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LARGE EDDY SIMULATIONS OF MAGNETIC FIELD EFFECT ON TURBULENT FLOW IN A SQUARE DUCT

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Abstract. *Numerical simulation of magnetohydrodynamic turbulent duct flow is important in the application of liquid metal fusion blanket. We develop a large eddy simulation magneto-hydrodynamic turbulent flow solver based on finite volume method and dynamic Smagorinsky subgrid model in OpenFOAM environment. The induced magnetic field is negligible as the magnetic Reynolds number is small. The electric potential Poisson equation has been used to solve the electromagnetic field. The consistent and conservative method has been used to solve the electric potential, the electric current and the Lorentz force. Periodic boundary condition has been used at the inlet and outlet. We investigate the effects of applied magnetic field on mean velocity field, fluctuation velocity, and turbulent kinetic energy of the liquid metal fluid flow. The secondary flow generated by the anisotropic turbulence stress has been shown obviously. As the Hartmann number increases the turbulent kinetic energy decreases, which shows the turbulence suppression effects of the external magnetic field.*

1 INTRODUCTION

The study of magnetohydrodynamic (MHD) turbulent flow is essential in the application of liquid metal pumps, MHD power generators, and liquid metal blankets of thermal nuclear fusion reactors[1]. Turbulent MHD duct flows have been investigated extensively by numerical simulation and experiment in these years [2-8]. Zikanov [9] reviewed the laminar-turbulent transition of MHD flows in duct, pipe, and channel.

Numerical investigation of MHD duct flow has been implemented by direct numerical simulation (DNS) [2], large eddy simulation (LES) [5, 7, 10] and Reynolds-averaged Navier-Stokes (RANS) method [11]. Researchers concerned the effects of the external magnetic field, wall conductance ratio, the cross-section of the duct on the MHD turbulent flow. Large eddy simulation is a promising method to investigate the MHD turbulent flow with limited grid and calculation time. There are different subgrid-scale(SGS) models in the simulation of MHD turbulent flow such as Smagorinsky model (SM), coherent structure Smagorinsky model (CSM) and dynamic Smagorinsky model (DSM) [6, 7]. The study of Kobayshi [6] and Kransnov [4] shows that the DSM can predict the MHD flow transformation accurately.

We have developed MHD solver with dynamic Smagorinsky model in OpenFOAM environment. MHD turbulent flow in a insulating square duct has been simulated to investigate the effect of the external magnetic field on the turbulent flow.

2 GOVERNING EQUATIONS AND NUMBERICAL METHODS

2.1 Governing equations

The incompressible MHD duct flow subject to an uniform magnetic field parallel to insulated walls is governed by the mass conservative equation, the Navier-Stokes equation with Lorentz force, the Ohm's law, electric current conservation and the electric potential Poisson equation,

$$\nabla \bullet \mathrm{u} = 0, \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nabla\cdot\nu\nabla\mathbf{u} + \frac{1}{\rho}\mathbf{j}\times\mathbf{B}, \quad (2)$$

$$\mathrm{j} = \sigma(-\nabla\phi + \mathrm{u}\times\mathrm{B}), \quad (3)$$

$$\nabla\cdot\mathrm{j} = 0, \quad (4)$$

$$\nabla^2\phi = \nabla\cdot(\mathbf{u}\times\mathbf{B}). \quad (5)$$

Here $\mathbf{u}$ is the velocity vector, t is the time, p is the pressure divided by the density, ν is the kinematic viscosity, $\mathbf{j}$ is the electric current density, ρ is the density, $\mathbf{B}$ is the applied external magnetic field, σ is the electrical conductivity and ϕ is the electric potential.

In the magnetohydrodynamic flow, there are several important dimensionless parameters. The magnetic Reynolds number $Re_m = \mu\sigma L v_0$, μ is the magnetic permeability. In fusion blanket application, the magnetic Reynolds number is less than 1. Therefore, we could neglect the induced magnetic field in the Ohm's law. The Reynolds number $Re = v_0 D/\nu$, D is hydrodynamic diameter, v_0 is character velocity, which in general is the mean velocity of the flow. The interaction parameter or Stuart number $N = \sigma LB^2/\rho v_0$. The Hartmann number is defined as $Ha = BL\sqrt{\sigma/\rho\nu} = \sqrt{Re \cdot N}$.

