Available online at www.eccomasproceedia.org
Eccomas Proceedia EUROGEN (2021) 95-106

ECCOMAS
Proceedia

EUROGEN 2021
14th ECCOMAS Thematic Conference on
Evolutionary and Deterministic Methods for Design, Optimization and Control
N. Gauger, K. Giannakoglou, M. Papadrakakis, J. Periaux (eds.)
Streamed from Athens, Greece, 28–30 June 2021

AN OPTIMIZATION ALGORITHM FOR THE DETECTION OF DAMAGE IN FRAME STRUCTURES

Annalisa Greco [1], Ilaria Fiore [1], Alessandro Pluchino [2]

[1] Department of Civil Engineering and Architecture, University of Catania
via S. Sofia 64, Catania, Italy
annalisa.greco@unict.it, ilaria.fiore@phd.unict.it

[2] Department of Physics and Astronomy "Ettore Majorana", University of Catania,
and INFN, Unit of Catania, via S. Sofia 64, Catania, Italy
alessandro.pluchino@ct.infn.it

Abstract

The aim of the study is the formulation of a computational method for the identification of the first damage in planar frames. The damage is considered to be concentrated in some opportune sections of beams and columns and many assume different intensities.
In order to identify the location and intensity of the damage when measured frequencies on the damaged frame are available, an original algorithm has been implemented assigning to each critical section an appropriate fitness function. This function indicates the difference between the measured frequencies of vibration and those obtained, by means of opportune numerical models, assuming that the damage is located in the considered critical section. The exact location will then be individuated seeking the optimal fitness function. An iterative procedure which allows reducing the computational effort is also presented with reference to a case study.

Keywords: Damage identification; Frames; Stiffness matrix; Frequencies of vibration.

Peer-review under responsibility of the organizing committee of EUROGEN 2021.
doi: 10.7712/140121.7953.18424

1 INTRODUCTION

As it can be observed from the scientific literature of recent decades, structural health monitoring of existing structures has received great interest and has inspired many dedicated studies. Many studies are devoted to the evaluation of the static or dynamic response of damaged structures and highlight their modification with respect to undamaged configurations [1-9]. The presence of structural damage reduces the bearing capacity of structures and therefore, in order to prevent its progressive expansion, which inevitably leads to structural failure, must be detected in its early stage. Many damage identification techniques are reported in the literature and require the measurement of some data on the existing structure. Different data can be taken into account for instance the variation of dynamic characteristics, such as natural frequencies [10-17], mode shapes [18,19], or static quantities, such as displacements or strains induced by applied loads [20-22]. Many studies are based on the solution of an inverse problem that compares numerical response data, evaluated on a model of the structure, to the corresponding experimentally measured ones [23-25]. Recent computational procedures based on genetic algorithms [26-28], particle swarm optimization [29], fuzzy cognitive maps [30] have been presented.

A crucial aspect concerns the model of damage and numerous attempts to quantify local defects are reported in the literature. All the models involve a reduction of the stiffness of the structural element by using one-dimensional continuum theories [31] or fracture mechanics methods [32,33].

The damage can be either localized or diffused on a certain portion of the length of the structural element and its intensity can be related to the reduction of the reactive area of the cross section, for example due to the presence of a notch. Contributions on crack modelling approaches and their effects on the response of beams can be found in [34,35].

The aim of the present paper is to propose a computational method for monitoring the structural integrity of planar frames. It is assumed that appropriate sensors are permanently located on the frame detecting its vibration properties with a certain periodicity. In this case it will be possible to immediately detect the appearance of the first structural damage indicated by a difference between some measured data and those known for the undamaged frame. The widely used assumption of a hinge with a rotational spring is used to model the damage and this is assumed to be concentrated, located in opportune critical sections and may have different intensities.

With the aim of identifying the location of the first damage, the frequencies of vibration of undamaged and damaged frames are calculated. In particular the frequencies of vibration are evaluated for each damage configuration of the frame corresponding to a possible location of damage among the considered critical sections. These frequencies of vibration are then used to identify the location and the intensity of the damage by means of the solution of an opportune optimization problem. The application on a case study allows to show the reliability of the proposed identification algorithm. Furthermore, an automatic procedure is illustrated which, assigning to each critical section an opportune fitness with respect to the possibility that the damage is located there, allows to iteratively converge to the exact position reducing the required computational effort.

