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FLAPPING THIN AIRFOIL INTERACTING WITH THE GROUND

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Abstract: *The rotary wing and the flapping wing technology for the MAVs (Micro Air Vehicle) demand the studies of unsteady aerodynamic effects on the vehicles performing near the ground. In this study, a fast and robust numerical method based on the general formulae for the lift and propulsive forces, acting on a heaving-plunging airfoil, induced by a vortex sheet and its ground image, is developed and implemented. The Keldish-Lavrentiev series expansion of the Kernel function is employed to find the effect of the image vortex sheet on the airfoil. Coupled with the unsteady aerodynamics, the unsteady boundary layer equations cast in the velocity-vorticity formulation are solved numerically to predict the viscous drag on the airfoil performance under the: i) restrained condition where the mean distance to the ground remains the same, and ii) unrestrained condition for which the airfoil ascends or descends freely during its interaction with the ground. