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A GAME FOR MULTI-SCALE TOPOLOGY OPTIMIZATION OF STATIC AND DYNAMIC COMPLIANCES OF TPMS-BASED LATTICE STRUCTURES

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Abstract. *In this work, we set up a game of minimizing static (player 1) and dynamic (player 2) compliance, respectively, by using a multi-scale topology optimization framework for TPMS-based lattice structures. Player 1 will find the optimal macro layout by minimizing the static compliance for a given micro layout delivered from player 2, and player 2 will find the optimal micro layout (grading of the TPMS-based lattice structure) by minimizing the dynamic compliance for a given macro layout from player 1. The two multi-scale topology optimization formulations are obtained by using two density variables in each finite element. The first variable is the standard topology optimization macro density variable, which defines if the element should be treated as a void or contain the graded lattice structure by letting this variable be governed by the rational approximation of material properties (RAMP) model. The second variable is the local relative lattice density, and it sets the effective orthotropic elastic properties of the element, which in turn are obtained by using numerical homogenization of representative volume elements of the TPMS-based lattice structure of interest. Player 1 follows the standard compliance problem formulated in the first density variable using graded lattice structures from player 2. Furthermore, player 2 grades the lattice structure by solving the dynamic compliance problem for a harmonic load formulated in the second variable for a macro-layout presented by player 1. The game is implemented for three-dimensional problems and the results are presented as STL-files using implicit-based geometry and marching cubes. It is demonstrated that the proposed game generates designs that have good performance for both the static and harmonic load cases and efficiently can avoid resonance at the frequencies of the harmonic loads.*

Keywords: A game approach, Multi-scale topology optimization, TPMS-based lattices

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1 INTRODUCTION

Advances in additive manufacturing make it possible to manufacture complex cellular structures with high quality [1]. In such manner, new ultra-lightweight multi-functional designs that are both strong and stiff can be manufactured, similar to ultra-lightweight designs found in nature such as e.g. bird skeletons [2]. One class of cellular structure that has proven suitable for this is the triply periodic minimal surface (TPMS)-based lattice structures [3], where the Gyroid structure probably is the most well-known. The TPMS-based surfaces have zero mean curvature at every point and are formulated as implicit-based surfaces. Two types of TPMS-based structures can be set up using Boolean operators: the shell-based and the frame-based, in fact one could even set up Honeycomb-based alternatives. An extensive study of the mechanical properties of several TPMS-based lattices can be found in the work by Maskery et al. [4]. Recently, a multi-scale topology optimization framework for finding optimal macro layouts with optimal grading of these TPMS-based structures was proposed with transversely isotropic elastic bulk properties [5]. Frame- and shell-based Gyroid, G-prime and Schwarz-D structures were implemented by establishing elastic properties as function of relative lattice density using numerical homogenization of representative volume elements of the TPMS-based structures [6].

If the frequency of a harmonic load is close to any of the natural frequencies of a component, then the phenomena of resonance occurs. Therefore, in the design of lightweight components, natural frequencies close to the driving frequency of the external load must be avoided. A design possibility to accomplish this could be to utilize TPMS-based structures and grade these in order to tune the natural frequencies such that resonance is avoided. This is the topic of the following paper. We suggest to set up a non-cooperative two players game, where one of the player minimize the static compliance by finding the optimal macro-layout for a given design of local graded TPMS-based lattice structures and the other player minimize the dynamic compliance by finding the optimal local grading of the TPMS-based lattice structure for a given design of the macro-layout. The game is formulated by using the multi-scale topology optimization framework for TMPS-based lattice structures presented recently in [5].

The outline of the paper is the following: in the next section the governing equations needed for the multi-scale topology optimization formulation are presented, in section 3 the strategies for the multi-scale players are formulated as two separate topology optimization problems, in section 4 the treatment of the multi-scale game using a Gauss-Seidel approach is presented, and, finally, some concluding remarks are given.

2 GOVERNING EQUATIONS

Let us consider a non-homogenous linear orthotropic elastic body with fixed displacements and prescribed external forces, which is discretized with linear finite elements. Two density variables, γ_e and ρ_e, are introduce for each element e; γ_e is the local relative density of lattice in the element and ρ_e is a standard macro density variable defining if the element should be treated as a void or filled with lattice structure.