2 FREQUENCIES OF VIBRATION IN UNDAMAGED AND DAMAGED FRAMES

In the present study planar regular frames, with columns clamped at the ground level, are considered. Once the mechanical and geometrical characteristics of each structural member of the frame have been assigned, the stiffness and mass matrices of the undamaged frame respectively denoted as $\mathbf{K}^u$ and $\mathbf{M}^u$ can be evaluated. In particular the mass matrix takes into account of the distributed masses in beams and columns. Neglecting the axial deformability of the structural members, the degrees of freedom of the frame, and therefore the size of the stiffness and mass matrices, correspond to the horizontal displacements of each floor and the rotation of the unconstrained nodes. The natural frequencies of vibration $\boldsymbol{\omega}_u$ of the undamaged frame can be evaluated solving the classic equation:

$$\det\left(\mathbf{K}^u - \omega_u^2 \mathbf{M}^u\right) = 0 \tag{1}$$

When the first damage occurs in a structural member the total stiffness decreases and consequently also the frequencies of vibration assume lower values with respect to the undamaged configuration.

In this study reference is made to a single damage which can be located in one of the critical sections of the frame corresponding to the base or top of each column at each floor or to the ends of each beam. With reference to the two corners at the top of the frame, the critical section will be located in correspondence to the structural member (beam or column) having the lower elastic moment.

In accordance with what is frequently adopted in the scientific community, the loss of resistance induced by the presence of the damage is modelled by means of a hinge with a rotational spring whose stiffness k_φ is related to the intensity of the damage.

In order to evaluate the natural frequencies of the frame having a damage located in one of the previously described critical sections, new stiffness and mass matrices must be assembled. For each location of the damage, stiffness and mass matrices, respectively denoted as $\mathbf{K}^d$ and $\mathbf{M}^d$, are evaluated. The correspondent frequencies of vibration for each location of the damage are evaluated solving the equation:

$$\det\left(\mathbf{K}^d - \omega_d^2 \mathbf{M}^d\right) = 0 \tag{2}$$

The stiffness of the rotational spring modelling the damage will be assumed equal to n_i discrete values which decrease with the intensity of the damage itself.
In particular, the stiffness of the rotational spring is expressed as:

$$k_{\varphi i} = \alpha_i \frac{EI_d}{L_d} \qquad i = 1, \dots, n_i \tag{3}$$

where α_i is an integer number and I_d and L_d are respectively the moment of inertia and the length of the structural element where the damage is located (beam or column).

3 DAMAGE IDENTIFICATION PROCEDURE

In this section the damage parameters, i.e. location and intensity, are evaluated by means of an optimization strategy based on the use of a certain number of vibration properties measured on the considered frame. For each possible damage location among the critical sections a fitness value is assigned for each damage intensity as follows:

$$F_{k,j} = F_{\max} - \sum_i^{nf} \left| \frac{\hat{\omega}_i - \omega_{i,kj}}{\omega_{iu}} \right| \qquad j = 1, \dots, n_i \tag{4}$$

where:

$F_{\max}$ is an arbitrary constant, chosen great enough to have always $F_k > 0$
n_f is the number of considered natural frequencies

n_i is the number of considered damage intensities
$\hat{\omega}_i$ is the measured *i-th* frequency of vibration
$\omega_{i,kj}$ is the calculated *i-th* frequency of vibration with intensity damage *j* located in critical section *k*
ω_{iu} is the *i-th* frequency of vibration for the undamaged frame

For each critical section the maximum fitness among those associated to different intensity values is then selected:

$$F_k = \max\left(F_{k,j}\right) \qquad j = 1, \ldots, n_i \tag{5}$$

Of course, among all the fitness functions associated to possible damage locations, only the one related to the exact position will reach its maximum value.

