ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

LOCALIZATION OF VIBRATORY ENERGY OF A LINEAR SYSTEM IN A CHAIN OF FOUR NONLINEAR OSCILLATORS

Simon Charlemagne[1], **Claude-Henri Lamarque**[1] **and Alireza Ture Savadkoohi**[1]

[1]Univ Lyon, ENTPE, LTDS UMR CNRS 5513
Rue Maurice Audin, F-69518, Vaulx-en-Velin Cedex, France
{simon.charlemagne,claude.lamarque,alireza.turesavadkoohi}@entpe.fr

Keywords: Nonlinear oscillators, vibratory energy, multi-scales method, passive control

Abstract. *In this paper, dynamics of a five degree-of-freedom system formed by a main linear oscillator coupled to four light nonlinear systems in series is studied. The aim is to control and/or to harvest the energy of the main structure under harmonic excitations around its resonance. A multiple scales method is used to derive the behavior of the system at different time scales. At fast time scale, detected slow invariant manifold gives an overall comprehension of the possible behaviors that the system can undergo. At slow time scale, the modulated behavior of the system around its invariant is described by traced equilibrium and singular points. The former predict periodic regimes, while the latter hint at strongly modulated responses characterized by persisting bifurcations of the system around its unstable zones. All analytical results are validated by numerical simulations.*

1 INTRODUCTION

Nonlinear oscillators are known to be efficient to passively attenuate vibratory energy of mechanical systems. Several works in the literature deal with two degree-of-freedom (dof) systems. Gendelman [1] studies energy pumping by means of energy transfer to a nonlinear mode of a system consisting of a linear and a nonlinear coupled oscillators. Kerschen et al. [2] make experimental investigations on energy transfer from a linear oscillator to a cubic nonlinear one. Passive control of nonlinear systems have also been studied. Dahl type and piece-wise linear systems coupled to a piece-wise linear oscillator are examined by Ture Savadkoohi and Lamarque [3] and Lamarque et al. [4], respectively. The present work aims to study passive control of a linear system by four nonlinear oscillators in series. This represents a step towards investigation of system with higher dof, i.e. a main system coupled to a chain of oscillators. It is organized as it follows: model of the system and methodology of treatment are given in Sect. 2. Study of the system behavior at fast and slow time scale, leading to detection of slow invariant manifold and equilibrium and singular points, respectively, is led in Sect. 3. Analytical predictions are compared to numerical results in Sect. 4. Finally, concluding remarks are summarized in Sect. 5.

2 MODEL DESCRIPTION AND SYSTEM EQUATIONS

The five dof system described in Fig. 1 is studied. A linear structure under external excitation $F(t)$ which has a mass M, a stiffness K and a damping $\hat{a}$ is coupled to four nonlinear oscillators in series via a linear spring ρ and a damper $\tilde{a}$. The nonlinear oscillators have a mass m, a damping $\tilde{a}$ and a nonlinear potential $\tilde{V}$. Their masses is very small compared to the main system one's ($m = \epsilon M$, $0 < \epsilon \ll 1$).

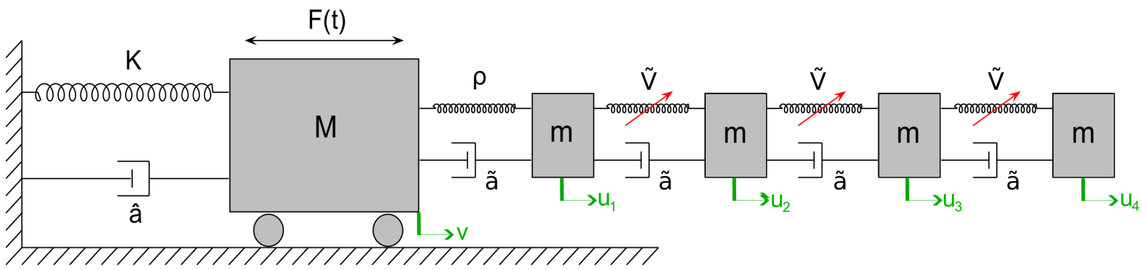


Figure 1: Five dof model studied consisting of a forced linear structure coupled to four nonlinear oscillators ($m = \epsilon M, 0 < \epsilon \ll 1$)

Displacements of the main system and the nonlinear oscillators are called as v and u_j, $j = 1, \ldots, 4$, respectively. Let us introduce w_l as new variables of the system:

$$\begin{aligned} w_1 &= u_1 - v \\ w_l &= u_l - u_{l-1} \quad l = 2, 3, 4 \end{aligned} \tag{1}$$

