Available online at www.eccomasproceedia.org
Eccomas Proceedia EUROGEN (2023) 285-299

ECCOMAS
Proceedia

EUROGEN 2023
15th ECCOMAS Thematic Conference on
Evolutionary and Deterministic Methods for Design, Optimization and Control
N. Gauger, K. Giannakoglou, M. Papadrakakis, J. Periaux (eds.)
Chania, Crete Greece, 1–3 June 2023

REDUCING THE NUMBER OF OBJECTIVES FOR MANY-OBJECTIVES OPTIMIZATION: EMPIRICAL ANALYSIS OF A MACHINE LEARNING APPROACH

António Gaspar-Cunha[1]*, Paulo Costa[2], Francisco Monaco[3] and Alexandre Delbem[3]

[1] Institute of Polymers and Composites, University of Minho
Campus de Azurém, Guimarães, Portugal
e-mail: agc@dep.uminho.pt

[2] Institute of Polymers and Composites, University of Minho
Campus de Azurém, Guimarães, Portugal
byic.mail@gmail.com

[3] Institute of Mathematics and Computer Science, University of São Paulo
São Carlos, Brazil
{acbd,monaco}@usp.br

Abstract

The practical need of solving real-world optimization problems is faced very often of dealing with many objectives, but from the beginning, a question arises: Are all the objectives really necessary? The answer to this question lies in the complex relations existing between the parameters of the process, i.e., not only between the objectives and the decision variables (DVs), but also between the DVs and DVs and between the objectives and objectives. Simultaneously, intense research is made to improve the performance of multi-objective population-based algorithms to deal with many objectives that, often, imply complex algorithms and time-consuming computations with complex results that experts on the field of the problem might not understand and, as a consequence, did not accept and apply in practice. A straightforward alternative is to infer the complex relations between the process parameters with the aim of reducing the number of objectives. The use of Machine Learning (ML) methodologies for that can be very useful since it is someway demonstrated in the literature on the subject of reducing the number of objectives. In this work, ML is used to reduce the number of objectives and the results are assessed empirically using a real-world application.

Keywords: Multi-objective optimization, Machine Learning, Evolutionary Algorithms, Reduction of the number of objectives.

Peer-review under responsibility of the organizing committee of EUROGEN 2023.
doi: 10.7712/140123.10205.18861

1 INTRODUCTION

The multiobjective nature of real-world optimization problems, in which multiple conflicting objectives exist, can be dealt with in two ways, either using scalarization functions or population-based algorithms. The limitations on the use of scalarization functions originated the investigation of population-based metaheuristics based on the concept of Pareto-dominance and niching to progress a population of solutions in the direction of the Pareto-optimal front [1,2].

Generally, three types of population-based algorithms are used to solve Multiobjective Optimization Problems MOPs, explicitly, evolutionary algorithms, swarm-based methods, and colony-based algorithms, which to evolve the population of solution in the direction of the Pareto front apply the dominance concept, the metric indicators, or the decomposition strategy [3]. In these types of algorithms, a random initial population of solutions (or a single solution) is generated and the operators of selection and variation are applied to obtain successively new populations until a stop criterion is met. In this way, the following populations progress towards, or to a good approximation of, the Pareto-optimal frontier.

It is clear from the literature that these algorithms only work well when the number of objectives is low, but when the number of objectives grows the percentage of non-dominated solutions decreases. In this way, it is very difficult that the algorithms based on Pareto-dominance to work effectively, a problem known as the curse of dimensionality. The number of objectives for which this problem occurs is not clear, in some cases, the authors indicate that this number is ten [4] and in others is four [5]. In practice, these problems appear when the number of objectives is four or more.

In literature, two different methods are reported to deal with this problem, namely, through the application of relaxed forms of Pareto optimality or by reducing the number of objectives [5]. The latter is very useful not only during the search process but also for the decision-making process during and/or at the end of the optimization.

Another important issue, mainly when dealing with real-world problems in which the objectives to use are not clear, consists in defining a methodology able to identify the relevant objectives from a, sometimes, very high set of objectives. Simultaneously, and this fact cannot be discarded, these populations are characterized by the existence of complex relations between the Decision Variables (DVs) and the objectives, as well as between DVs and DVs and objectives and objectives. Thus, make sense the use of data mining methodologies to tackle this problem [4].

Therefore, the aim of this work is to present a methodology able to tackle multiobjective problems in two different ways. First to select the relevant objectives to be used from a high pool of potential objectives, and, then to optimize the problem, but, simultaneously, the Decision Maker (DM) must have a full understanding of the process of optimization, i.e., it must be explainable.

This work is organized as follows. In section 2 the state-of-the-art will be presented. In section 3 the methodology proposed is presented in detail. Section 4 is dedicated to describing the real problem under study. The results will be presented and discussed in section 5. Finally, in section 6 the conclusions are stated.

2 STATE-OF-THE-ART

The work related to objective reduction for many objectives optimization existent in literature can be subdivided into five different categories. The methods in which the aim is to maintain the relation of the dominance of the non-dominated solutions [6,7], the methods based on unsupervised feature selection [8], the methods based on a comparative analysis between the results obtained when the number of objectives is reduced [9]; the methods based on data mining [5,10–12]; and methods based on the use of multi-objective formulations [13].

The first work was developed by Brockoff and Zitzler [6,7]. The authors proposed the use of two different approaches for objectives reduction, each one based on the definition of a different type of problem. In the first problem, the aim was to obtain the minimum objective subset of solutions that produces a certain error (δ) defined previously, named δ-MOSS problem (δ- Minimum Objective Subset problem). In the second problem, the aim was to obtain an objective subset of solutions with a predefined size (k) and with the minimum possible error, named k-EMOSS problem. The authors developed two different algorithms, an exact and a greedy algorithm, for each one of these types of problems, which were characterized for maintaining the dominance relation. The algorithms were tested using different knapsack problems and the DTLZ2, DTLZ5, and DTLZ7 benchmark problems considering different numbers of objectives.

López et al. [8] proposed a methodology based on unsupervised feature selection to address the δ-MOSS and k-EMOSS problems. To divide the objective set into homogeneous neighborhoods a correlation matrix obtained from the non-dominated set is used. They considered that the objectives are more conflicting if the distance between those objectives is higher. In this way, only the objectives in the center of the neighborhoods found are kept (and the others discarded). The results obtained were compared with those of the reference [7] in order to validate the proposed algorithm.

An algorithm named Pareto Corner Search Evolutionary Algorithm (PCSEA) was proposed by Singh et al. [9]. In this algorithm, the search was made for the solutions on the corners of the Pareto front based on a ranking scheme instead of searching for the complete Pareto front. The relevant objectives are determined by those solutions, while the others are discarded. The performance of the methodology proposed was tested with some benchmark problems and two engineering problems.

Principal Component Analysis (PCA) was proposed by Deb and Saxena as an approach for the same purpose of objectives reduction named PCA-NSGAII [10]. This was based on the hypothesis that if two objectives are negatively correlated, they are conflicting. Thus, the objectives that can explain most of the variance in the objective space were kept, which correspond to the ones that are the most positive and the most negative of the eigenvectors of the correlation matrix. However, a problem with the misunderstanding of the data when it lies in sub-manifolds appears and the authors made a new proposal based on nonlinear dimensionality reduction [11]. In this way, two new algorithms to replace the linear PCA were implemented, one based on correntropy [14] and the other on Maximum Variance Unfolding (MVU). Nevertheless, still some problems persist. For example, the method does not provide evidence on the means by which objective reduction modifies the dominance structure, it cannot assure the conservation of the dominance relation and it does not provide a measure to specify how much the dominance relation changes when objectives are eliminated. The methodology was tested using DTLZ2 and DTLZ5 benchmark problems with different numbers of objectives.

Due to those limitations, a different framework was proposed later by the same group (Saxena et al. [5]), in which linear and nonlinear objective reduction algorithms are used, namely, L-PCA and NL-MVU-PCA. Two different machine learning techniques, PCA and MVU, were used to remove the secondary higher-order dependencies in the non-dominated solutions. This was very similar to the previous work of the same authors [10,11], the difference being that this time they proposed a reduction of the number of algorithm parameters and an error measure. The same methodology was used by Sinha et al. [15] to develop an iterative procedure to reduce the objectives in which a Decision Maker (DM) chose the best solutions. The methodology was applied to solve some real-world problems, namely storm drainage and car-side impact. The methodology presented in reference [5] was modified by Duro et al. [12] to rank all objectives by order of preference. They solved the δ-MOSS and k-EMOSS problems by obtaining, respectively, the smallest set of objectives that can originate the same POF, and the smallest objective set corresponding to a minimum pre-defined error and the objective sets of a certain size that originates a minimum error.

Finally, a methodology based on the use of multi-objective evolutionary algorithms was proposed by Yuan et al. to solve a MOOP formulation [13]. The approach was used to solve some benchmark problems and two real optimization problems. The limitation of this approach lies in the difficulty of application for problems where the computation cost to evaluate the solutions cannot be neglected, which is not the case with the problems we want to study

Gaspar-Cunha et al. [16] pointed out some disadvantages of the methodologies based on PCA, namely the need to use a kernel and, concomitantly, the need to optimize the kernel parameters. The authors proposed a method for reducing the objectives based on data mining that: i) can be applied independently on the type and the size of the data and the shape of the Pareto-optimal front; ii) is independent of the choice/definition of the algorithm parameters; iii) considers the relations DVs-DVs and objectives-objectives (and not only the relations between the DVs and objectives), and iv) can provide explainable results for a DM that is a non-expert in optimization or machine learning. The characteristics of the NL-MVU-PCA methodology were compared with the approach proposed. A summary of this comparison will be provided in the next section.

The contribution of the present work is to extend the previous methodology of reference [16] to solve problems with many objectives (in the problem solved here the number of initial objectives is 21) in order to choose the better objectives to use during the optimization. Also, the proposed methodology will be accessed by comparing the optimization results of 11 runs obtained with a different number of objectives using the Hypervolume (HV) and the Inverted Generational Distance (IGD) metrics.

3 METHODOLOGY FOR REDUCING THE NUMBER OF OBJECTIVES

3.1 Required characteristics

The principal aim of the works mentioned above was to find a reduced set of objectives that could precisely reproduce the results from the original set, which implies that, after the reduction procedure, only the redundant objectives could be discarded. However, the aim of the present paper is to apply the proposed methodology to complex problems where the relations between DVs and the objectives are complex and the objectives are partially redundant [16].

Therefore, the methodology developed aims to capture those complex relations and to define the relative significance of the objectives based on the determination of the objectives–objectives relations. This allows the definition of the objectives that can be discarded, assuming that a certain error is allowed when compared with the approximation to the optimal Pareto front when using all the objectives. This will aid the optimization algorithms in finding a POF estimate and makes it easier to explain the results found to the DM.

On the contrary, the method proposed by Duro et al. [12], named NL-MVU-PCA, aims at finding the essential objective set in MaOPs. In this method, PCA runs with the aim to improve the objectives' preference ranking based on the objective–function correlation matrix by maximizing the variance in objective space while preserving the local isometry. The optimization of the Kernel matrix values is made by the non-linear (NL) approach and the matrix values are obtained by minimizing the Maximum Variance Unfolding (MVU).

In this work, the aim is to use DAMICORE, which is a framework based on the estimation of distances by compression algorithms able to assist the analysis that is an important characteristic of the present work, i.e., a small amount of data is available and the system can be dependent on external effects [17, 18]. This method is the core of the Feature Sensitivity Optimization based on Phylogram Analysis (FS-OPA) tools that are used to work directly with raw data involving the sequence of three main steps [17, 18, 19]: i) the generation of a distance matrix from the data using the Normalized Compression Distance (NCD) metric [20]; ii) the generation of evolutionary trees using phylogram based modelling for which a distance reconstruction algorithm called Neighbour Joining (NJ) is used and, simultaneously, the quality of the models is improved by a systematic resampling strategy [21]; iii) to apply a Complex Network approach called Fast Newman (FS) to perform community detection by analyzing the phylograms found previously and to extract significant and reliable information from them [22]. From this analysis, subgroups of data that share common information (DNA), designated by clades, identifying the communities are obtained.

For the purpose of the present work, the implementation of DAMICORE contemplates two different levels of learning [23]:

1. **First level learning**. The learning approach finds clades, where each one is a cluster of variables and objectives that share information. For optimization purposes, each cluster shows a set of variables with significant interactions. For example, they can correspond to correlated covariates in regression techniques. The output is a Table with a list of variables (one cluster) per line.
2. **Second level learning**. The potential contribution of each clade, i.e., the decision variables for the objectives, for example, is estimated. The calculation of all these distances allows for determining the output of the second learning level, which has two matrices: one with the phylogenetic distances from variables to objectives and another with the relative phylogenetic distances between objectives.

In reference [16] a comparison between the relevant properties of NL-MVU-PCA and FS-OPA to solve multi-objective optimization problems was made. The most important conclusions are as follows. Concerning the type of analysis, the FS-OPA is able to take into account associations objective-objective, variable-variable and variable-objective, while the NL-MVU-PCA only take into consideration the relations objective-objective. However, the NL-MVU-PCA is able to take into account objective space reduction. The FS-OPA does not need the use of a kernel function or the apriori kernel parameter optimization. Finally, the explainability in the NL-MVU-PCA is implicit while in the FS-OPA is explicit. This is an important characteristic since being "explicit" means providing a knowledge representation to help the decision-making process, while "implicit" is related to the capacity to determine the relative importance of the objectives.

3.2 Proposed framework

Figure 1 presents the flowchart of the iterative procedure using FS-OPA to determine the best objectives to use in the optimization based on the previous methodology developed for reducing the number of objectives [16]. The idea is to start this process using a high number of objectives characteristic of some real-world optimization problems.

After the random generation and the evaluation of an initial set of solutions, the FS-OPA determines the phylogram and the distance table objectives-objectives. Then, two different options can be pursued.

In the automatic procedure, the selection of the number of objectives to be used in the optimization is made automatically, using the table of the distance objectives-objectives and applying the following rules [16]:

1. choose the objective(s) of the less distant clusters;
2. choose one objective of the more distant (single) cluster;
3. choose one objective from each of the remaining clusters.

In the second procedure, the selection of the objectives is made by the DM(s) using both the tables of distances objectives-objectives and the phylogram as follows:

1. choose the objective(s) of the less distant clusters;
2. choose one objective of the more distant (single) cluster;
3. choose objective(s) from each of the remaining clusters taking into account, also, the phylogram and the knowledge of the DM(s) about the process.

Then, if the predefined number of objectives is not reached the process continues, but now using the objectives defined in the previous iteration. Otherwise, if the number of objectives is satisfactory the iterative process stops and the optimization is done with the reduced number of objectives.

The rules used in this procedure can be explained by taking into consideration that the less distant cluster is the one that has more information concerning the process, since it is near of most of the decision variables, while the more distant clusters also have some information that cannot be lost. Additionally, the intermediate clusters, chosen using rule 3, have some information regarding the process that is already present in the objectives selected by the other rules 1 and 2 and, thus, only one objective is chosen from these clusters.

Using the first procedure the final solution is obtained directly without the intervention of the DM(s), while in the second procedure, the DM takes part in the process, and therefore, the solution found can be straightforwardly explainable to it.

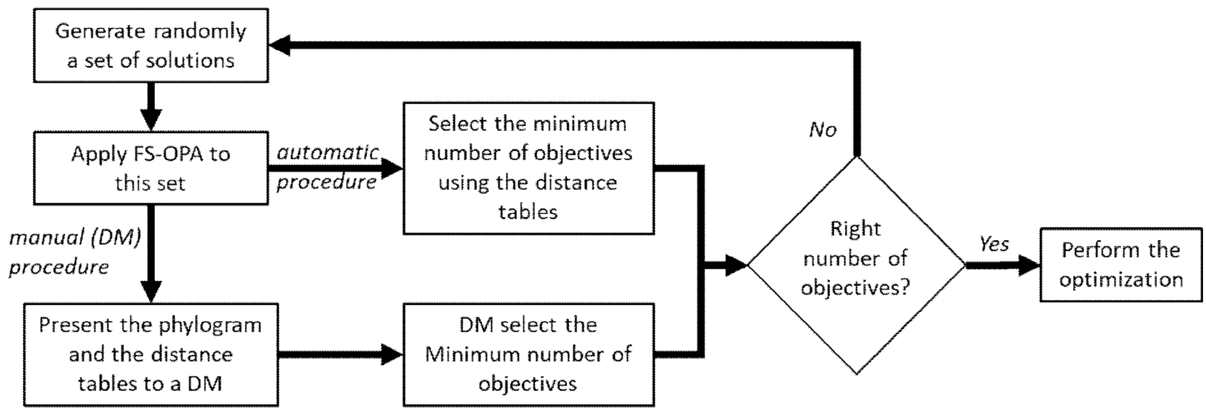

Figure 1. The general procedure of FS-OPA for the reduction of the number of objectives.

4 PROBLEM TO SOLVE

The methodology proposed will be applied to the polymer extrusion process illustrated in Figure 2. The extruder is constituted by a heated barrel where a screw rotates. In this polymer processing technology, a solid polymer is fed into the hopper falling into the barrel due to gravity. Then, the solid polymer by the action of the screw rotation is transported to the heated barrel zone where it melts and, after, this melted polymer is pressurized and forced to cross the die that gave the final shape to the product. In this way, the following five functional zones were developed [24, 25]: i) solids conveying in the hopper; ii) solids conveying in the initial turns of the screw due to friction between the polymer and the screw and barrel walls; iii) melting of the polymer, where a specific mechanism develops; iv) melt conveying of the melted polymer; and v) melt conveying in the die.

To model this process these different steps must be linked coherently, i.e., the data obtained in a certain step is the input data for the subsequent. This modelling will depend on the thermal, physical and rheological polymer properties, on the system geometry and on the operating conditions (screw speed and barrel temperature profile). All these aspects were considered in the modelling program used in the present calculations, for which the details can be found elsewhere [25, 26].

The aim here is to optimize both, the operating conditions, namely, barrel temperature profile (Tbi) and screw speed (N) and the screw geometry. For the latter, two different screws can be fitted in the extruder, a Conventional Screw (CS) and a barrier screw, specifically, a Maillefer Barrier Screw (MBS) [24, 26, 27]. The aim of the MBS is to allow for a better and faster melting of the polymer. For more details, see references [26, 28].

A Low Density Polyethylene (LDPE) is used in the calculations for which the relevant properties can be found elsewhere [26].

Figure 2 shows the global system geometry where the parameters to optimize are also identified. The range of variation of the operating conditions are as follows: N $\in$ [40, 80] rpm; Tb1 $\in$ [140, 160] °C, Tb2 $\in$ [150, 170] °C and Tb3 $\in$ [160, 200] °C. The geometrical parameters are presented in Table 1. Since only one type of screw can be fitted in the extruder, an additional variable is created, identified by “case”, ranging in the interval [0,1], if “case” is lower or equal to 0.5 the screw to be considered is a CS, otherwise, the screw is an MBS. Also, both screws are characterized by some common geometrical parameters (Table 1): length of the feed zone (L1) and compression zone (L2), the height of the channel in the feed zone (H1) and metering zone (H3), the pitch of the screw (P) and flight channel width (e). For the MBS two additional parameters are necessary: the channel height (Hf) and the width (wf) of the additional flight. The variables are repeated for the two types of screws in order that during the evolutionary process of the multiobjective optimization algorithm used no information will be lost.

The most important aspect to be analyzed within this work is related to the objectives. Twenty-one objectives were defined as described in Table 2. These are the objectives defined taking into account the knowledge of a DM concerning the thermomechanical behavior of the polymer inside the machine as well some local specifications related to the functioning of the machine, such as for example the melting length and the viscous dissipation.

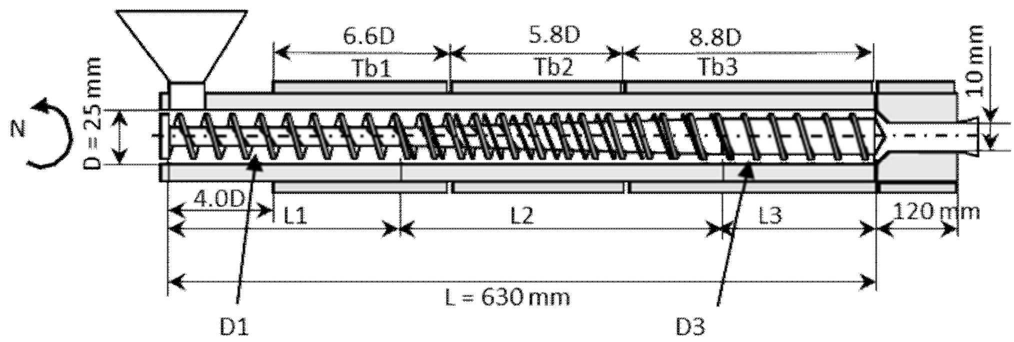

Figure 2. Geometry and operating parameters of the extruder to be optimized.

Screw Type	Decision Variables								
CS MBS	case	L1 L1_	L2 L2_	H1 H1_	H3 H3_	P P_	e e_	 Hf	 wf
Interval	[0, 1]	[100, 400]	[170, 400]	[18, 22]	[22, 26]	[25, 35]	[3, 4]	[0.1, 0.6]	[3, 4]

Table 1. Geometrical parameters of both CS and MBS screws.

ID	Description	Aim	Minimum	Maximum
Q	Mass output (kg/hr)	max	1	30
L	Length for melting (m)	min	0.1	0.85
T	Melt temp. at extruder exit (ºC)	min	160	230
Power	Total mechanical power consumption (W)	min	0	9200
WATS	Mixing degree	max	0	1000
TTb	Average viscous dissipation	min	1	2
TmaxTb	Maximum viscous dissipation	min	1	2
Shear_max	Maximum shear rate (s^{-1})	max	0	1000
Ntimes	Number of times the temperature is higher than Tb+10 ºC	min	0	300
Pexit	Pressure at extruder exit (MPa)	max	0	30
SME	Specific mechanical energy (Power*3600/Q)	min	300000	2000000
Z_Ztotal	Z/Ztotal, proportional to L above	min	0.1	0.85
Shear_avg	Average shear rate (s^{-1})	max	0	1000
Visco_avg	Average viscosity (Pa.s)	min	50000	200000
Davg_max	Viscous dissipation of the maximum values in each interval	min	1	2
NCam	CAMERON number	max	0.005	0.5
NPec	PECLET number	min	100000	3000000
NBri	BRINKMAN number	min	2	150
Nnam	NAME number	min	100000	4000000
Qd	Drag volumetric flow at melt conveying zone (m^3/s)	max	2000000	10000000
Qp	Pressure volumetric flow at melt conveying zone (m^3/s)	min	5000000	1.5E+08

Table 2. Objectives, aim of optimization and range of variation.

5 RESULTS AND DISCUSSION

This section presents and discusses the results obtained from the case study presented in the previous section taking into account the methodology proposed in Figure 1. For that, in the first iteration, all 21 objectives (Table 1) will be considered.

1st iteration: Figure 3 and Table 3 show the results after the application of the FS-OPA to a random initial population of solutions. For that purpose, the FS-OPA was applied 10 times by changing the order of the solutions randomly and the average is calculated. The phylogram of figure 3 presents, in different colors, the clusters obtained and the number of times the location of each variable or objective is in the same position represented by the number in the circles. From the phylogram the distance between the objectives is calculated, representing the distance shown in the phylogram as shown in Table 3, however, due to the size of the Table with all objectives included, only the average is presented for this case.

Then, from the application of the rules presented in section 3 the following two sets of objectives, after reduction, were obtained:

- 8 objectives: Pexit, Shear_max, Shear_avg, TTb, WATS, Q, Ntimes, NCam;
- 6 objectives: Pexit, Shear_max, TTb, WATS, Q, NCam.

2nd iteration: Figure 4 and Table 4 present the results obtained using the FS-OPA for the case with 8 objectives while Figure 5 and Table 5 for the case with 6 objectives, as defined by the first iteration.

These phylograms and tables allow a further reduction and the following set of objectives were considered:

- 4 objectives – A: Pexit, TTb, Q, Ntimes;
- 4 objectives – B: Pexit, Shear_max, TTb, Q;
- 3 objectives: Shear_max, TTb, Q.

Finally, the methodology for the reduction of the number of objectives was assessed by performing 11 optimizations for each one of the sets of objectives: 21, 8, 6, 4-A, 4-B and 3 objectives. Then, a statistical comparison using the hypervolume (HV) and the Inverted Generational Distance (IGD) metrics was made [29,30,31]. To calculate IGD, all Pareto-optimal solutions found in each run were put together in a pool and the non-dominated solutions of this pool were used for comparison.

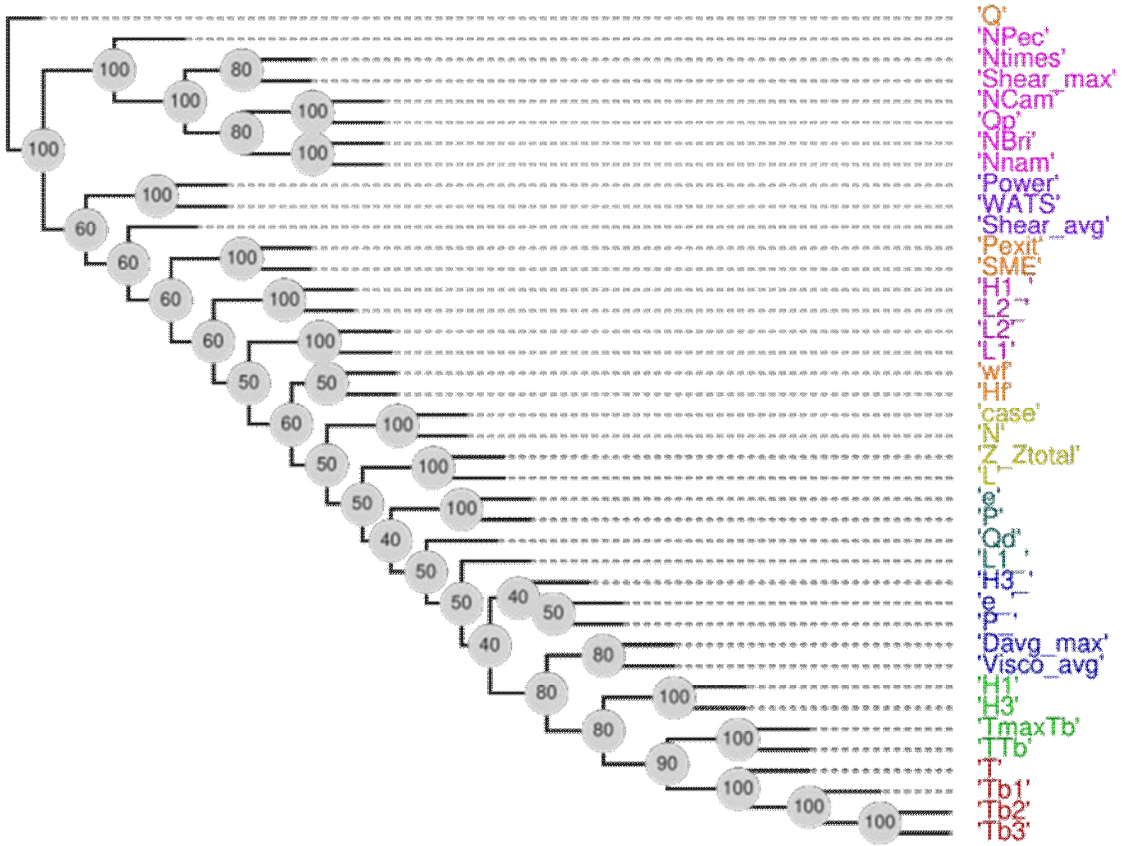

Figure 3. Phylogram obtained considering 21 objectives (the colors identify the clusters found).

The procedure adopted was the following: i) to do 11 optimization runs for 21, 8, 6, a-A, 4-B and 3 objectives; ii) to all cases the 21 objectives were used for all sets of objectives; iii) to calculate the HV and the IGD metrics: iv) to determine the average, standard deviation and the percentage of reduction/increase relative to the runs with 21 objectives.

The results are presented in Table 6. If HV increases and IGD decreases means that the optimal Pareto fronts are better. Thus, better optimization results were obtained for the case with 6 objectives A, i.e., for the case with objectives Pexit, Shear_max, TTb, WATS, Q and NCam. This can be understood due to the fact that the use of MOEAs with many objectives causes some difficulties to the algorithms and better results can be obtained when using a lower number of objectives, even if the comparison is made using 21 objectives for an optimization run with only 6 objectives.

However, further study is necessary to compare better the results obtained with less than 6 objectives and to determine a better way for comparing the results obtained using, for example, the runs with 6 objectives.

	Average
'Q'	0.38
'L'	0.38
'T'	0.56
'Power'	0.35
'WATS'	0.35
'TTb'	0.55
'TmaxTb'	0.55
'Shear_max'	0.38
'Ntimes'	0.38
'Pexit'	0.34
'SME'	0.34
'Z_Ztotal'	0.38
'Shear_avg'	0.31
'Visco_avg'	0.49
'Davg_max'	0.49
'NCam'	0.41
'NPec'	0.33
'NBri'	0.41
'Nnam'	0.41
'Qd'	0.39
'Qp'	0.41

Table 3. Average of the distance between objectives considering 21 objectives.

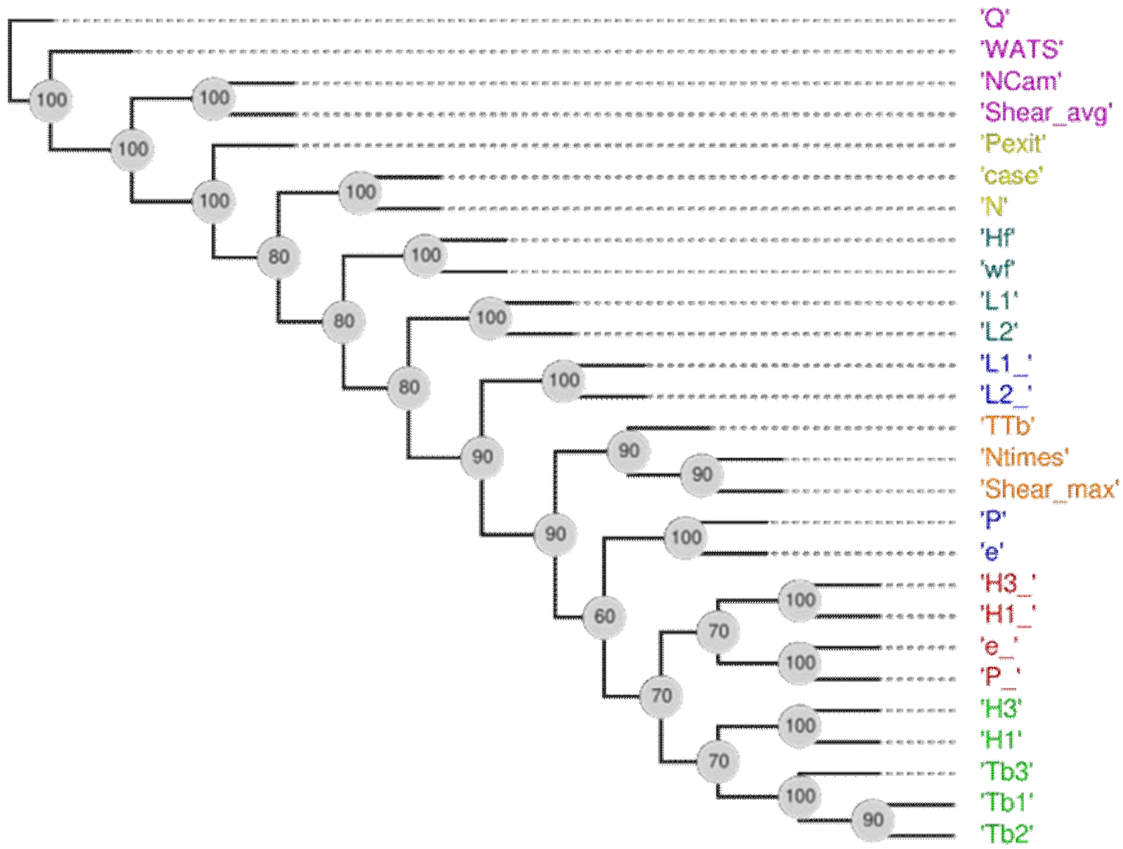

Figure 4. Phylogram obtained considering 8 objectives (the colors identify the clusters found).

	'Q'	'WATS'	'TTb'	'Shear_max'	'Ntimes'	'Pexit'	'Shear_avg'	'NCam'	Average
'Q'	0.00	0.07	0.64	0.71	0.71	0.21	0.21	0.21	0.34
'WATS'	0.07	0.00	0.64	0.71	0.71	0.21	0.21	0.21	0.34
'TTb'	0.64	0.64	0.00	0.14	0.14	0.50	0.64	0.64	0.41
'Shear_max'	0.71	0.71	0.14	0.00	0.07	0.56	0.71	0.71	0.45
'Ntimes'	0.71	0.71	0.14	0.07	0.00	0.56	0.71	0.71	0.45
'Pexit'	0.21	0.21	0.50	0.56	0.56	0.00	0.21	0.21	0.30
'Shear_avg'	0.21	0.21	0.64	0.71	0.71	0.21	0.00	0.07	0.34
'NCam'	0.21	0.21	0.64	0.71	0.71	0.21	0.07	0.00	0.34

Table 4. Distance between objectives for the case with 8 objectives.

	'Q'	'WATS'	'TTb'	'Shear_max'	'Pexit'	'NCam'	Average
'Q'	0.00	0.07	0.73	0.20	0.27	0.27	0.25
'WATS'	0.07	0.00	0.73	0.20	0.27	0.27	0.25
'TTb'	0.73	0.73	0.00	0.73	0.80	0.80	0.63
'Shear_max'	0.20	0.20	0.73	0.00	0.13	0.13	0.23
'Pexit'	0.27	0.27	0.80	0.13	0.00	0.07	0.25
'NCam'	0.27	0.27	0.80	0.13	0.07	0.00	0.25

Table 5. Distance between objectives for the case with 6 objectives.

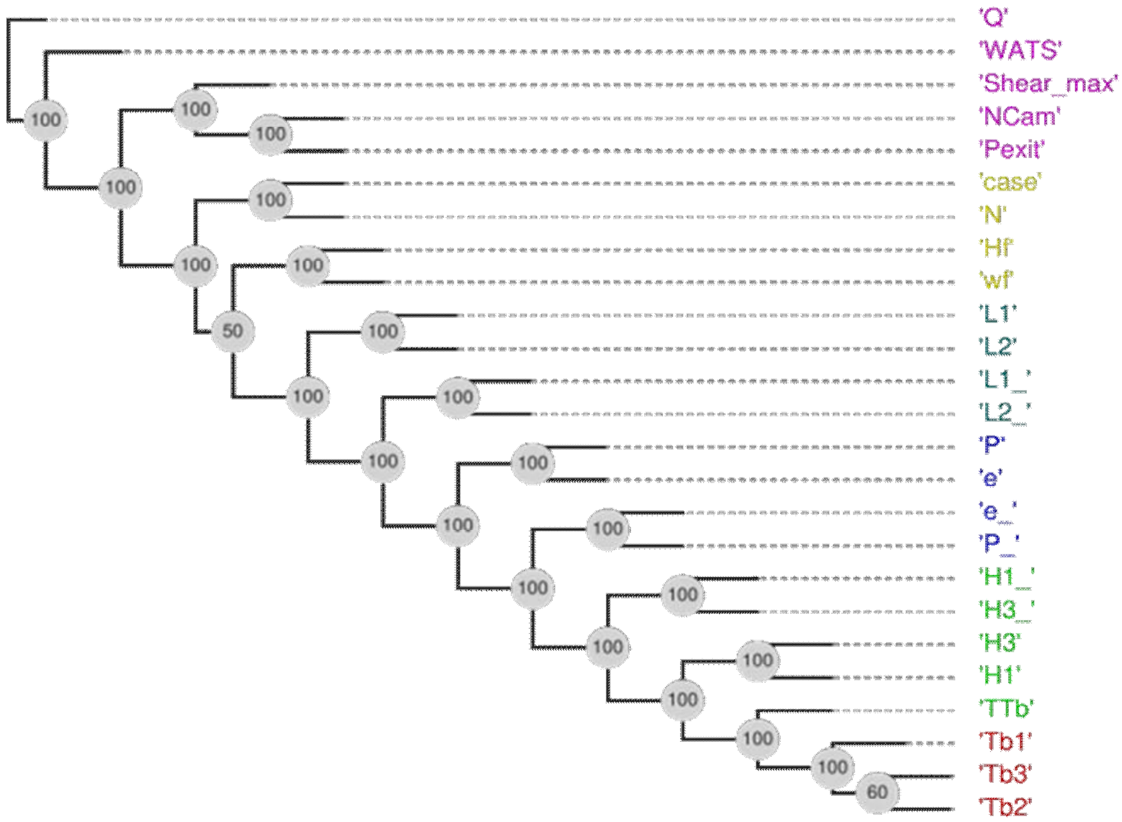

Figure 5. Phylogram obtained considering 6 objectives (colors identify the clusters found).

	21 Objs		8 Objs			6 Objs		
	AVG	STD	AVG	STD	Δ	AVG	STD	Δ
HV	9.9E-06	6.9E-06	4.6E-05	4.5E-05	3.6E+02	**5.8E-04**	**7.0E-05**	**5.7E+03**
IGD	9.6E-01	1.6E-01	8.6E-01	1.3E-01	-1.0E+01	**4.0E-01**	**1.3E-02**	**-5.9E+01**

Table 6. Performance comparison using Hypervolume and IGD for the total number of objectives (21) and the automatic reduction to eight, six, four and three objectives (Δ represents the gain or loss percentage relative to 21 objectives).

	4 Objs - A			4 Objs - B			3 Objs		
	AVG	STD	Δ	AVG	STD	Δ	AVG	STD	Δ
HV	8.7E-05	2.9E-06	7.8E+02	5.0E-06	2.0E-05	-5.0E+01	2.1E-06	2.9E-07	-7.9E+01
IGD	7.5E-01	3.1E-02	-2.2E+01	8.2E-01	2.2E-02	-1.4E+01	9.5E-01	4.0E-02	-8.6E-01

Table 6. (continuation).

6 CONCLUSIONS

An efficient methodology for reducing the number of objectives is proposed to help the DM to solve real-world optimization problems. Starting from a big set of objectives it was possible to conclude that the better way to optimize a multi-objective optimization problem is using the most relevant objectives, i.e., a reduced set of objectives.

Further work will concentrate on finding a better framework to deal with the higher number of objectives, mainly in what concerns the comparison of Pareto fronts after optimization

REFERENCES

[1] K. Deb, Multi-Objective Optimization using Evolutionary Algorithms, Wiley, Chichester, UK, 2001.

[2] A. Carlos, C. Coello, B.L. Gary, A.V.V. David, Evolutionary Algorithms for Solving Multi-Objective Problems, 2nd ed., Springer: New York, NY, USA, 2007.

[3] I. Boussaïd, J. Lepagnot, P. Siarry, A survey on optimization metaheuristics. Inf. Sci., 237, 82–117, 2013.

[4] S. Bandaru, A.H.C. Ng, K. Deb, Data mining methods for knowledge discovery in multi-objective optimization: Part A—Survey. Expert Syst. Appl., 70, 139–159, 2017. doi:10.1016/j.eswa.2016.10.015.

[5] D.K. Saxena, J.A. Duro, A. Tiwari, K. Deb, Q. Zhang, Objective Reduction in Many-Objective Optimization: Linear and Nonlinear Algorithms in IEEE Trans. Evol. Comput., 17, 77–99, 2013. doi:10.1109/tevc.2012.2185847.

[6] D. Brockhoff, E. Zitzler, Are All Objectives Necessary? On Dimensionality Reduction in Evolutionary Multiobjective Optimization in Lecture Notes in Computer Science, Springer: Berlin/Heidelberg, Germany, 533–542, 2006. doi:10.1007/11844297_54.

[7] D. Brockhoff, E. Zitzler, Objective Reduction in Evolutionary Multiobjective Optimization: Theory and Applications, Evol. Comput., 17, 135–166, 2009. doi:10.1162/evco.2009.17.2.135.

