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MAXIMIZATION OF DAMPING ENERGY DISSIPATION USING TOPOLOGY OPTIMIZATION WITH DISCRETE VARIABLES CONSIDERING TRANSIENT LOADS

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Abstract. *Damping structures under transient dynamic loads are essential in many industrial applications, such as viscoelastic dampers. Topology optimization methods have previously been applied to design structures using the time-domain approach without considering damping, or an extra layer of damping material is added to the final design to improve the damping energy dissipation. The objective of this work is to maximize damping energy dissipation, subjected to a volume constraint in the solid materials, considering a multimaterial topology optimization with discrete design variables. Aluminum and rubber-like material are considered in the structure's design, and a linear elastic material model is adopted. Aluminum is used as the structural material, while rubber-like material is utilized as the damping material. A transient rectangular step load is applied to the structure to introduce a transient motion and assess the inertial effects of the structure. The Rayleigh damping model is adopted, and the damping energy dissipation is calculated by dynamic analysis. The sensitivity numbers are calculated using the adjoint variable method based on the discretize-then-differentiate approach. A bi-clamped beam is studied to show the effectiveness of the proposed methodology. The results demonstrate the effectiveness of the developed methodology in the design of damped structures.*

Keywords: BESO method, Transient dynamic loads, Structural damping

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1 INTRODUCTION

Structures with damping energy dissipation are fundamental in many industries, where their damping characteristics are desired to provide a minimum vibration amplitude in a specific period of time. Due to this, researchers have shown an increased interest in designing structures with desired damped dynamic behavior [1]

The topology optimization methods deal with structural design by optimally distributing material taking into account the applied load with specific design constraints. Studies over the past three decades have provided important information on the topology optimization applied to enhance static-related characteristics of structures. However, little research has considered time-domain dynamics-related topology optimization.

Most studies in dynamics-related topology optimization have been carried out dealing with specific eigenfrequencies [2, 3], frequency gaps [4, 5, 6], or steady-state frequency responses[7, 8, 9, 10]. However, few studies have investigated time-domain dynamic topology optimization where transient conditions are important. There are some problems when the objective function is a transient response, such as treating time-dependent constraints, calculating design sensitivity, approximation, and higher computational cost [11].

To date, several studies have investigated time-domain dynamic topology optimization problems utilizing objective functions based on displacement, such as the displacement of a target degree of freedom, dynamic compliance, and strain energy [12, 13, 14], with and without using the equivalent static loads method (ESLM) [15, 16]. Regardless, such studies are focused on the minimization of the amplitude displacement related objective function.

Consequently, a methodology to design damped structures is developed and applied in this work to maximize the damping energy dissipation as the objective function. The discretize-then-differentiate approach is used to calculate the sensitivity numbers because it does not lead to the inconsistencies presented with the differentiate-then-discretize approach [17].

This paper is divided into seven parts. Section 2 deals with the governing equations, the Hilber-Hughes-Taylor numerical integration algorithm, HHT$-\alpha$, and the calculation of the damping energy dissipation. Section 3 presents the topology optimization problem. Section 4 is concerned with material interpolation. Section 5 presents the sensitivity analysis, and the Bi-directional Evolutionary Structural Optimization procedure is shown in Section 6. Then, numerical application considering a bi-clamped beam is presented in Section 7. Finally, the conclusions are presented in Section 8.

2 GOVERNING EQUATIONS

The boundary problem for linear elastodynamics considering a finite element discretized damped system is given by:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{f}t, \quad t \in [0, T_f] \tag{1}$$

$$\boldsymbol{u}(0) = \boldsymbol{u}_0, \quad \dot{\boldsymbol{u}}(0) = \dot{\boldsymbol{u}}_0 \tag{2}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are mass-, damping- and stiffness matrices, $\boldsymbol{f}(t)$ is an external load vector, $[0, T_f]$ is the time interval of interest, $\boldsymbol{u}(t)$ is the displacement vector, superimposed dots indicate time differentiation, and the subscript zero relates to the initial condition.

The damping model used in this work is proportional damping, such that,

$$\boldsymbol{C} = \alpha^c \boldsymbol{M} + \beta^c \boldsymbol{K} \tag{3}$$

where α^c and β^c are the Rayleigh damping parameters.