Besides, new parameters are defined as followed: $\frac{K}{M} = \omega_0^2$, $\frac{\rho}{M} = \epsilon r$, $\frac{\tilde{V(z)}}{M} = \epsilon(Bz + Dz^3)$, $\frac{\hat{a}}{M} = \epsilon a$, $\frac{\tilde{a}}{M} = \epsilon a_1$ and $\frac{F(t)}{M} = \epsilon f(t)$. External forcing is assumed to be sinusoidal, oscillating at the angular frequency ω: $f(t) = f^0 \sin(\omega t)$. We study the system around resonance of the main structure: $\omega^2 = \omega_0^2(1 + \sigma\epsilon)$. Finally, a multiple scale method [5] is used by embedding time to different scales, i.e. fast time scale $\tau_0 = t$ and slow time scales $\tau_j = \epsilon^j t,\ \ j = 1, 2 \ldots$ Governing equations of the system read:

$$\ddot{\mathbf{y}} + (\mathbf{C}_0 + \epsilon \mathbf{C}_1)\dot{\mathbf{y}} + (\mathbf{A}_0 + \epsilon \mathbf{A}_1)\mathbf{y} + \mathcal{N}(\mathbf{y}) = \epsilon \mathcal{F}(t) \tag{2}$$

where

$$\mathbf{A}_0 = \begin{pmatrix} \omega_0^2 & 0 & 0 & 0 & 0 \\ -\omega_0^2 & r & -B & 0 & 0 \\ 0 & -r & 2B & -B & 0 \\ 0 & 0 & -B & 2B & -B \\ 0 & 0 & 0 & -B & 2B \end{pmatrix} \quad \mathbf{A}_1 = \begin{pmatrix} 0 & -r & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{C}_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & a_1 & -a_1 & 0 & 0 \\ 0 & -a_1 & 2a_1 & -a_1 & 0 \\ 0 & 0 & -a_1 & 2a_1 & -a_1 \\ 0 & 0 & 0 & -a_1 & 2a_1 \end{pmatrix} \quad \mathbf{C}_1 = \begin{pmatrix} a & -a_1 & 0 & 0 & 0 \\ -a & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\mathcal{N}(\mathbf{y}) = D \begin{pmatrix} 0 \\ -w_2^3 \\ 2w_2^3 - w_3^3 \\ -w_2^3 + 2w_3^3 - w_4^3 \\ -w_3^3 + 2w_4^3 \end{pmatrix} \quad \mathbf{y} = \begin{pmatrix} v \\ w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} \quad \mathcal{F}(t) = \begin{pmatrix} f^0 \sin(\omega t) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

As $\mathbf{A}_0$ is diagonalizable, there exists a matrix $\mathbf{P}_0$ such as $\mathbf{P}_0^{-1}\mathbf{A}_0\mathbf{P}_0 = \mathbf{\Lambda}_0^2$, i.e. $\mathbf{A}_0$ and $\mathbf{\Lambda}_0^2$ are similar matrices. Furthermore, we assume that $\mathbf{\Lambda}_0^2 = \omega^2 \mathbf{Id}_5 + \tilde{\mathbf{\Lambda}}_0^2$, where $\mathbf{Id}_5$ is the 5×5 identity matrix. System (2) can be rewritten:

$$\ddot{\mathbf{y}} + \omega^2 \mathbf{y} + (\mathbf{C}_0 + \epsilon \mathbf{C}_1)\dot{\mathbf{y}} + \mathbf{P}_0 \tilde{\mathbf{\Lambda}}_0^2 \mathbf{P}_0^{-1}\mathbf{y} + \epsilon \mathbf{A}_1 \mathbf{y} + \mathcal{N}(\mathbf{y}) = \epsilon \mathcal{F}(t) \tag{4}$$

Following complex variables of Manevitch [6] are introduced to the system (4):

$$\mathbf{\Phi} e^{i\omega t} = \dot{\mathbf{y}} + i\omega \mathbf{y} \quad \text{with} \quad \mathbf{\Phi} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \\ \varphi_5 \end{pmatrix} \tag{5}$$

In order to keep only first harmonics and to truncate higher order ones, a Galerkin method is used by applying following formula on each equations $s_j,\ j = 1, \ldots, 5$ of system (4):

$$S_j = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} s_j(\tau_0) e^{-i\omega\tau_0} d\tau_0 \tag{6}$$

Φ is supposed to be independent of τ_0, which will be eventually verified at least for an asymptotic state. Finally, system (4) reads:

$$\dot{\mathbf{\Phi}} + \frac{1}{2}(\mathbf{C}_0 + \epsilon \mathbf{C}_1)\mathbf{\Phi} + \frac{1}{2i\omega}(\mathbf{P}_0 \tilde{\mathbf{\Lambda}}_0^2 \mathbf{P}_0^{-1} + \epsilon \mathbf{A}_1)\mathbf{\Phi} - \frac{3iD}{8\omega^3}\mathcal{N}_m(\mathbf{y}) = \epsilon \mathcal{F}_m(t) \tag{7}$$

$$\mathcal{N}_m(\mathbf{y}) = \begin{pmatrix} 0 \\ -|\varphi_2|^2\varphi_2 \\ 2|\varphi_2|^2\varphi_2 - |\varphi_3|^2\varphi_3 \\ -|\varphi_2|^2\varphi_2 + 2|\varphi_3|^2\varphi_3 - |\varphi_4|^2\varphi_4 \\ -|\varphi_3|^2\varphi_3 + 2|\varphi_4|^2\varphi_4 \end{pmatrix} \quad \mathcal{F}_m(t) = \begin{pmatrix} \frac{f^0}{2i} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