[8] J.A. López, C.C.A. Coello, D. Chakraborty, Objective reduction using a feature selection technique in Proceedings of the 10th Annual Conference on Genetic and Evolutionary Computation—GECCO '08, Atlanta, GA, USA, 12–16, 2008. doi:10.1145/1389095.1389228

[9] H.K. Singh, A. Isaacs, T. Ray, A Pareto Corner Search Evolutionary Algorithm and Dimensionality Reduction in Many-Objective Optimization Problems, IEEE Trans. Evol. Comput., 15, 539–556, 2011. doi:10.1109/tevc.2010.2093579.

[10] K. Deb, D.K. Saxena, Searching for Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems in Proceedings of the 2006 IEEE Congress on Evolutionary Computation (CEC'2006), IEEE: Vancouver, BC, Canada, 3353–3360, 2006.

[11] D.K. Saxena, K. Deb, Non-linear Dimensionality Reduction Procedures for Certain Large-Dimensional Multi-Objective Optimization Problems: Employing Correntropy and a Novel Maximum Variance Unfolding in Evolutionary Multi-Criterion Optimization. S. Obayashi, K. Deb, C. Poloni, T. Hiroyasu, T. Murata, eds. Springer: Berlin/Heidelberg, Germany, Vol. 4403, 2007. https://doi.org/10.1007/978-3-540-70928-2_58

[12] J.A. Duro, K.D. Saxena, K. Deb, Q. Zhang, Machine learning based decision support for many-objective optimization problems in Neurocomputing, 146, 30–47, 2014. doi:10.1016/j.neucom.2014.06.076

[13] Y. Yuan, Y.-S. Ong, A. Gupta, H. Xu, Objective Reduction in Many-Objective Optimization: Evolutionary Multiobjective Approaches and Comprehensive Analysis. IEEE Trans. Evol. Comput., 22, 189–210, 2018. doi: 10.1109/TEVC.2017.2672668.

[14] A. Gunduz, J.C. Principe, Correntropy as a novel measure for nonlinearity tests. Signal Processing, 89, 14–23, 2009. doi:10.1016/j.sigpro.2008.07.005.

[15] A. Sinha, D.K. Saxena, K. Deb, A. Tiwari, Using objective reduction and interactive procedure to handle many-objective optimization problems. Appl. Soft Comput., 13, 415–427, 2013. doi:10.1016/j.asoc.2012.08.030.

[16] A. Gaspar-Cunha, P. Costa, F. Monaco, A. Delbem, Many-Objectives Optimization: A Machine Learning Approach for Reducing the Number of Objectives. Math. Comput. Appl., 28, 17, 2023. https://doi.org/10.3390/mca28010017

[17] A. Sanches, J.M. Cardoso, A.C. Delbem, Identifying merge-beneficial software kernels for hardware implementation in Proceedings of the International Conference on Reconfigurable Computing and FPGAs (ReConFig), Cancun, Mexico, 74–79, 2011.

[18] F.G.Z. Kharrat, F., N.S.B. Miyoshi, J. Cobre, J. Mazzoncini De Azevedo-Marques, P. Mazzoncini de Azevedo-Marques, A.C.B. Delbem, Feature sensitivity criterion-based sampling strategy from the Optimization based on Phylogram Analysis (Fs-OPA) and Cox regression applied to mental disorder datasets in PLOS ONE, Vol. 15, 2020.

[19] A. Soares, R. Râbelo, A.C.B. Delbem, Optimization based on phylogram analysis in Expert Systems with Applications, Vol. 78, 32-50, 2017.

[20] J.P. Martins, C.M. Fonseca, A.C.B. Delbem, On the performance of linkage-tree genetic algorithms for the multidimensional knapsack problem in Neurocomputing, Vol. 146, 17-29, 2014.

[21] R. Cilibrasi, P.M.B. Vitanyi, Clustering by Compression in IEEE Trans. Inf. Theory, Vol. 51, 1523-1545, 2005.

[22] M.E.J. Newman, Fast algorithm for detecting community structure in networks Phys. Rev. E., Vol. 69, 066133, 2004.

[23] A. Sanches, J. M. P. Cardoso, A. C. B. Delbem, Identifying Merge-Beneficial Software Kernels for Hardware Implementation, 2011 International Conference on Reconfigurable Computing and FPGAs, 2011. doi:10.1109/reconfig.2011.51

[24] C. Rauwendaal, Polymer Extrusion, Carl Hanser Verlag, Munich, 2001.

[25] A. Gaspar-Cunha, Modelling and Optimisation of Single Screw Extrusion Using Multi-Objective Evolutionary Algorithms, 1st ed. Koln, Germany, Lambert Academic Publishing, 2009.

[26] A. Gaspar-Cunha, J. A. Covas, The Plasticating Sequence in Barrier Extrusion Screws Part I: Modeling, Polymer Engineering and Science, 54(8), 1791–1803, 2014. doi:10.1002/pen.23722/full

[27] C. Rauwendaal, Extruder screws with barrier sections, Polymer Engineering and Science, 26(18), 1245–1253, 1986. doi:10.1002/pen.760261804

[28] A. Gaspar-Cunha, J. A. Covas, The Plasticating Sequence In Barrier Extrusion Screws Part II: Experimental Assessment. Polymer-Plastics Technology and Engineering, 53(14), 1456–1466, 2014. doi:10.1080/03602559.2014.909482

[29] I. Hisao, M. Hiroyuki, T. Yuki, N. Yusuke, Modified distance calculation in generational distance and inverted generational distance in Evolutionary Multi-Criterion Optimization. A. Gaspar-Cunha, C.H. Antunes, C. Coello Coello, eds. Springer International Publishing: Cham, Switzerland, 110–125, 2015.

[30] C.M. Fonseca, L. Paquete, M. López-Ibáñez, An improved dimension sweep algorithm for the hypervolume indicator in Proceedings of the 2006 Congress on Evolutionary Computation (CEC 2006), Vancouver, BC, Canada, 16–21, 1157–1163, 2006. doi:10.1109/CEC.2006.1688440.

[31] pymoo: Multi-objective Optimization in Python. Available online: https://pymoo.org/misc/indicators.html#nb-hv (accessed on 5 November 2022).