The effective elastic properties of each element e are obtained by numerical homogenization of representative volume elements (RVEs) of the TPMS-based lattice structure of interest such that the effective elasticity matrix in Voigt notation is given by

$$\boldsymbol{C}_e = \boldsymbol{C}_e(\gamma_e) = \begin{bmatrix} f_{11}c_{11} & f_{21}c_{21} & f_{31}c_{31} & 0 & 0 & 0 \\ f_{21}c_{21} & f_{22}c_{11} & f_{32}c_{31} & 0 & 0 & 0 \\ f_{31}c_{31} & f_{32}c_{31} & f_{33}c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & f_{44}c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & f_{55}c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & f_{66}c_{66} \end{bmatrix}, \tag{1}$$

where $f_{ij} = f_{ij}(\gamma_e)$ are material interpolation (regression) laws as function of the relative lattice density $0 \leq l_b \leq \gamma_e \leq u_b \leq 1$ of element e, where l_b and u_b are prescribed lower and upper limits on γ_e, respectively. Furthermore, c_{ij} in (1) are the elastic properties of the bulk material of the lattice structure, which are assumed to be governed by transversely isotropic elasticity, i.e. $\boldsymbol{C}_e(1)^{-1} =$

$$\begin{bmatrix} 1/E & -\nu/E & -\nu_{13}/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu_{13}/E & 0 & 0 & 0 \\ -\nu_{13}/E & -\nu_{13}/E & 1/E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu)/E \end{bmatrix}, \tag{2}$$

where E and E_{33} are Young's moduli, ν and ν_{13} are Poisson's ratios and G_{23} is the out-of-plane shear moduli.

The corresponding local effective finite element stiffness matrix for element e is denoted by

$$\boldsymbol{k}_e = \boldsymbol{k}_e(\gamma_e). \tag{3}$$

In the global assembly procedure, the RAMP model is utilized in order to define if the finite element e should be considered to be a void or filled with lattice structure. Thus, the global stiffness matrix is generated by

$$\boldsymbol{K} = \boldsymbol{K}(\boldsymbol{\rho}, \boldsymbol{\gamma}) = \bigcap_e \frac{\rho_e}{1 + n(1-\rho_e)} \boldsymbol{k}_e(\gamma_e), \tag{4}$$

where, $\boldsymbol{\rho} = \{\rho_e\}$, $\boldsymbol{\gamma} = \{\gamma_e\}$, $\bigcap$ represents an assembly operator, n is the RAMP factor and $\epsilon \leq \rho_e \leq 1$ is the relative macro density of lattice structure for element e, where ϵ is a small number ϵ in order to avoid singular stiffness matrices.

The global mass matrix is given by

$$\boldsymbol{M} = \boldsymbol{M}(\boldsymbol{\rho}, \boldsymbol{\gamma}) = \bigcap_e \rho_e \gamma_e \boldsymbol{m}_e, \tag{5}$$

where $\boldsymbol{m}_e$ is local element mass matrix for an element e with $\rho_e = \gamma_e = 1$.

The total volume of bulk material generated in the assembly procedure is given by

$$V^{\text{bulk}} = V^{\text{bulk}}(\boldsymbol{\rho}, \boldsymbol{\gamma}) = \sum_e \rho_e \gamma_e V_e, \tag{6}$$

where V_e represents the total volume of each element e when $\rho_e = \gamma_e = 1$. By using (6), the following different volume measures can be identified:

$$\begin{aligned} V^{\text{design}} &= V^{\text{bulk}}(\mathbf{1}, \mathbf{1}), \\ V^{\text{macro}} &= V^{\text{macro}}(\boldsymbol{\rho}) = V^{\text{bulk}}(\boldsymbol{\rho}, \mathbf{1}), \\ V^{\text{lattice}} &= V^{\text{lattice}}(\boldsymbol{\gamma}) = V^{\text{bulk}}(\mathbf{1}, \boldsymbol{\gamma}), \end{aligned} \tag{7}$$

where V^{design} is the total volume of the design domain, V^{macro} is the total volume of the macro layout filled with lattice structure, V^{lattice} is the total volume of lattice in the design domain and $\mathbf{1}$ is a vector of ones. The volume of lattice in the macro layout of lattice structure V^{macro} is given by

$$V^{\text{lattice}}_{\text{macro}} = V^{\text{lattice}} - (V^{\text{design}} - V^{\text{macro}})l_b, \tag{8}$$

which will converge towards the volume of bulk material V^{bulk}, because $\rho_e = \epsilon$ in the void region.

3 THE MULTI-SCALE PLAYERS

In this section, the two players with their strategies for the non-cooperative game presented in the next section are defined. The first player minimize the static compliance c_s by finding the optimal macro-layout $\boldsymbol{\rho}^*$, and the second player minimize the dynamic compliance c_d by finding the optimal local grading of the TPMS-based lattice $\boldsymbol{\gamma}^*$.

Thus, the two players' strategies, P_1 and P_2, are defined by the following multi-scale topology optimization problems:

$$P_1 \begin{cases} \text{Given } \boldsymbol{\gamma}^*: \\ \min\limits_{(\boldsymbol{\rho}, \boldsymbol{d})} c_s = \boldsymbol{F}^T \boldsymbol{d} \\ \text{s.t.} \begin{cases} \boldsymbol{K}(\boldsymbol{\rho}, \boldsymbol{\gamma}^*)\boldsymbol{d} = \boldsymbol{F}, \\ V^{\text{macro}}(\boldsymbol{\rho}) \leq \hat{V}^{\text{macro}}, \\ \epsilon\mathbf{1} \leq \boldsymbol{\rho} \leq \mathbf{1}, \end{cases} \end{cases} \tag{9}$$

$$P_2 \begin{cases} \text{Given } \boldsymbol{\rho}^*: \\ \min\limits_{(\boldsymbol{\gamma}, \boldsymbol{u})} c_d = \frac{1}{2}\left(\boldsymbol{F}^T \boldsymbol{u}\right)^2 \\ \text{s.t.} \begin{cases} \left(-\Omega^2 \boldsymbol{M}(\boldsymbol{\rho}^*, \boldsymbol{\gamma}) + \boldsymbol{K}(\boldsymbol{\rho}^*, \boldsymbol{\gamma})\right)\boldsymbol{u} = \boldsymbol{F}, \\ V^{\text{lattice}}(\boldsymbol{\gamma}) \leq \hat{V}^{\text{lattice}}, \\ l_b\mathbf{1} \leq \boldsymbol{\gamma} \leq u_b\mathbf{1}, \end{cases} \end{cases} \tag{10}$$

where $\boldsymbol{d}$ and $\boldsymbol{u}$ are the displacement vectors, $\boldsymbol{F}$ contains the external forces, $\boldsymbol{Kd} = \boldsymbol{F}$ is the static equilibrium equation, $-\Omega^2\boldsymbol{Mu} + \boldsymbol{Ku} = \boldsymbol{F}$ is the dynamic equilibrium equation, where Ω is the angular frequency for the harmonic load $\boldsymbol{F}\sin(\Omega t)$, and $\hat{V}^{\text{macro}}$ and $\hat{V}^{\text{lattice}}$ are the upper limits on the macro layout volume and the lattice volume, respectively.

The players' strategies in (9) and (10) are solved using sequential linear programming (SLP) using the sensitivities and filters presented below. The sensitivities of the compliances c_s and c_d, using the adjoint method, are given by

$$s^s_e = \frac{\partial c_s}{\partial \rho_e} = -\boldsymbol{d}^T \frac{\partial \boldsymbol{K}}{\partial \rho_e}\boldsymbol{d}, \tag{11}$$

$$s_e^d = \frac{\partial c_d}{\partial \gamma_e} = -\left(\boldsymbol{F}^T\boldsymbol{u}\right)\boldsymbol{u}^T\left(-\Omega^2\frac{\partial \boldsymbol{M}}{\partial \gamma_e} + \frac{\partial \boldsymbol{K}}{\partial \gamma_e}\right)\boldsymbol{u}, \tag{12}$$

where

$$\frac{\partial \boldsymbol{K}}{\partial \rho_e} = \frac{n+1}{(1+n(1-\rho_e))^2}\boldsymbol{k}_e(\gamma_e), \tag{13}$$

$$\frac{\partial \boldsymbol{M}}{\partial \gamma_e} = \rho_e \boldsymbol{m}_e, \tag{14}$$

and

$$\frac{\partial \boldsymbol{K}}{\partial \gamma_e} = \frac{\rho_e}{1+n(1-\rho_e)}\frac{\partial \boldsymbol{k}_e}{\partial \gamma_e}, \tag{15}$$

where $\partial f_{ij}/\partial \gamma_e$ are needed in (15).

The sensitivities of the volume V^{macro} and V^{lattice} are, of course,

$$\frac{\partial V^{\text{macro}}}{\partial \rho_e} = \frac{\partial V^{\text{lattice}}}{\partial \gamma_e} = V_e. \tag{16}$$

The sensitivities are treated by a density filter for both ρ_e and γ_e [7], and, in addition, ρ_e is passing a smooth Heaviside function [8]. Thus, ρ_e is treated by

$$\rho_e^{\text{filt}} = \frac{\sum_{g=1}^{n_{\text{el}}}\delta_g V_g \rho_g}{\sum_{f=1}^{n_{\text{el}}}\delta_f V_f}, \tag{17}$$

where n_{el} is the number of finite elements,

$$\delta_f = \delta_f(e) = (r_{\text{min}} - \text{dist}(e,f))_+\,, \tag{18}$$

$\text{dist}(e,f)$ denotes the distance between the center of element e and f, and r_{min} is the filter radius, which is set to 1.5 times the characteristic length of the finite elements in the numerical examples. (17) is also applied on γ_e. Furthermore,

$$\rho_e^{\text{heav}} = \frac{\tanh(\beta\eta) + \tanh(\beta(\rho_e^{\text{filt}} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1-\eta))}, \tag{19}$$

where η defines the threshold and β sets the slope of the smooth Heaviside filter. In the numerical examples, $\eta = 0.5$, β is ramped from 1 to 20, and the filter is activated after 100 SLP iterations.

4 THE MULTI-SCALE GAME

The non-cooperative game is to find an equilibrium point $(\boldsymbol{\rho}^*, \boldsymbol{\gamma}^*)$ that is optimal simultaneously for the players P_1 and P_2 in (9) and (10). In this work, we try to find such equilibrium points by applying a sequential Gauss-Seidel approach with sequential linear programming. A recent paper on solving a two-player game in topology optimization using a Gauss-Seidel approach is given in [9].

The corresponding LP-problems for player P_1 and player P_2 read

$$P_1 \begin{cases} \text{Given } \boldsymbol{\gamma}^k: \\ \min\limits_{\boldsymbol{\rho}} \sum\limits_e s_e^s \rho_e \\ \text{s.t.} \begin{cases} \sum\limits_e V_e \rho_e \leq \hat{V}^{\text{macro}}, \\ \boldsymbol{\rho}^k - \boldsymbol{\delta}_\rho \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}^k + \boldsymbol{\delta}_\rho, \end{cases} \end{cases} \tag{20}$$

$$P_2 \begin{cases} \text{Given } \boldsymbol{\rho}^k: \\ \min\limits_{\boldsymbol{\gamma}} \sum\limits_e s_e^d \gamma_e \\ \text{s.t.} \begin{cases} \sum\limits_e V_e \gamma_e \leq \hat{V}^{\text{lattice}}, \\ \boldsymbol{\gamma}^k - \boldsymbol{\delta}_\gamma \leq \boldsymbol{\gamma} \leq \boldsymbol{\gamma}^k + \boldsymbol{\delta}_\gamma, \end{cases} \end{cases} \tag{21}$$

where $\boldsymbol{\delta}_\rho$ and $\boldsymbol{\delta}_\gamma$ contain the move limits.

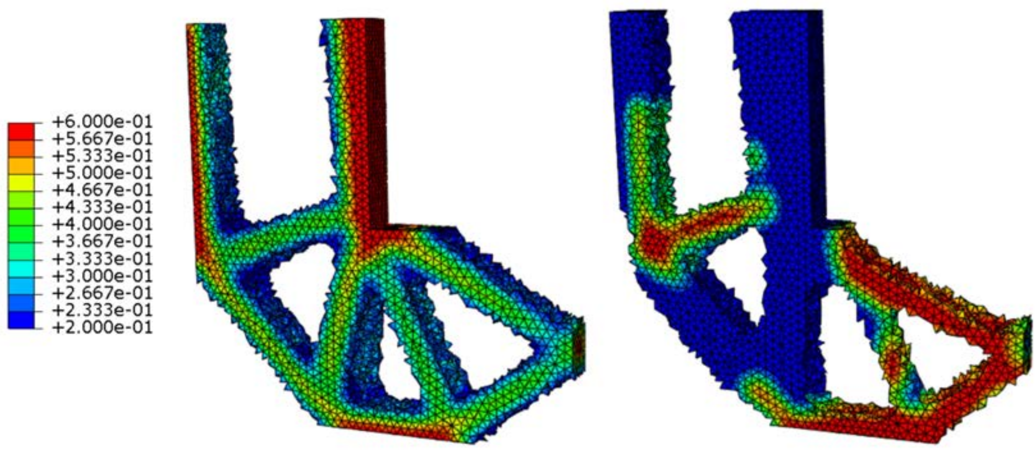

Figure 1: *The plots show the optimal macro-layout with the optimal graded lattice density field. Left: the driving frequency is set to zero, right: the driving frequency of the harmonic load is 2910.5 Hz, which corresponds to the fifth natural frequency of the left design.*