The linear elastodynamic problem presented in Equations 1 and 2 are solved using the equations of the HHT$-\alpha$ method [18], and the Newmark$-\beta$ [19] method. Then, the acceleration,$\ddot{\boldsymbol{u}}_i$, the velocity, $\dot{\boldsymbol{u}}_i$, and the displacement, $\boldsymbol{u}_i$, are computed by solving the following equation discretized in the time domain:

$$\boldsymbol{M}_1\ddot{\boldsymbol{u}}_i = -\boldsymbol{M}_0\ddot{\boldsymbol{u}}_{i-1} - \boldsymbol{C}_0\dot{\boldsymbol{u}}_{i-1} - \boldsymbol{K}\boldsymbol{u}_{i-1} + (1-\alpha)\,\boldsymbol{f}_i + \alpha\boldsymbol{f}_{i-1}, \quad for \quad i = 1, ..., N \tag{4}$$

where,

$$\boldsymbol{M}_1 = \boldsymbol{M} + (1-\alpha)\,\gamma h\boldsymbol{C} + (1-\alpha)\,\beta h^2\boldsymbol{K} \tag{5}$$

$$\boldsymbol{M}_0 = (1-\alpha)\,(1-\gamma)\,h\boldsymbol{C} + (1-\alpha)\left(\tfrac{1}{2}-\beta\right)h^2\boldsymbol{K} \tag{6}$$

$$\boldsymbol{C}_0 = \boldsymbol{C} + (1-\alpha)\,h\boldsymbol{K} \tag{7}$$

To obtain a second-order accurate and unconditionally stable characteristics, $\alpha = 0.05$, $\beta = (1+\alpha)^2/4$, and $\gamma = (1+2\alpha)/2$. h is the time step and N is the number of time steps. The initial acceleration $\ddot{\boldsymbol{u}}_0$ is calculated using the initial displacement, $\boldsymbol{u}_0$, and the initial velocity, $\dot{\boldsymbol{u}}_0$, as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_0 = \boldsymbol{f}_0 - \boldsymbol{C}\dot{\boldsymbol{u}}_0 - \boldsymbol{K}\boldsymbol{u}_0 \tag{8}$$

The damping energy dissipated in a time interval of interest is used in this work as objective function, in order to design damped structures. The objective function, E_d, can be calculated as follows:

$$E_d(\dot{\boldsymbol{u}}(t), \boldsymbol{X}) = \int_0^{T_f} \dot{\boldsymbol{u}}(t)\boldsymbol{C}(\boldsymbol{X})\dot{\boldsymbol{u}}(t)dt \tag{9}$$

where $\boldsymbol{X}$ is the matrix of the design variables.

3 TOPOLOGY OPTIMIZATION PROBLEM

The topology optimization problem consists in finding the material distribution that maximizes the damping energy dissipation for a given time interval, subject to volume constraints on the solid material phases. Aluminum, rubber-like material, and void are considered inside the structure during the topology optimization process, and its distribution is given by the design variable $\boldsymbol{X}$. The topology optimization problem can be stated as:

$$\begin{aligned}
&\text{Find: } \boldsymbol{X}\\
&\text{Maximize: } \int_0^{T_f} E_{d_i}(\boldsymbol{X}, \dot{\boldsymbol{u}})dt\\
&\text{Subject to: } V_s^* - \sum_{e=1}^{nel} V_e X_{e1} = 0\\
&\qquad V_1^* - \sum_{e=1}^{nel} V_e X_{e2} = 0\\
&\qquad \boldsymbol{M}(\boldsymbol{X})\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}(\boldsymbol{X})\,\dot{\boldsymbol{u}}(t) + \boldsymbol{K}(\boldsymbol{X})\,\boldsymbol{u}(t) = \boldsymbol{f}(t)\\
&\qquad X_{ej} = X_{\min}\; or\; 1; \quad j = 1, 2 \quad \text{and} \quad e = 1, ..., nel
\end{aligned} \tag{10}$$

where $X_{\min}$ is a small value to avoid singularity, and V_s^* is the final volume of the solid material, sum of the final volume fraction of material 1, V_1^*, and material 2, V_2^*. The discrete design variable for the eth element, X_{ej}, is defined as follows

$$X_{e1} = \begin{cases} 1 & \text{Solid material} \\ X_{\min} & \text{Void} \end{cases} \tag{11}$$

$$X_{e2} = \begin{cases} 1 & \text{Aluminum} \\ X_{\min} & \text{Rubber-like material} \end{cases} \tag{12}$$