3 SYSTEM BEHAVIOR AT DIFFERENT TIME SCALES

3.1 Fast time scale τ_0

The system behavior at fast time scale is described by system (7) derived at ϵ^0 order:

$$\begin{aligned} &\frac{\partial \mathbf{\Phi}}{\partial \tau_0} + \frac{1}{2}\mathbf{C}_0\mathbf{\Phi} + \frac{1}{2i\omega_0}\mathbf{P}_0\tilde{\mathbf{\Lambda}}_0^2\mathbf{P}_0^{-1}\mathbf{\Phi} - i\mathcal{D}\mathcal{N}_m(\mathbf{y}) = 0 \\ &\Leftrightarrow \frac{\partial \mathbf{\Phi}}{\partial \tau_0} + \begin{pmatrix} 0 \\ \mathcal{H}_2(\mathbf{\Phi}) \\ \mathcal{H}_3(\mathbf{\Phi}) \\ \mathcal{H}_4(\mathbf{\Phi}) \\ \mathcal{H}_5(\mathbf{\Phi}) \end{pmatrix} = 0 \end{aligned} \tag{9}$$

where $\mathcal{D} = \frac{3D}{8\omega_0^3}$.

Fix points of system (9) verify $\lim_{\tau_0 \to +\infty} \frac{\partial \tilde{\mathbf{\Phi}}}{\partial \tau_0} = 0$ where $\tilde{\mathbf{\Phi}} = \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_4 & \phi_5 \end{pmatrix}^T$. They define the slow invariant manifold (SIM) of the system:

$$\mathcal{H} = \begin{pmatrix} \mathcal{H}_2(\tilde{\mathbf{\Phi}}) \\ \mathcal{H}_3(\tilde{\mathbf{\Phi}}) \\ \mathcal{H}_4(\tilde{\mathbf{\Phi}}) \\ \mathcal{H}_5(\tilde{\mathbf{\Phi}}) \end{pmatrix} = 0 \tag{10}$$

Introducing polar coordinates as $\phi_j = N_j e^{i\delta_j},\ j = 1, \ldots, 5$, SIM can be written after some cumbersome algebra:

$$\begin{pmatrix} \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{pmatrix} = \frac{\phi_1}{g(N_3^2, N_4^2, N_5^2)} \begin{pmatrix} h_2(N_3^2, N_4^2, N_5^2) \\ h_3(N_3^2, N_4^2, N_5^2) \\ h_4(N_3^2, N_4^2, N_5^2) \\ h_5(N_3^2, N_4^2, N_5^2) \end{pmatrix} \tag{11}$$

$h_j(N_3^2, N_4^2, N_5^2)$ and $g(N_3^2, N_4^2, N_5^2)$ are complex functions. From system (11), SIM can be plotted in the $(N_1, N_2, N_3, N_4, N_5)$ space. This manifold houses all possible asymptotic states of the system.

Stability of the SIM can be computed by introducing an infinitesimal perturbation of $\mathbf{\Phi}$ in system (9) around the SIM as:

$$\mathbf{\Phi} \to \tilde{\mathbf{\Phi}} + \mathbf{\Delta}\tilde{\mathbf{\Phi}} \quad , \quad \mathbf{\Delta}\tilde{\mathbf{\Phi}} = \begin{pmatrix} 0 & \Delta\phi_2 & \Delta\phi_3 & \Delta\phi_4 & \Delta\phi_5 \end{pmatrix}^T \tag{12}$$

Linearizing the obtained system leads to the following matrix system:

$$\begin{pmatrix} \dfrac{\partial \mathbf{\Delta} \tilde{\mathbf{\Phi}}}{\partial \tau_0} \\ \\ \dfrac{\partial \mathbf{\Delta} \tilde{\mathbf{\Phi}}^*}{\partial \tau_0} \end{pmatrix} = \mathbf{\Sigma} \begin{pmatrix} \mathbf{\Delta} \tilde{\mathbf{\Phi}} \\ \\ \mathbf{\Delta} \tilde{\mathbf{\Phi}}^* \end{pmatrix} \tag{13}$$

where $\mathbf{\Sigma}$ is an 10×10 matrix. Points where real parts of the eigenvalues of $\mathbf{\Sigma}$ are negative determine unstable zones of the SIM.