The sequential Gauss-Seidel algorithm reads

Set $k = 0$ and guess an initial state $(\boldsymbol{\rho}^0, \boldsymbol{\gamma}^0)$ and then solve the following steps sequential until convergence:

1. Let player 1 solve (20), and let $\boldsymbol{\rho}^{k+1}$ be equal to the optimal solution.
2. Set $\boldsymbol{\rho}^k = \boldsymbol{\rho}^{k+1}$, let player 2 solve (21), and let $\boldsymbol{\gamma}^{k+1}$ be equal to the optimal solution.
3. Let k=k+1 and go to step 1.

After a certain number of loops the algorithm is halted and the convergence is checked. If a solution is believed to be close to an equilibrium point, then algorithm is stopped. Otherwise, the algorithm is restarted from $(\boldsymbol{\rho}^k, \boldsymbol{\gamma}^k)$ and continues until a new check of convergence is performed.

5 ONE EXAMPLE

In this section, the established L-shaped benchmark used in topology optimization investigations is studied in order to demonstrate the proposed multi-scale non-cooperative game approach. First, the frequency of the harmonic load is set to zero and the design presented to the left in Figure 1 is obtained. The lower and upper limits on γ_e are 0.2 and 0.6, respectively. The limit on the macro volume fraction is set to 0.4 and the limit on the lattice volume is 0.52, implying a volume fraction of bulk material of 0.16. The natural frequency of the corresponding fifth mode, which is presented to the left in Figure 2, is 2910.5 Hz.

Next, we let the driving frequency of the harmonic load be equal to the natural frequency of 2910.5 Hz. A new design is generated with a different macro-layout as well as different optimal local grading of the TPMS-based lattice structure, see the right plot in Figure 1. The natural frequency of this design for the fifth mode is decreased to 2124.1 Hz, which is significantly different from the driving frequency of 2910.5 Hz. The corresponding fifth mode of this new design is presented to the right in Figure 2.

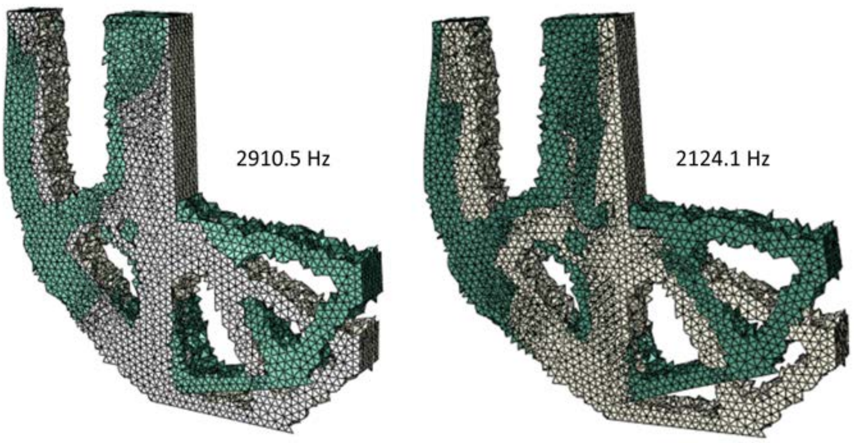

Figure 2: *The 5th mode of the two design. The natural frequency of the left design is 2910.5 Hz and 2124.1 Hz for the right design. The external driving frequency of the harmonic force of the right design is 2910.5 Hz.*

6 CONCLUSIONS

In this paper, we propose, implement and demonstrate a multi-scale non-cooperative two player game for avoiding resonance for given harmonic loads. Player one minimize the static compliance and player two minimize the dynamic compliance. The game is solved using SLP and a Gauss-Seidel approach, and it is demonstrated the an equilibrium design with optimal macro layout including optimal local grading of TPMS-based lattice structures is generated with natural frequencies significantly different from the driving frequency of the harmonic load.

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