4 MATERIAL INTERPOLATION

The alternative material interpolation proposed by [20], mixed with the SIMP model, is used to interpolate the material properties because it allows us to consider two solid materials and void in the design domain without the presence of localized modes in void regions. Then, the material interpolation of the elemental density, ρ^e, and the elemental elasticity matrix, $\boldsymbol{K}^e$, are given by the following equations:

$$\rho^e\left(X_{e1}, X_{e2}\right) = X_{e1}\left[\left(X_{e2}\right)\rho_1 + \left(1 - X_{e2}\right)\rho_2\right] \tag{13}$$

$$\boldsymbol{K}^e\left(X_{e1}, X_{e2}\right) = \left[\tfrac{X_{\min} - X_{\min}^p}{1 - X_{\min}}\left(1 - X_{e1}^p\right) + X_{e1}^p\right]\left[X_{e2}^p \boldsymbol{K}_1^e + \left(1 - X_{e2}^p\right)\boldsymbol{K}_2^e\right] \tag{14}$$

where p is the penalty exponent, ρ_1 and ρ_2 are the densities of aluminum and rubber-like material, respectively. The elemental damping matrix, $\boldsymbol{C}^e$, is interpolated using the SIMP based interpolation as follow:

$$\boldsymbol{C}^e = X_{e1}\left[X_{e2}\boldsymbol{C}_1 + \left(1 - X_{e2}\right)\boldsymbol{C}_2\right]. \tag{15}$$

where $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are the damping matrices calculated using Equation 3 and the Rayleigh damping parameters of aluminum and the rubber-like material, respectively.

5 SENSITIVITY ANALYSIS

The final topology is obtained using a relative ranking of the sensitivity numbers in the BESO method. The sensitivity numbers are calculated by the differentiation of the objective function with respect to the design variables. These numbers are calculatd using the adjoint variable method (AVM) [11] based on the discretize-then-differentiate approach because it produces consistent sensitivities numbers [17].

In order to calculate the sensitivity numbers, the objective function 9 is discretized in the time domain, as follows:

$$\phi(\boldsymbol{X}, \dot{\boldsymbol{u}}_0, ..., \dot{\boldsymbol{u}}_N) = \sum_{i=1}^{N} E_{d_i}(\boldsymbol{X}, \dot{\boldsymbol{u}}) = \sum_{i=1}^{N} \frac{1}{2}\dot{\boldsymbol{u}}_i^T \boldsymbol{C}\dot{\boldsymbol{u}}_i \tag{16}$$

Then, the residual form of the motion equation and Newmark equations premultiplied by the adjoint variables, $\boldsymbol{\lambda}$, are added to the discretized objective function, ϕ. Finally, the modified objective function, which is already discretized in the time domain, is derivated with respect to each design variable,X_{ej}, of the eth element and the jth interpolation, yielding the following equation:

$$\frac{\partial \phi}{\partial X_{ej}} = \sum_{i=0}^{N}\left(\frac{1}{2}\dot{\boldsymbol{u}}_i^T \frac{\partial \boldsymbol{C}}{\partial X_{ej}}\dot{\boldsymbol{u}}_i\right) + \boldsymbol{\lambda}_0^T \frac{\partial \boldsymbol{M}}{\partial X_{ej}}\ddot{\boldsymbol{u}}_0 + \sum_{i=1}^{N}\left[\boldsymbol{\lambda}_i^T\left(\frac{\partial \boldsymbol{M}}{\partial X_{ej}}\ddot{\boldsymbol{u}}_i + \frac{\partial \boldsymbol{C}}{\partial X_{ej}}\dot{\hat{\boldsymbol{u}}}_i + \frac{\partial \boldsymbol{K}}{\partial X_{ej}}\hat{\boldsymbol{u}}_i\right)\right] \tag{17}$$

for $j = 1, 2$, where $\hat{\boldsymbol{u}}_i$ and $\hat{\dot{\boldsymbol{u}}}_i$ are given by:

$$\hat{\boldsymbol{u}}_i = (1-\alpha)\,\beta h^2 \ddot{\boldsymbol{u}}_i + (1-\alpha)\left(\frac{1}{2}-\beta\right) h^2 \ddot{\boldsymbol{u}}_{i-1} + (1-\alpha)\,h\dot{\boldsymbol{u}}_{i-1} + \boldsymbol{u}_{i-1}, \tag{18}$$

and

$$\hat{\dot{\boldsymbol{u}}}_i = (1-\alpha)\,\gamma h \ddot{\boldsymbol{u}}_i + (1-\alpha)\,(1-\gamma)\,h\ddot{\boldsymbol{u}}_{i-1} + \dot{\boldsymbol{u}}_{i-1}, \tag{19}$$

respectively.