Once the position of the damage is identified, it is possible to evaluate its intensity comparing all the sets of frequencies related to variable intensity to the measured values.

The first step of the proposed algorithm concerns with the localization of the damaged section and represents the main goal of the proposed approach. Undoubtably from a health monitoring point of view the most crucial aspect is to identify the presence and the location of the damage. Once the damage is detected it is important to repair as soon as possible the involved structural zones in order to prevent subsequent worsening, independently on the damage intensity.

In principle all the natural frequencies of all the possible configurations of damaged frames can be calculated and the correct location of the damage can be identified minimizing the fitness function. Anyway, it will be shown that it is sufficient to take into account only some natural frequencies in order to identify the correct damage location. In the applicative section the number of considered frequencies of vibration has been set equal to 4. For large frames with high number of critical sections in order to reduce the number of calculated fitness values, an original iterative procedure described in the following can be applied.

It is worth pointing out that the proposed procedure applies both for damage identification in symmetric and unsymmetric frames. Anyway, in case of symmetric frames it is not possible to discern between symmetric damage positions. However, in order to carry out maintenance and restoration interventions, this circumstance does not appear to be a major limitation as it allows to drastically reduce the sections to be considered to two only. Anyway, if one needs to univocally identify the damaged section in a symmetric frame, some information on mode shapes could be added in the objective function.

4 CASE STUDY AND NUMERICAL APPLICATIONS

The five storeys frame assumed as case study for the proposed damage identification procedure is shown in Figure 1. In the same figure the length and the profiles of the structural members are also reported. In Table 1 moments of inertia and the ratio between yield moments of the cross sections and the yield stress are reported together with the values of the distributed masses. The value of Young modulus for the steel under consideration is assumed to be: $E = 210000000\left[\frac{kN}{m^2}\right]$

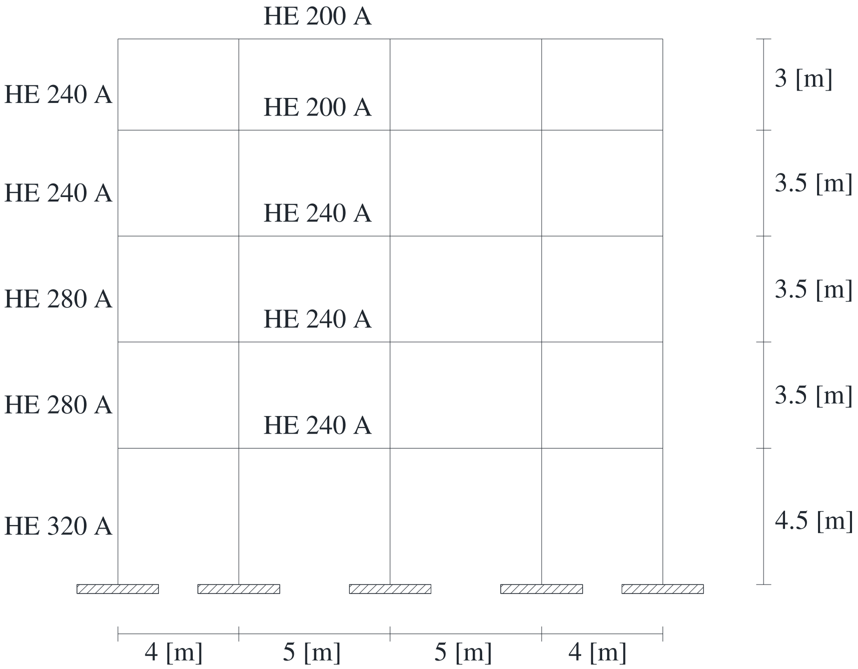

Figure 1 - Frame with five floors and four columns

Profile	I_x [m^4]	M_y/σ_y[m^3]	m [$kN*s^2/m^2$]
HE320A	$22930 \cdot 10^{-8}$	$1479 \cdot 10^{-6}$	$97.6 \cdot 10^{-3}$
HE280A	$13670 \cdot 10^{-8}$	$1013 \cdot 10^{-6}$	$76.4 \cdot 10^{-3}$
HE240A	$7763 \cdot 10^{-8}$	$675.1 \cdot 10^{-6}$	$60.3 \cdot 10^{-3}$
HE200A	$3692 \cdot 10^{-8}$	$388.6 \cdot 10^{-6}$	$42.3 \cdot 10^{-3}$