The spatial filtering of unsteady Navier-Stokes equation is:

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u}_i \overline{u}_j\right) = -\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{1}{\rho}[\mathbf{j} \times \mathbf{B}]_i , \tag{6}$$

where τ_{ij} is the subgrid-scale stresses,

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j = \tau_{ij}^* + \frac{1}{3}\delta_{ij}\tau_{kk} . \tag{7}$$

2.2 Dynamic Smagorinsky SGS model

In LES, the SGS stress is modeled as a single SGS turbulence model. Germano [12] proposed the dynamic Smagorinsky SGS model. In the DSM, τ_{ij}^* is modeled with the filter width $\overline{\Delta}$ as:

$$\tau_{ij}^* = -2\nu_t \overline{S_{ij}} = -2\left(C_{SGS}\Delta_1\right)^2 \left|\overline{S}\right| \overline{S_{ij}} . \tag{8}$$

Here, C_{SGS} is the Smagorinsky constant, the velocity-strain tensor for the resolved component $\overline{S_{ij}}$ and its magnitude $\left|\overline{S}\right|$ are defined by

$$\overline{S_{ij}} = \frac{1}{2}\left(\frac{\partial \overline{u_j}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_j}\right), \tag{9}$$

$$\left|\overline{S}\right| = \sqrt{2\overline{S_{ij}}\,\overline{S_{ij}}} . \tag{10}$$

The model parameter of the DSM is determined using a least square procedure proposed by Lilly [13] with an average in homogeneous directions,

$$C_{SGS}^2 = \frac{1}{2}\frac{\left\langle L_{ij} M_{ij} \right\rangle}{\left\langle M_{ij} M_{ij} \right\rangle} . \tag{11}$$

The angular brackets $\langle\ \rangle$ indicate an average procedure over the cell face. L_{ij} and M_{ij} are given by:

$$L_{ij} = \widehat{\overline{u}_i \overline{u}_j} - \hat{\overline{u}}_i \hat{\overline{u}}_j \tag{12}$$

$$M_{ij} = \Delta_1^2 \widehat{\left|\overline{S}\right| \overline{S}_{ij}} - \Delta_2^2 \left|\hat{\overline{S}}\right| \hat{\overline{S}}_{ij} \tag{13}$$

$$\Delta_1^3 = \Delta_x \Delta_y \Delta_z , \qquad \Delta_2 = 2\Delta_1 . \tag{14}$$

Here, Δ_x, Δ_y, Δ_z are element sizes in x, y, z directions, respectively.

The Van Driest function is used for the wall damping corrections, which is expressed as:

$$f = 1 - \exp(-y^+ / 26) . \tag{15}$$

2.3 Numerical model and boundary conditions

An incompressible electric conducting flow in a square duct subject to external uniform magnetic field has been simulated. The duct is electric insulating. The magnetic field is applied in y direction. The x, y and z are streamwise, Hartmann layer and side layer directions, respectively. The calculation domain is $\left(2\pi,1,1\right)$. The Reynolds number is $\mathrm{Re}=6400$, where $D=1$. The Hartmann number equals 8, 20 and 25.6. Therefore, the corresponding ratios of $R=Re/Ha$ are 800, 320 and 220 respectively.

At the wall, no-slip boundary condition is applied:

$$\mathbf{u}=0\,. \tag{16}$$

As the wall is electric insulating, no electric current penetrates the wall,

$$\partial\phi/\partial n=0\,, \tag{17}$$

here n is the normal direction of the wall.

At the inlet and outlet, periodic boundary condition is used for the velocity, the pressure and the electric potential.

The domain is divided by $256\times96\times96$ grids. A uniform grid is used along the streamwise direction and nouniform grids are used along y, z direction. The spatial resolutions of Δy^{+}, Δz^{+} are ranged from $1.03\sim8.224$ with stretching 3%. The time step is 1.0×10^{-3} seconds, and the statistic properties are obtained by averaging 500 seconds data after the flow reaches a statistically steady state.