The derivative of the stiffness matrix, the mass matrix and the damping matrix with respect to the design variables of the first, $j = 1$, and the second, $j = 2$, interpolations are given by the following equations:

$$\frac{\partial \boldsymbol{M}}{\partial X_{e1}} = \frac{\partial \rho}{\partial X_{e1}} \boldsymbol{M}_e = A_\rho \boldsymbol{M}^e, \tag{20}$$

$$\frac{\partial \boldsymbol{M}}{\partial X_{e2}} = \frac{\partial \rho}{\partial X_{e2}} \boldsymbol{M}_e = \left[X_{e1}\left(\rho_1 - \rho_2\right)\right] \boldsymbol{M}^e, \tag{21}$$

$$\frac{\partial \boldsymbol{K}}{\partial X_{e1}} = pX_{e1}^{p-1}\left(1 - \frac{x_{\min} - x_{\min}^p}{1 - x_{\min}}\right) \boldsymbol{A}_{\boldsymbol{K}}, \tag{22}$$

$$\frac{\partial \boldsymbol{K}}{\partial X_{e2}} = pX_{e2}^{p-1} A_x \left(\boldsymbol{K}_1^e - \boldsymbol{K}_2^e\right), \tag{23}$$

$$\frac{\partial \boldsymbol{C}}{\partial X_{e2}} = A_\alpha \left[X_{e1} A_\rho \boldsymbol{M}^e + X_{e1} \frac{\partial \boldsymbol{M}}{\partial X_{e1}}\right] + A_\beta A_x \boldsymbol{A}_{\boldsymbol{K}} + A_\beta X_{e1} \frac{\partial \boldsymbol{K}}{\partial X_{e1}}, \quad \text{and} \tag{24}$$

$$\frac{\partial \boldsymbol{C}}{\partial X_{e2}} = X_{e1}^2 \left(\alpha_1^c - \alpha_2^c\right) A_\rho \boldsymbol{M}^e + X_{e1} A_\alpha \frac{\partial \boldsymbol{M}}{\partial X_{e2}} + X_{e1}\left(\beta_1^c - \beta_2^c\right) A_x \boldsymbol{A}_{\boldsymbol{K}} + X_{e1} A_\beta \frac{\partial \boldsymbol{K}}{\partial X_{e2}}, \tag{25}$$

where,

$$A_\alpha = \left[X_{e2}\alpha_1^c + \left(1 - X_{e2}\right)\alpha_2^c\right], \tag{26}$$

$$A_\beta = \left[X_{e2}\beta_1^c + \left(1 - X_{e2}\right)\beta_2^c\right], \tag{27}$$

$$A_\rho = \left[X_{e2}\,\rho_1 + \left(1 - X_{e2}\right)\rho_2\right], \tag{28}$$

$$A_x = \left[\frac{x_{\min} - x_{\min}^p}{1 - x_{\min}}\left(1 - X_{e1}^p\right) + X_{e1}^p\right], \quad \text{and} \tag{29}$$

$$\boldsymbol{A}_{\boldsymbol{K}} = \left[X_{e2}^p \boldsymbol{K}_1^e + \left(1 - X_{e2}^p\right) \boldsymbol{K}_2^e\right] \tag{30}$$

(31)

6 Bi-directional Evolutionaty Structural Optimization procedure (BESO)

BESO method is used to solve the topology optimization problem described in this work. It allows the addition and remotion of material based on the sensitivity numbers and was implemented as proposed by [21], following the next steps:

1. Define the design domain, boundary conditions and finite element mesh.
2. Define the $HHT - \alpha$ parameters.
3. Define the BESO parameters (ER, AR_{max1},AR_{max2}, V_1^*, V_2^*, R_{min}, τ and N).
4. Calculate the nodal displacements, velocities and accelerations usign Equations 4 to 8.
5. Calculate the sensitivity numbers using Equations 17 to 31.
6. Pos-processing the sensitivity numbers.
7. Calculate the next targeted volume for both materials.
8. Update X_{e1} for $e = 1, ..nel$.
9. Update X_{e2} in the elements with $X_{e1} == 1$.
10. Run steps 4 to 9 until the stop criterion and the final volume of both materials are satisfied.
11. The final topology is obtained.