Table 1 - Characteristics of the structural elements

For this frame the numbers of degrees of freedom in the undamaged configuration is equal to 30 and therefore 30 frequencies of vibration can be calculated solving the related dynamic eigenvalue problem once the matrices $\mathbf{K}^u$ and $\mathbf{M}^u$ are evaluated.

The total number of critical sections for the considered frame is 88 but since it is symmetric, 49 critical sections must be taken into account. For each possible location of the damage, and for each value of its intensity, 31 frequencies of vibration can be calculated. Considering, eight levels of intensity of the damage, 392 sets of 31 frequencies of vibration have been evaluated. With the aim of solving the inverse problem which concerns the identification of damage parameters, some examples of location and intensity of the damage will be developed, and the related natural frequencies will be used as pseudo-experimental data in the identification procedure.

4.1 The Identification Algorithm

The identification algorithm has been realized within the software environment NetLogo [36], where the five storey frame can be reproduced in a virtual metric space, as shown in panel (a) of Figure 2. The critical sections S_k (k = 1,…,88), represented by yellow squares, are labeled with an increasing ID number. Among them, let us call S_T the target (damaged) section.

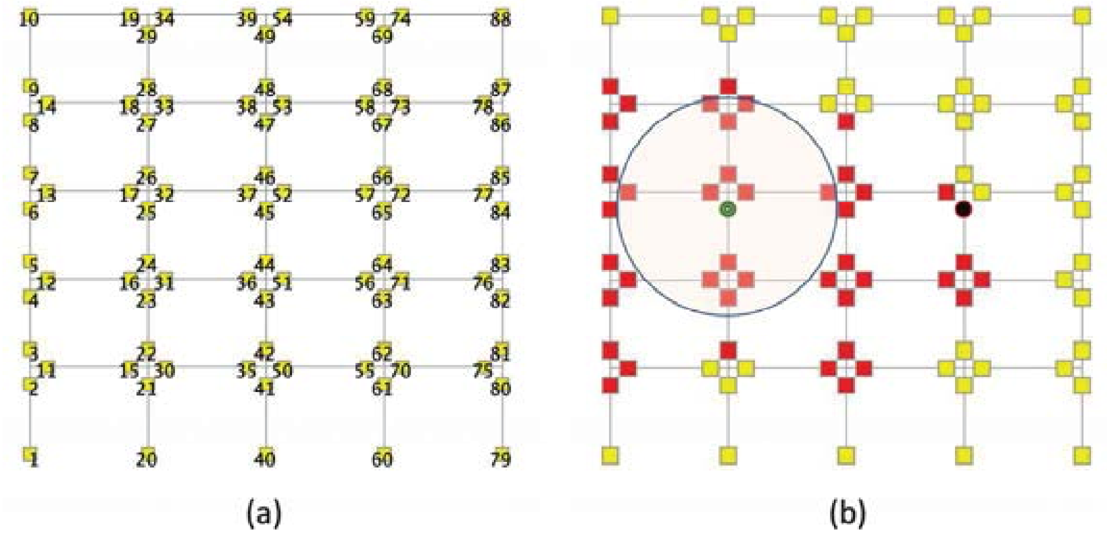

Figure 2 – (a) Frame with five floors and five columns, where each critical section is labeled with an increasing integer ID number. (b) The same frame at the end of a single run of the identification procedure, with the jumper that has reached one of the two symmetric target sections, after a path along a subset of the explored sections (colored in red). The exploration circle is visible around the jumper. See text for more details.