Consistent and conservative methods on a rectangular collocated grid [14] have been used to solve the electric current, the electric Potential equation and Lorentz force. The the mass conservative equation and the momentum equation are solved by the PISO algorithm in OpenFOAM.

In order to keep a constant flow rate, a constant pressure gradient corrected by the flow rate at the cross section is added in the Navier-Stokes equation.

The initial field is perturbed by adding 2% random fluctuation of the average velocity in three directions.

3 RESULTS AND DISCUSSION

The instantaneous axial velocity contours with transverse velocity vectors and the time-averaged axial velocity with transverse velocity vectors at the cross section at the middle of the duct are presented in Figure 1 and Figure 2 for $Ha=20$. Figure 1 shows that there is a high speed axial velocity core in the centre of the duct. The secondary velocities are generated especially near the parallel duct wall, which accompanies the reduction of the axial velocity. Figure 2 shows the time statistical information of the velocity. It shows that the secondary velocities direct from centre to the four corners and form eight vortexes at the corners. As the external magnetic field is along y direction, the velocity component along z is reduced remarkably because of the Lorentz force. The vortex formed by the secondary flow is bigger near the Hartmann walls than that near the side walls.

Figure 3 shows the mean axial velocity along horizontal bisector. With the same Reynolds number, when the Hartmann number is small, the MHD effect is not apparently, the difference of the mean velocity distributions along y, z is quite small. With the external magnetic field increasing, the MHD effect results that the distribution of the mean axial velocity along the parallel external magnetic field is more circular than that along the vertical magnetic field direction.

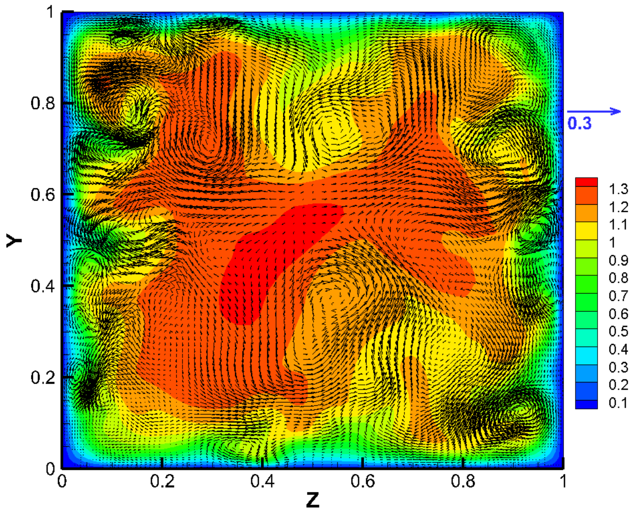

Figure 1 Instantaneous axial velocity contours and secondary velocity vectors for *Re*=6400, *Ha*=20.

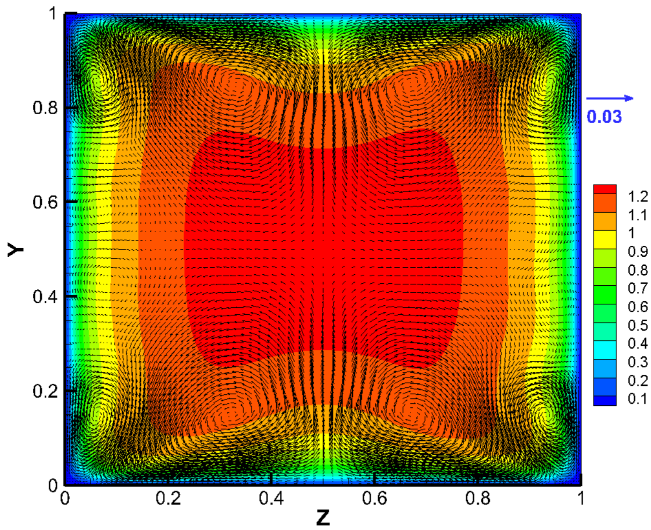

Figure 2 Mean axial velocity contours and secondary velocity vectors for *Re*=6400, *Ha*=20.