The topology optimization problem is parameter-dependent, where the BESO parameters are set by performing numerical examples. The BESO parameters used in this work are the evolutionary ratio, ER, which represents the volume percentage of aluminum removed from the design domain in each iteration. The maximum addition ratio, AR_{max}, which limits the number of void elements that turn solid elements (AR_{max1}), and the number of elements filled by rubber-like material that turn aluminum (AR_{max2}) in each iteration. The filter radius, R_{min}, is used to avoid checkerboard and mesh-deéndency. The convergence tolerance, τ; and the convergence parameter, N.

7 NUMERICAL APPLICATION

This example uses the bi-clamped beam, presented in Figure 1, to demonstrate the proposed methodology's effectiveness for designing highly damped structures under transient loads. The gray part represents the design domain, surrounded by a fixed domain border. A rectangular step load of 1×10^3 N is applied in the middle of the bottom edge of the structure. The two materials considered are aluminum and a rubber-like material with Young's Modulus $E_1 = 69 \times 10^9\, Pa$ and $E_2 = 22 \times 10^9\, Pa$, Poisson ratio $\nu_1 = 0.3$ and $\nu_2 = 0.49$ and, density $\rho_1 = 2700\, Kg/m^3$ and $\rho_2 = 980\, Kg/m^3$, respectively.

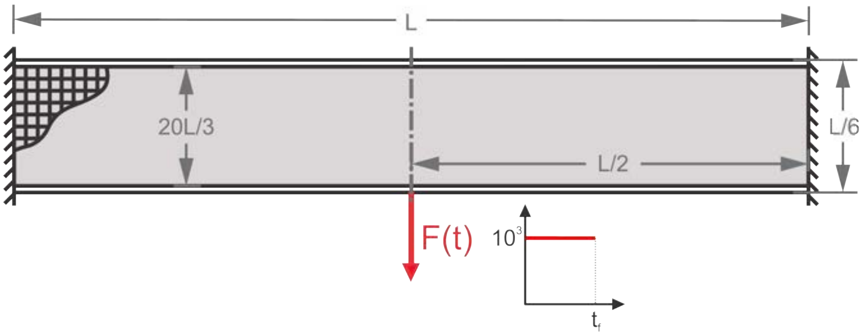
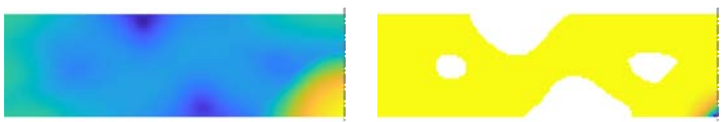


Figure 1: Bi-clamped beam

The right half of the domain is considered by symmetry and discretized into 150×50 bilinear isoparametric four-node plane stress elements. The aluminum and rubber-like material's final volume fractions are 75% and 5%, respectively. The BESO parameters used are an evolutionary ratio of $ER = 1\%$, an addition ratio of $AR_{\text{max1}} = AR_{\text{max2}} = 1\%$, a filter radio of $r_{\text{min}} = 0.15m$, a convergence tolerance of $\tau = 1 \times 10^{-2}$, and a convergence parameter of $N = 5$. The time interval of interest is from zero to $T_f = 0.1\, s$, and it is discretized into 200 constant steps. For the $HHT - \alpha$ method, it is used an $\alpha = 0.05$.

Figure 2(a) presents the final topology, and Figures 2(b) and 2(c) are the sensitivity maps on the last iteration related to the design variables $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$, respectively. The black and pink areas represent the aluminum and the rubber-like material, respectively. Figure 2(a) shows that the algorithm concentrates the rubber-like material near the load point and the supports where the structure deformation is important.

(a)



(b) (c)

Figure 2: Topology optimization results: (a)Final topology; (b) Sensitivity numbers with respect to $\boldsymbol{X}_1$; (c) Sensitivity numbers with respect to $\boldsymbol{X}_2$

Figure 3 shows the history of the objective function and of the material volume along the iterations. The design domain starts full of aluminum. Then, the aluminum is removed from the design domain, while the rubber-like material is added. When the final volume of the rubber-like material is satisfied, the algorithm keeps removing the aluminum until its final volume is satisfied, allowing the void to be part of the domain, as observed in Figure 3. Figure 3 also shows that the variation of the objective function is smooth along the iterations.