The searching procedure can be easily implemented by creating a “jumper”, able to move from one section to another, starting at $t = 0$ from a given (randomly selected or fixed by the user) section S_i on the frame, for which the fitness F_i is calculated. Jumper is represented by a concentric green circle. Then, the algorithm goes on according to the following subsequent steps:

1. At the next iteration step $t > 0$, the jumper explores the frame by calculating the fitness values associated to all the neighboring sections included within an “exploration circle” of a given radius R (expressed using spatial units arbitrarily chosen as equal to the distance between two floors). Sections for which the fitness has been calculated are colored in red.
2. Then, two options are available for the jumper:

   - if some of these neighboring sections have a fitness $F_k > F_i$, the jumper moves on the section with the maximum fitness among them;
   - if all the neighboring sections have a fitness $F_k < F_i$, with a certain probability $p_m < 1$ (called “moving probability”, which introduces in the system a sort of noise useful to avoid local maxima) the jumper is forced to move on one of those sections, chosen at random, otherwise the procedure is stopped.

3. If the procedure has not been stopped at step 2, the algorithm came back to step 1 and prosecutes until it is stopped. Notice that a given section can be visited more than one time during the entire procedure but, of course, its fitness is calculated only the first time the section has been included in the exploration circle.
4. If the section reached by the jumper when it stops coincides with the target section $S_{T,}$ or with its symmetric section (both these sections are represented as black circles), we say that the run has been successful.

In panel (b) of Figure 2 the situation at the end of a generic successful run is reported, with the jumper over the target. An example of exploration circle, centered on the jumper, is also shown for allowing the user to visualize it. The percentage E% of explored (red) sections over the total is visibly around 50% and this tell us that the algorithm was not only effective (since the jumper reached the target) but also efficient, since we were able to identify the damaged section without needing to calculate the fitness for all the sections.

Of course, in order to have more reliable information about the effectiveness and the efficiency of the proposed algorithm for the identification of a given (fixed) target section, one needs to repeat N times (with $N >> 1$) the entire procedure starting, for each run, from a different random section. At the end of the N repetitions, the percentage $P_{\%}$ of successful runs and the average percentage E% of explored sections will represent the quantities that we have to take into account for evaluating the performance of the algorithm.

4.2 Results for the five storeys frame

In this paragraph we discuss the numerical results obtained by applying our algorithm to the considered five storeys frame. The damage can be located in each of the 49 critical sections and eight damage intensities, represented by the rotational stiffness given by equation (3) with $\alpha = [1000 \quad 800 \quad 700 \quad 500 \quad 300 \quad 100 \quad 50 \quad 25]$, have been taken into account.

Considering different arbitrary sets of location and intensity of the damage and assuming the correspondent first four frequencies of vibration as pseudo-experimental data, the fitness function (4) has been previously evaluated for each of the 49 critical sections observing that it always reached a zero value in correspondence of the exact damage configuration.

Successively in order to implement the searching procedure described in the previous section (and therefore to avoid the calculation of the fitness for all the sections), the two percentages $P_{\%}$ (of successful runs) and $E_{\%}$ (of explored sections) have been evaluated assuming different damage locations and intensities.

In order to evaluate an appropriate value of the radius of the exploration able to provide reliable results still maintaining a significant reduction in the number of nodes in which the fitness function has to be calculated, a preliminary analysis has been developed. Figure 3 shows the results obtained assuming for example the damage located in sections 1 and 15 (involving respectively one column and one beam) with intensity $\alpha = 300$ and evaluating the two percentages $P_{\%}$ and $E_{\%}$ for increasing values of R. As it can be observed, increasing the value of the radius, higher values of both $P_{\%}$ and $E_{\%}$ are reached. A fair compromise between the need of reaching high values of $P_{\%}$ without increasing too much $E_{\%}$ can be obtained assuming R=2,2, so this value will be chosen for the simulations. Figure 4 shows the effect of the forcing moving probability p_m in the identification of the damage located again in sections 1 and 15 with intensity $\alpha = 300$. While for the damage located on the beam the two percentages $P_{\%}$ and $E_{\%}$ show to be almost independent on p_m, for the damaged column a sensible increase with the forcing moving probability can be observed. In order to choose a unique value, in the following p_m has been set equal to 0,9.

