposed model (using equations (6) for the effective stresses in (1)) with the results based on the classical thin shell theory formula (using equation (2) for the effective stress). In figure 1 the dependencies of $|\sigma|$ on $t$ are plotted for the cylindrical tube with the middle radius $R_c = 100$ [$l_c$] and the initial thickness $h_0 = 4$ [$l_c$]; $t_0 = 0$. Here, $l_c$ is a certain unit of length. Corrosion kinetics constants are $a_r = a_R = 0.16$ [$l_c/t_c$] and $m_r = m_R = 0.008$ [$l_c/(t_c p_c)$]). Here and below, $t_c$ is an appropriate unit of time; $p_c$ is a given unit of stress.

In order to demonstrate the effect of hydrostatic pressure $p = \min\{p_r, p_R\}$, four sets of the internal and external pressure values were used:

- $p_r = 1$ [$p_c$], $p_R = 0$ (green dotted line);
- $p_r = 0$, $p_R = 1$ [$p_c$] (blue dotted line);
- $p_r = 6$ [$p_c$], $p_R = 5$ [$p_c$] (green dashed line);
- $p_r = 5$ [$p_c$], $p_R = 6$ [$p_c$] (blue dashed line).

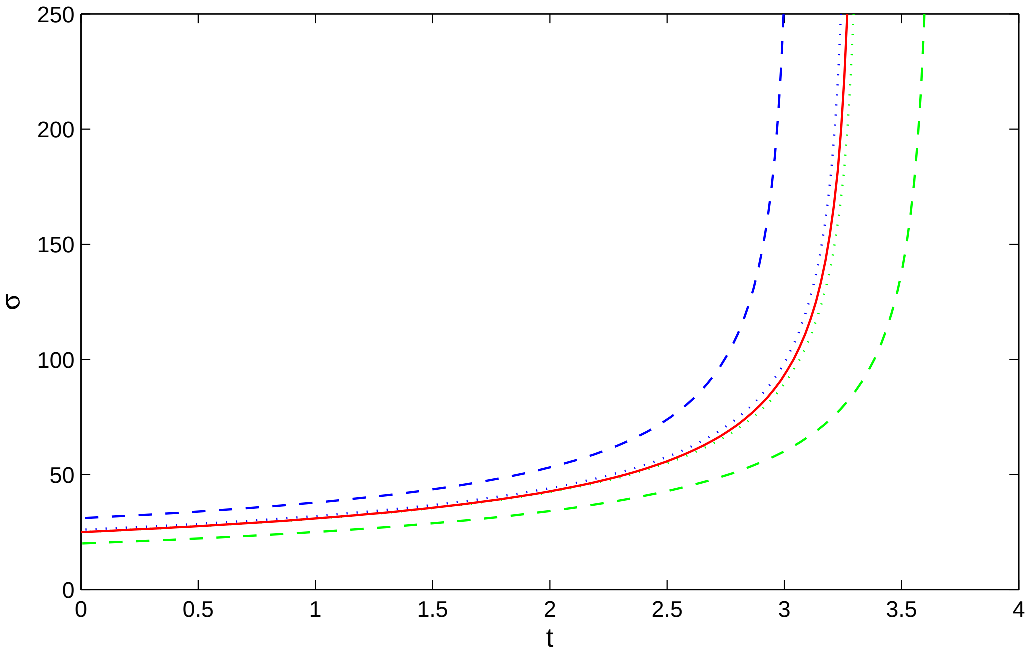


Figure 1: Evolution of $|\sigma(t)|$ for $|\Delta p| = 1$.

These sets are chosen in such a way that despite various values of the pressures $p_r$ and $p_R$, the absolute value of the pressure difference remains constant: $|\Delta p| = 1$ [$p_c$].

It is evident that when the effective stress is given by equation (2), the dependencies of $|\sigma|$ on the time $t$ for all the above sets of pressure values are the same. In other words, for $|\Delta p| = \text{const}$, the classical thin shell theory formula provides only one curve; it is the red solid line in figure 1.

For the refined model, we have different curves for each of the considered sets of pressure values. Blue curves in figure 1 correspond to the case when $\Delta p < 0$, while green ones correspond to positive differences, $\Delta p > 0$.

It is seen from the figure that the curves for both models are close to each other at zero hydrostatic pressure, $p = \min\{p_r, p_R\} = 0$ (compare solid line and dotted lines). However, the difference between the classical thin shell model (solid line) and our refined model (dashed lines) increases with the growing $p$. Therefore, using the classical thin shell model is not reasonable for modelling the mechanochemical corrosion under high hydrostatic pressure (especially when $|\Delta p| << p$).

In the framework of our model it was revealed that the durability of the tube increases as $p$ grows if $\Delta p > 0$ (see green lines), and decreases if $\Delta p < 0$ (see blue lines).

At the same time, the computational results [11] based on the Lamé's formula and proposed here are hardy differ for any pressure values.

## 5 CONCLUSIONS

- A new mathematical model for the double-sided mechanochemical corrosion of a thin-walled cylindrical tube has been developed.
- An analytical solution of the problem has been obtained.

- The computational results for the classical thin shell model and for the proposed here refined model are very close to each other for zero hydrostatic pressure, $p = \min\{p_r, p_R\} = 0$.
- As compared to the classical thin shell model, the durability of the tube increases as $p$ grows if $p_r > p_R$, and decreases if $p_r < p_R$.
- Using the classical thin shell model is not reasonable for modelling the mechanochemical corrosion under high hydrostatic pressure.
- The solutions based on the Lamé's formula and proposed here are hardy differ for any pressure values.

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