3.2 Slow time scale τ_1

Let us consider first equation of system (7) derived at ϵ^1 order:

$$\frac{\partial \varphi_1}{\partial \tau_1} + \left(\frac{i\sigma\omega_0}{2} + \frac{a}{2} \right) \varphi_1 + \left(\frac{ir}{2\omega_0} - \frac{a_1}{2} \right) \varphi_2 + \frac{if^0}{2} = 0 \tag{14}$$

This equation provide necessary additional information to detect equilibrium and singular points at slow time scale. Equilibrium points predict periodic regimes whereas singular points are hints of strongly modulated response (SMR) [7] which is characterized by persistent bifurcations of the system around its unstable zones. Separating its real and imaginary parts and using system (11), Eq. (14) becomes:

$$\begin{gathered} \frac{\partial \varphi_1}{\partial \tau_1} + \underbrace{\left(\frac{i\sigma\omega_0}{2} + \frac{a}{2} + \frac{h_2}{g} \left(\frac{ir}{2\omega_0} - \frac{a_1}{2} \right) \right)}_{\alpha} \varphi_1 + \frac{if^0}{2} = 0 \\ \Leftrightarrow \begin{cases} \dfrac{\partial N_1}{\partial \tau_1} = -\alpha_r N_1 - \dfrac{f^0}{2} \sin(\delta_1) = E_1 \\ \dfrac{\partial \delta_1}{\partial \tau_1} = -\alpha_i - \dfrac{f^0}{2N_1} \cos(\delta_1) = E_2 \end{cases} \end{gathered} \tag{15}$$

where $._r$ and $._i$ stand for real and imaginary parts, respectively. Moreover, from systems (10) and (15), one can obtain:

$$\begin{gathered} \frac{\partial \mathcal{H}}{\partial \tau_1} = 0 \\ \Rightarrow \det(\mathbf{S}_2)\mathbf{Id}_8 \begin{pmatrix} \dfrac{\partial N_2}{\partial \tau_1} \\ \vdots \\ \dfrac{\partial N_5}{\partial \tau_1} \\ \\ \dfrac{\partial \delta_2}{\partial \tau_1} \\ \vdots \\ \dfrac{\partial \delta_5}{\partial \tau_1} \end{pmatrix} = \underbrace{-\mathrm{adj}(\mathbf{S}_2)\mathbf{S}_1 \begin{pmatrix} E_1 \\ \\ E_2 \end{pmatrix}}_{\mathcal{G}} \end{gathered} \tag{16}$$

where $\mathrm{adj}(.)$ stands for the adjugate matrix and:

$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_1 \\ \vdots \\ \mathcal{G}_8 \end{pmatrix} \mathbf{S}_1 = \begin{pmatrix} \frac{\partial \mathcal{H}_{2r}}{\partial N_1} & \frac{\partial \mathcal{H}_{2r}}{\partial \delta_1} \\ \frac{\partial \mathcal{H}_{2i}}{\partial N_1} & \frac{\partial \mathcal{H}_{2i}}{\partial \delta_1} \\ \vdots & \vdots \\ \frac{\partial \mathcal{H}_{5r}}{\partial N_1} & \frac{\partial \mathcal{H}_{5r}}{\partial \delta_1} \\ \frac{\partial \mathcal{H}_{5i}}{\partial N_1} & \frac{\partial \mathcal{H}_{5i}}{\partial \delta_1} \end{pmatrix} \quad \mathbf{S}_2 = \begin{pmatrix} \frac{\partial \mathcal{H}_{2r}}{\partial N_2} & \cdots & \frac{\partial \mathcal{H}_{2r}}{\partial N_5} & \frac{\partial \mathcal{H}_{2r}}{\partial \delta_2} & \cdots & \frac{\partial \mathcal{H}_{2r}}{\partial \delta_5} \\ \frac{\partial \mathcal{H}_{2i}}{\partial N_2} & \cdots & \frac{\partial \mathcal{H}_{2i}}{\partial N_5} & \frac{\partial \mathcal{H}_{2i}}{\partial \delta_2} & \cdots & \frac{\partial \mathcal{H}_{2i}}{\partial \delta_5} \\ \vdots & & \vdots & \vdots & & \vdots \\ \frac{\partial \mathcal{H}_{5r}}{\partial N_2} & \cdots & \frac{\partial \mathcal{H}_{5r}}{\partial N_5} & \frac{\partial \mathcal{H}_{5r}}{\partial \delta_2} & \cdots & \frac{\partial \mathcal{H}_{5r}}{\partial \delta_5} \\ \frac{\partial \mathcal{H}_{5i}}{\partial N_2} & \cdots & \frac{\partial \mathcal{H}_{5i}}{\partial N_5} & \frac{\partial \mathcal{H}_{5i}}{\partial \delta_2} & \cdots & \frac{\partial \mathcal{H}_{5i}}{\partial \delta_5} \end{pmatrix} \tag{17}$$

Equilibrium points verify:

$$\begin{cases} \det(\mathbf{S}_2) \neq 0 \\ E_1 = E_2 = 0 \end{cases} \Leftrightarrow \begin{cases} \det(\mathbf{S}_2) \neq 0 \\ \mathcal{G}_1 = \ldots = \mathcal{G}_8 = 0 \end{cases} \tag{18}$$

Singular points verify:

$$\begin{cases} \det(\mathbf{S}_2) = 0 \\ \mathcal{G}_1 = \ldots = \mathcal{G}_8 = 0 \end{cases} \tag{19}$$

4 NUMERICAL RESULTS

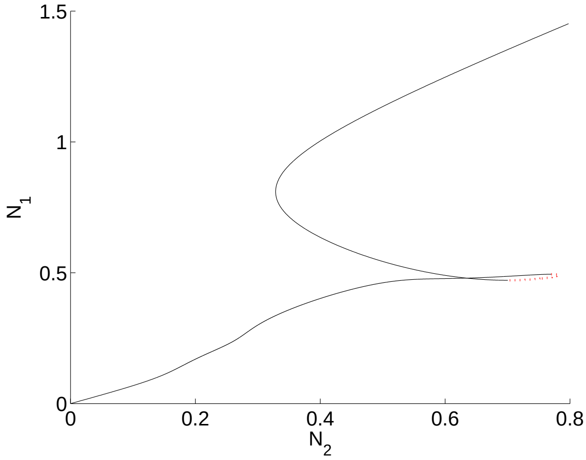
Analytical developments are compared in this section with numerical simulations obtained for direct integration of system (2). This task has been carried out thanks to function *ode45* of Matlab. Following parameters are fixed all along this section: $\omega_0 = 1$, $a = 0.2$, $\epsilon = 0.001$, $B = 0.2$, $r = 0.5$, $a_1 = 0.2$ and $D = 10$. Assumed initial conditions are: $v = 1$ and $\dot{v} = u_1 = \dot{u}_1 = u_2 = \ldots = \dot{u}_4 = 0$.

4.1 SIM

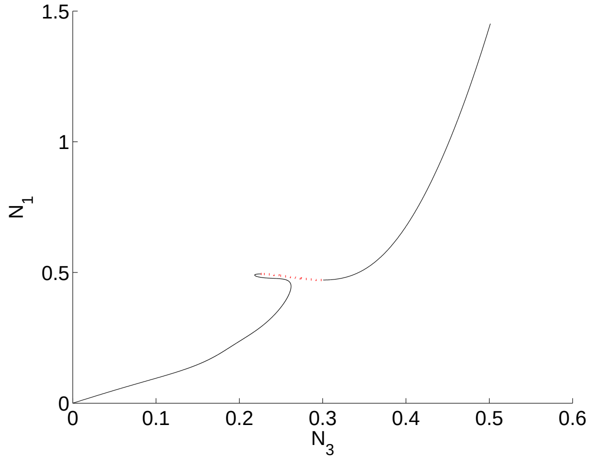
SIM of the system is illustrated in Fig. 2. Figures 2(a)-(d) represent N_1 versus N_2, N_3, N_4 and N_5, respectively. Unstable zones are plotted in dotted red line. The SIM presents two local extrema for N_1 which delimit an unstable zone. Furthermore, one can see what appears to be a double point in Fig. 2(a). This is an effect of the projection in the plane (N_1, N_2), as shown by the three-dimensional flow of the SIM in the (N_1, N_2, N_3)-space in Fig. 2(e).

4.2 $\mathbf{f^0 = 0.2}$ and $\sigma = 0$ (exact resonance)

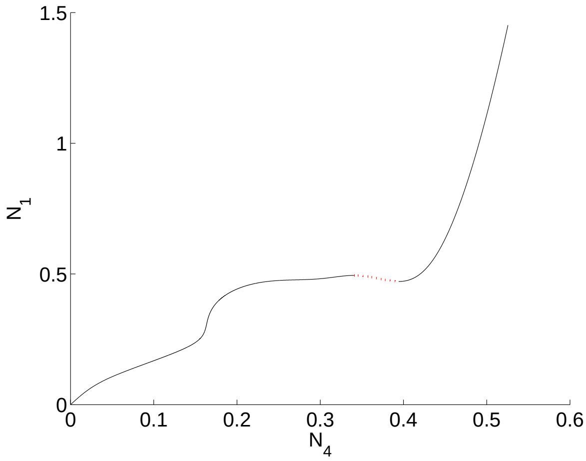
Position of equilibrium and singular points are given in Fig. 3. The system possesses one stable equilibrium point (no. 1). SIM of the system and corresponding numerical results are given in Figs. 4(a) and (b). We picked two representations, N_1 versus N_2 and N_1 versus N_5, among four possible. Corresponding time series are given in Figs. 4(c),(d) and (e), where amplitudes of predicted equilibrium point no. 1 (see Fig. 3) are depicted by a blue dashed line. Besides, the amplitude that the main system would have without additional nonlinear oscillators is plotted in solid black line in Fig. 4(c). One can see that numerical results match analytical predictions, as the system behavior is instantaneously attracted to the SIM and stabilizes around the above-cited equilibrium point. Moreover, vibratory energy has been localized in the nonlinear oscillators, as the main system's amplitude equates to around 27% of the amplitude it would have on its own.