With the aim of investigating on the effects of a damage located at the base of a column or on a beam at each floor, the sections 1, 3, 5, 7, 9 (base columns) and 15, 16, 17, 18, 19 (right end of beams of the first span) have been taken into account. Several sets of $N = 1000$ runs have been performed and the results in terms of $P_{\%}$ and $E_{\%}$ have been reported in Figure 5.

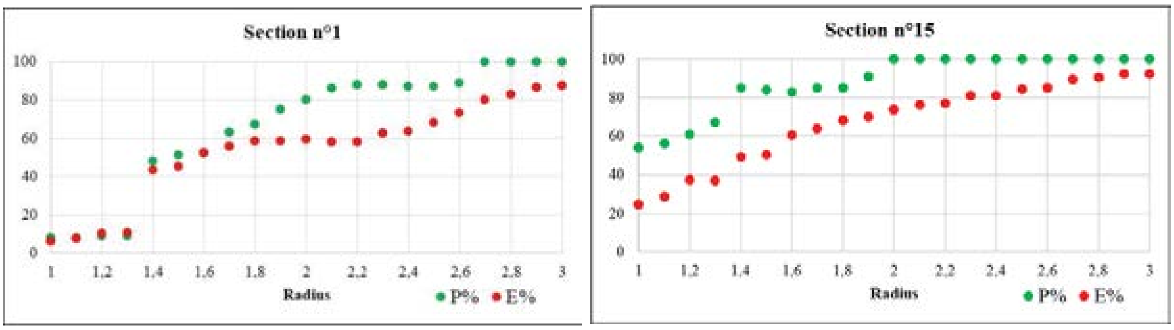

Figure 3 – P% and E% for damage located at the base of a column (1) and on a beam (15) of the first floor with stiffness intensity parameter α=300 for different values of the radius parameter

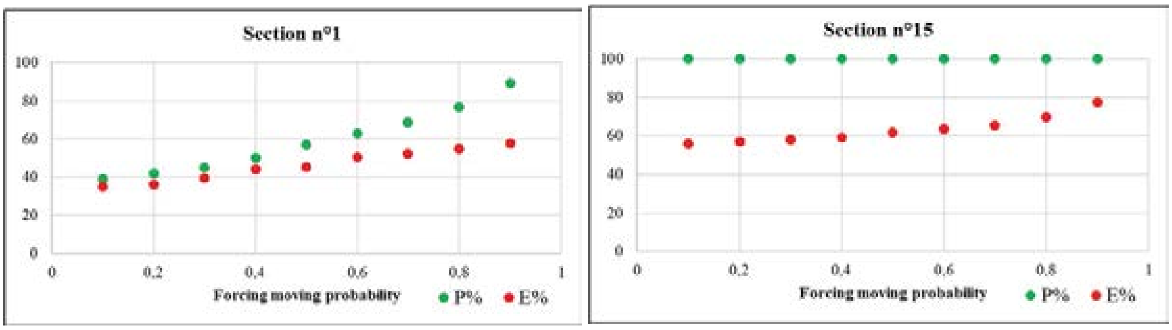

Figure 4 – P% and E% for damage located at the base of a column (1) and on a beam (15) of the first floor with stiffness intensity parameter α=300 for different values of the forcing moving probability

As it can be observed from Figure 5 the damage identification procedure gives excellent results when the damage is located on one of the considered beams. In these cases, in fact, independently on the considered section (and therefore on the floor) the percentage of successful runs turns out to be always 100% while the percentage of explored sections is about 80%. On the other hand, the identification of damages located at the base of the external columns provide values of P% around 85% with E% always smaller than 60%

Therefore, we can conclude that, for the considered five storeys frame, the proposed identification procedure provided satisfactory results, thus confirming its good performance and reliability.

5 CONCLUSIONS

- A computational method for monitoring the structural integrity of planar frames is presented.
- The first damage is identified by means of the frequencies of vibration of each damaged configuration of the frame corresponding to a possible location of damage among the considered critical sections.
- The solution of the damage identification problem is achieved by means of an optimization algorithm which assigns to each possible damage location, among the critical sections, a fitness value and then seeks the maximum one.