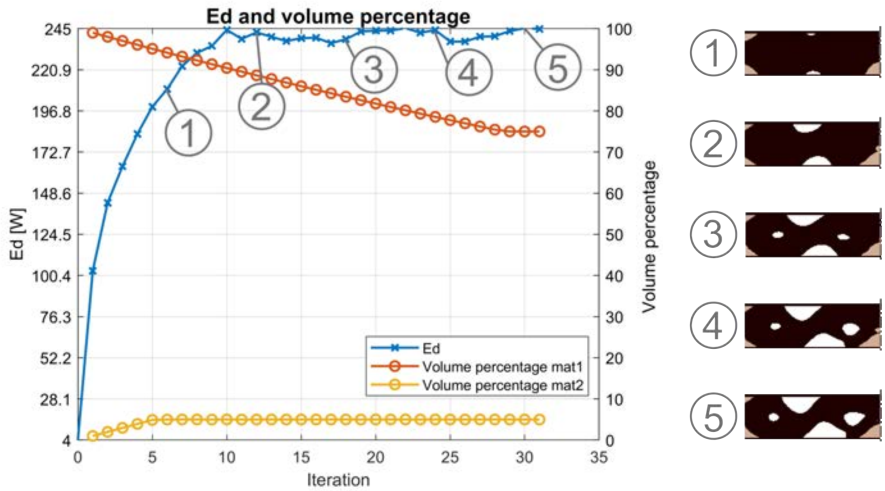
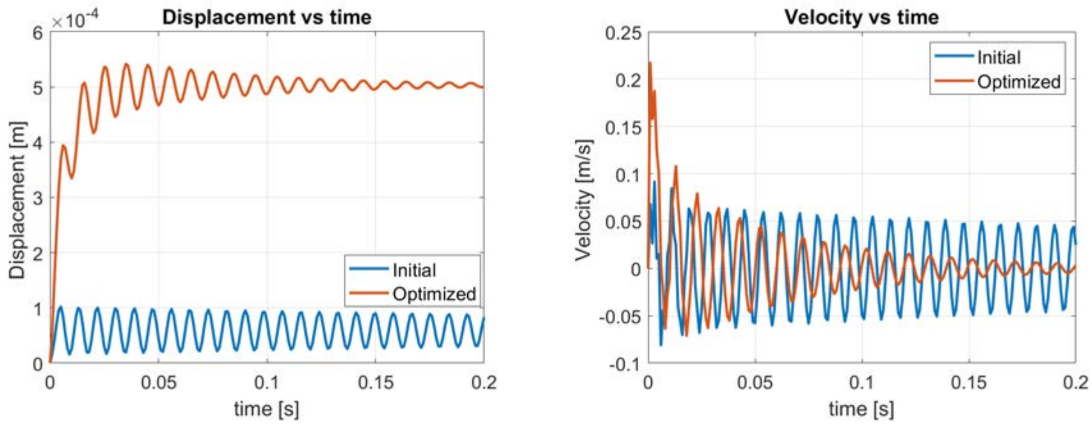


Figure 3: Evolutionary histories of the objective function and volume fraction for the bi-clamped beam using the discretize-then-differentiate approach

Figures 4(a) and 4(b) compare the displacement and velocity of the load point of the initial and final topologies. These figures show that the final topology presents a higher damping behavior but is less rigid when compared with the initial topology. It is expected to have a less rigid final topology because the stiffer material, the aluminum, is removed gradually during the topology optimization adding a rubber-like material and void.



(a) (b)

Figure 4: Displacement and velocity histories of the load point of the initial and final topologies

8 CONCLUSIONS

This work develops a BESO-based methodology to design highly damped structures subjected to arbitrary time-dependent loads. The HHT-α is used to solve the linear elastodynamic problem because it dampens the contribution of high modes in the dynamic response. Two materials and void are considered in the topology optimization process. The sensitivity numbers are calculated using the discretize-then-differentiate approach. The material interpolation considered in this work does not induce artificial modes in the void regions. The results demonstrate the effectiveness of the developed methodology in the design of highly damped structures when considering multiple material phases. The proposed design methodology places the rubber-like material near the supports and the point of the force application where the structure deformation is important.

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