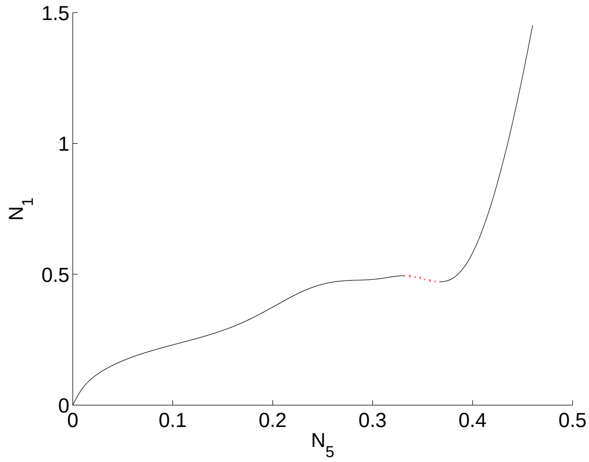
(a)



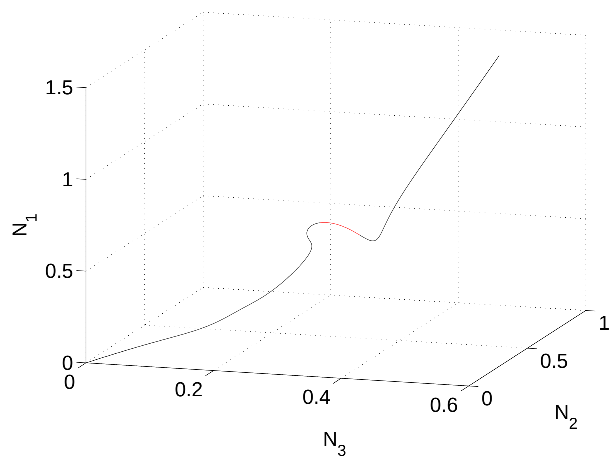
(b)



(c)



(d)



(e)

Figure 2: SIM of the system. (a) N_1 versus N_2, (b) N_1 versus N_3, (c) N_1 versus N_4, (d) N_1 versus N_5 and (e) Three-dimensional flow of the SIM in the (N_1, N_2, N_3)-space. Stable and unstable zones of the SIM are plotted in black solid line and dotted red line, respectively

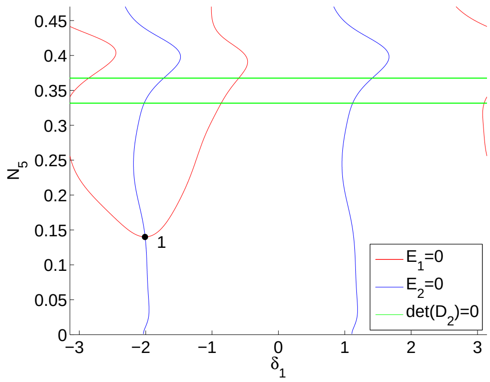


Figure 3: Position of equilibrium and singular points of the system with $f^0 = 0.2$ and $\sigma = 0$. The system possesses one equilibrium point (no. 1). Black points represent points where $\mathcal{G}_1 = \ldots = \mathcal{G}_8 = 0$ (see Sect. 3.2)