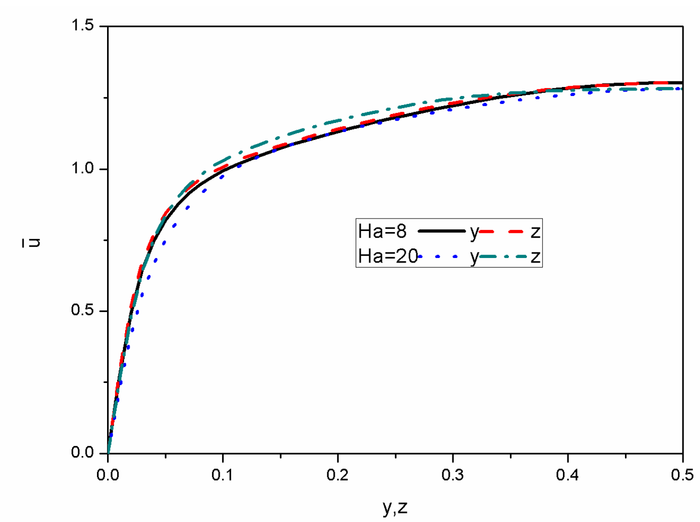

Figure 3 Mean axial velocity along horizontal bisector.

Figure 4 shows the turbulent kinetic energy along horizontal bisector. As $Ha = 8$, the turbulent kinetic energy is almost the same along y, z. The effect of the magnetic field is not obvious. As Hartmann number increases, the turbulent kinetic energy decreases obviously. Furthermore, the turbulent kinetic energy along y is remarkably small than that along z directions. This is caused by the MHD effects. The main induced electric current returns through the Hartmann layer, while the main induced electric current parallel to the external magnetic field in the side layers. The MHD effect results that the fluctuations of the velocity are suppressed by the Lorentz force.

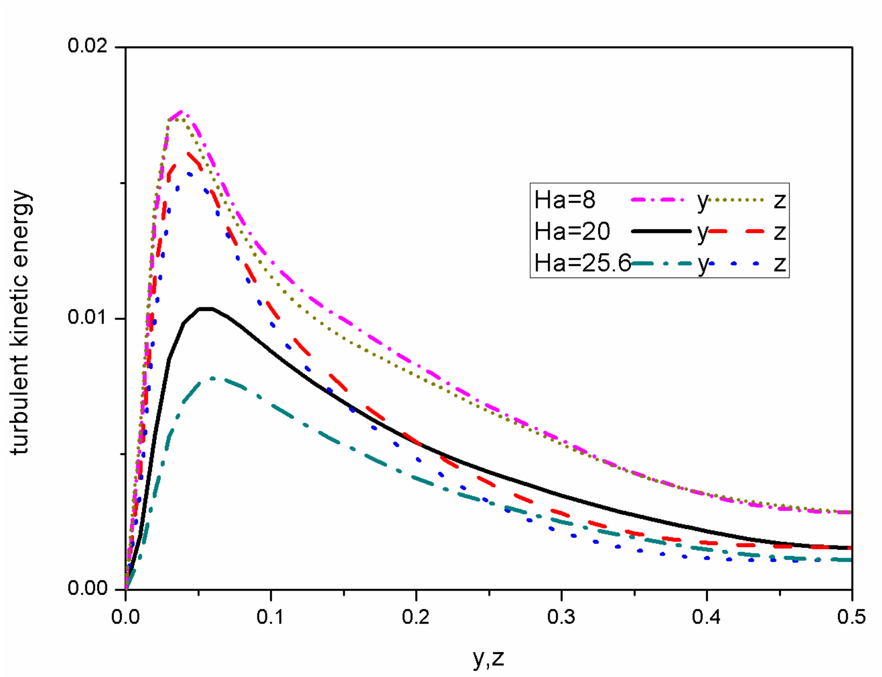

Figure 4 Turbulent Kinetic energy along horizontal bisector.

4 CONCLUSIONS

- A MHD turbulent LES solver has been developed in OpenFOAM environment.
- Dynamic Smagorinsky model has been used to simulate the MHD square duct flow with electric insulating walls.
- MHD duct flows with the same Reynolds number and different Hartmann numbers have been numerically simulated to investigate the effect of the external magnetic field on the turbulent flow.
- External magnetic field depresses the turbulent fluctuation especially on the Hartmann layer direction.

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