Figure 5 – P% and E% for assigned damage location and variable intensity
Left side: damage located at the base of an external column at each floor starting from the base
Right side: damage located at the right end of each beam in the first bay starting from the bottom

- An automatic procedure has been implemented within the software environment NetLogo and it has been shown how it is possible to iteratively converge to the exact damage position reducing the number of calculations.
- The application to a case study allows to show the reliability of the proposed identification algorithm.

REFERENCES

[1] T. Chondros and A. G. Dimagoronas, "Dynamic sensitivity of structures to cracks," *Journal of Vibration and Acoustics,* vol. 111, pp. 251-256, 1989.

[2] M.-H. Shenn and C. Pierre, "Natural modes of Bernoulli-Euler beams with symmetric cracks," *Journal of Sound and Vibration,* vol. 138, no. 1, pp. 115-134, 1990.

[3] W. Ostachowicz and M. Krawczuk, "Analysis of the effects of cracks on the natural frequencies of a cantilever beam," *Journal of Sound and Vibration,* vol. 150, no. 2, pp. 191-201, 1991.

[4] I. Caliò, D. D'Urso and A. Greco, "The influence of damage on the eigen-properties of Timoshenko spatial arches," *Computers & Structures,* vol. 190, pp. 13-24, 2017.

[5] B. Bozvigit, Y. Yesilce and M. Wahab, "Free vibration and harmonic response of cracked frames using a single variable shear deformation theory," *Structural Engineering and Mechanics,* vol. 74, no. 1, pp. 33-54, 2020.

[6] K. Mazanoglu and E. Kandemir-Mazanoglu, "A new simplified method for anti-symmetric mode in-plane vibrations of frame structures with column cracks," *Journal of Vibration and Control,* vol. 24, no. 24, pp. 5794-5810, 2018.

[7] U. Eroglu and E. Tufekci, "Free Vibration of Damaged Frame Structures Considering the Effects of Axial Extension, Shear Deformation and Rotatory Inertia: Exact Solution," *International Journal of Structural Stability and Dynamics,* vol. 17, no. 10, 2017.

[8] S. Caddemi and I. Caliò, "The exact explicit dynamic stiffness matrix of multi-cracked Euler-Bernoulli beam and applications to damaged frame structures," *Journal of Sound and Vibration,* vol. 332, no. 12, pp. 3049-3063, 2013.

[9] A. Ibrahim, H. Ozturk and M. Sabuncu, "Vibration analysis of cracked frame structures," *Structural Engineering and Mechanics,* vol. 45, no. 1, pp. 33-52, 2013.

[10] M. Cerri and F. Vestroni, "Identification of damage due to open cracks by changes of measured frequencies," in *Proceedings of XVI AIMETA Congress of Theoretical and Applied Mechanics;*, 2003.

[11] A. Morassi, "Structural identification of a steel frame from dynamic test-data," *Structural Engineering and Mechanics,* vol. 11, no. 3, pp. 237-258, 2001.

[12] F. Vestroni and D. Capecchi, "Damage detection in beam structures based on frequency measurements," *Journal of Engineering Mechanics,* vol. 126, pp. 761-768, 2000.

[13] A. Pau, A. Greco and F. Vestroni, "Numerical and experimental detection of concentrated damage in a parabolic arch by measured frequency variations," *Journal of Vibration and Control,* pp. 1-11, 2010.

[14] M. Dilena and A. Morassi, "Reconstruction method for damage detection in beams based on natural frequency and antiresonant frequency measurements," *Journal of Engineering*

Mechanics, vol. 136, no. 3, pp. 329-344, 2010.

[15] E. M. N. Giordano, M. Masciotta, F. Clementi, N. Sadeghi, R. Silva and D. Oliveira, "Expeditious damage index for arched structures based on dynamic identification testing," *Construction and Building Materials,* vol. 265, 2020.

[16] A. Labib, D. Kennedy and C. Featherston, "Crack localisation in frames using natural frequency degradations," *Computers and Structures,* vol. 157, pp. 51-59, 2015.