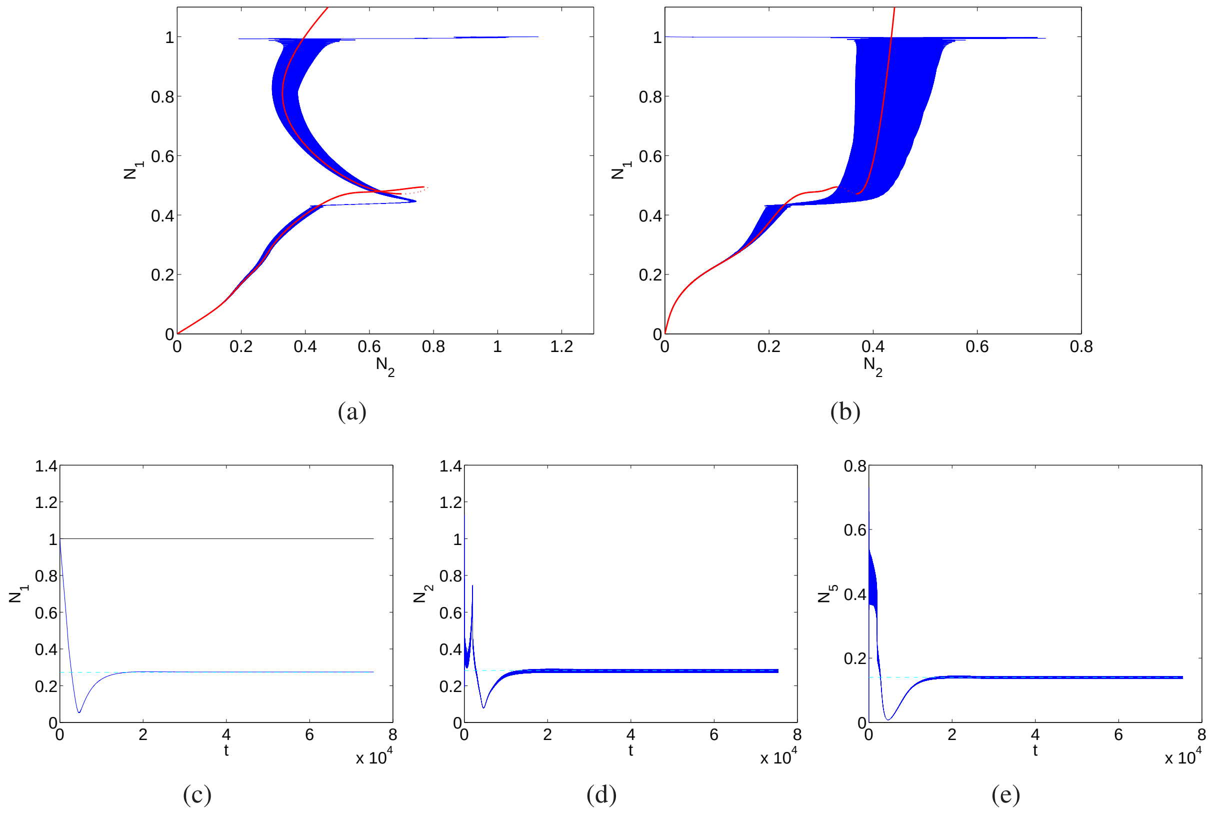


Figure 4: $f^0 = 0.2$ and $\sigma = 0$. (a) SIM of the system in red and corresponding numerical results in blue: N_1 versus N_2, (b) SIM of the system in red and corresponding numerical results in blue: N_1 versus N_5, (c) N_1 versus time, (d) N_2 versus time and (e) N_5 versus time. Amplitude of predicted equilibrium point no. 1 (see Fig. 3) is depicted by a blue dashed line. Amplitude that the main system would have without nonlinear oscillators is plotted in black

4.3 $\mathbf{f^0 = 0.5}$ and $\sigma = 0.6$

Position of equilibrium and singular points are given in Fig. 5. The system possesses one unstable equilibrium point (no. 1) and four singular points (no. 2, no. 3, no. 4 and no. 5). Consequently, SMR is expected. Figures 6(a)-(e) depict SIM of the system, corresponding

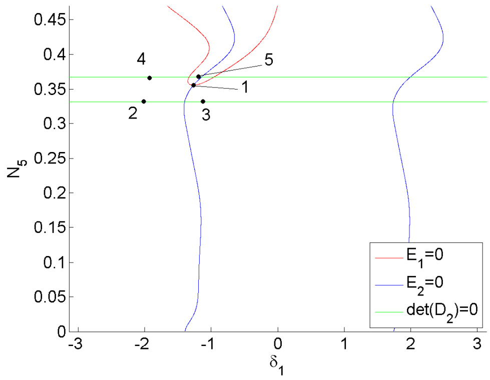


Figure 5: Position of equilibrium and singular points of the system with $f^0 = 0.5$ and $\sigma = 0.6$. The system possesses one equilibrium point (no. 1) and four singular points (no. 2, no. 3, no. 4 and no. 5). Black points represent points where $\mathcal{G}_1 = \ldots = \mathcal{G}_8 = 0$ (see Sect. 3.2)

numerical results and evolution versus time of the same variables as in the previous section. This time, the system faces repeated bifurcations around its unstable zone, i.e. it undergoes SMR. The system behavior during SMR is plotted in green in Figs. 6(a)-(b). Again, passive control has been operated on the main system as shown by the difference of its amplitude with nonlinear oscillators and without (marked by a solid black line in Fig. 6(c)).

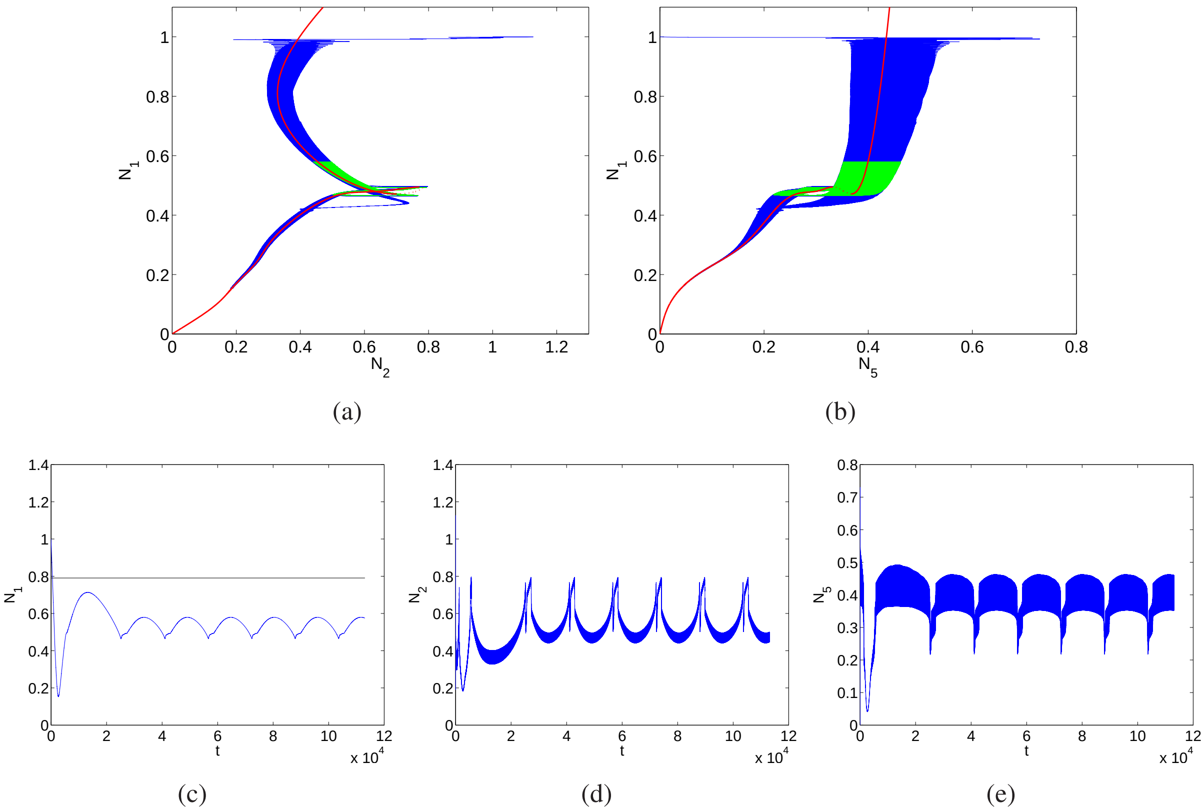


Figure 6: $f^0 = 0.2$ and $\sigma = 0$. (a) SIM of the system in red and corresponding numerical results in blue: N_1 versus N_2, (b) SIM of the system in red and corresponding numerical results in blue: N_1 versus N_5, (c) N_1 versus time, (d) N_2 versus time and (e) N_5 versus time. Amplitude that the main system would have without nonlinear oscillators is plotted in black

5 CONCLUSION

A five degree-of-freedom system consisting of a linear system coupled to four nonlinear oscillators is studied. Theoretical approaches such as implementation of Galerkin technique, complexification and time multi-scale methods are endowed for detection of fast and slow dynamics of the system. Numerical simulations and analytical predictions are in good agreement and provide evidence of passive control of the main system. In prospect, a similar method will be applied on a system with higher degree-of-freedom, in order to compare efficiency of passive control according to the number of nonlinear oscillators. Moreover, it will be interesting to study the behavior of the chain (propagation of a wave, localization in a few degrees of freedom, etc.) and to compare the results obtained with different approaches such as the search of nonlinear modes.

REFERENCES

[1] O.V. Gendelman, Transition of Energy to a Nonlinear Localized Mode in a Highly Asymmetric System of Two Oscillators. *Nonlinear Dynamics*, **25**, 237–253, 2001.

[2] G. Kerschen, D.M. McFarland, J.J. Kowtko, Y.S. Lee, L.A. Bergman, A.F. Vakakis, Experimental demonstration of transient resonance capture in a system of two coupled oscillators with essential stiffness nonlinearity. *Journal of Sound and Vibration*, **299**, 822–838, 2007.

[3] A. Ture Savadkoohi, C.-H. Lamarque, Dynamics of coupled Dahl type and nonsmooth systems at different scales of time. *International Journal of Bifurcation and Chaos*, **7**, 1350114 (14 p.), 2013.

[4] C.-H. Lamarque, A. Ture Savadkoohi, E. Etcheverria, Multi-scales dynamics of two coupled nonsmooth systems. *International Journal of Bifurcation and Chaos*, **22**, 1250295 (18 p.), 2012.

[5] A.H. Nayfeh, D.T. Mook, *Nonlinear oscillations*. John Wiley and Sons, 1979.

[6] L.I. Manevitch, The description of localized normal modes in a chain of nonlinear coupled oscillators using complex variables. *Nonlinear Dynamics*, **25**, 95–109, 2001.

[7] Y. Starosvetsky, O.V. Gendelman, Strongly modulated response in forced 2dof oscillatory system with essential mass and potential asymmetry. *Physica D*, **324**, 916–939, 2008.