[17] L. Rubio, J. Fernández-Sáez and A. Morassi, "Identification of an open crack in a beam with variable profile by two resonant frequencies," *Journal of Vibration and Control,* vol. 24, no. 5, pp. 839-859, 2018.

[18] M. Diaferio and V. Sepe, "Modal identification of damaged frames," *Structural Control and Health Monitoring,* vol. 23, no. 1, pp. 82-102, 2016.

[19] M. Ge, E. Lui and A. Khanse, "Non-proportional damage identification in steel frames," *Engineering Structures,* vol. 32, no. 2, pp. 523-533, 2010.

[20] A. Greco and A. Pau, "Detection of a concentrated damage in a parabolic arch by measured static displacements," *Structural Engineering and Mechanics,* vol. 39, pp. 751-765, 2011.

[21] S. Caddemi and A. Greco, "The influence of instrumental errors on the static identification of damage parameters for elastic beams," *Computer and Structures,* vol. 84, pp. 1696-1708, 2006.

[22] J. Guo, L. Wang and I. Takewaki, "Static damage identification in beams by minimum constitutive relation error," *Inverse Problems in Science and Engineering,* vol. 27, no. 10, pp. 1347-1371, 2019.

[23] M. Friswell, "Damage identification using inverse methods," *Philosophical Transactions of the Royal Society,* vol. 365, pp. 393-410, 2007.

[24] P. Nikolakopoulos, D. Katsareas and C. Papadopoulos, "Crack identification in frame structures," *Computers and Structures,* vol. 64, pp. 389-406, 1997.

[25] A. Greco and A. Pau, "Damage identification in Euler frames," *Computers and Structures,* Vols. 92-93, pp. 328-336, 2012.

[26] A. Greco, D. D'Urso, F. Cannizzaro and A. Pluchino, "Damage identification on spatial Timoshenko arches by means of genetic algorithms," *Mechanical Systems and Signal Processing,* vol. 105, pp. 51-67, 2018.

[27] A. Yu, J. Ji and S. Sun, "An improved multi-objective genetic algorithm and data fusion in structural damage identification," *International Journal of Security and Networks,* vol. 14, no. 2, pp. 95-102, 2019.

[28] A. Greco, A. Pluchino, F. Cannizzaro, S. Caddemi and I. Caliò, "Closed-form solution based genetic algorithm software: Application to multiple cracks detection on beam structures by static tests," *Applied Software Computing Journal,* vol. 64, pp. 35-48, 2018.

[29] F. Kang, J.-J. Li and Q. Xu, "Damage detection based on improved particle swarm optimization using vibration data," *Applied Software Computing Journal,* vol. 12, no. 8, pp. 2329-2335, 2012.

[30] P. Beena and R. Ganguli, "Structural damage detection using fuzzy cognitive maps and Hebbian learning," *Applied Software Computing Journal,* vol. 11, no. 1, pp. 1014-1020, 2011.

[31] S. Christides and D. Barr, "One-dimensional theory of cracked Bernoulli-Euler beams," *International Journal of Mechanics Science,* vol. 26, no. 11-12, pp. 639-648, 1984.

[32] T. Chondros, "The continuous crack flexibility model for crack identification," *Fatigue and Fracture of Engineering Materials and Structure,* vol. 24, pp. 643-650, 2001.

[33] T. Chondros, A. Dimarogonas and J. Yao, "A continuous cracked beam vibration theory," *Journal of Sound and Vibration,* vol. 215, no. 1, pp. 17-34, 1998.

[34] A. Dimarogonas, "Vibration of cracked structures: a state of the art review," *Engineering Fracture Mechanics,* vol. 55, no. 5, pp. 831-857, 1996.

[35] I. Caliò, A. Greco and D. D'Urso, "Structural models for the evaluation of eigen-properties in damaged spatial arches: a critical appraisal," *Archive of Applied Mechanics,* vol. 86, pp. 1853-1867, 2016.

[36] Wilensky, U. (1999). *NetLogo*. http://ccl.northwestern.edu/netlogo/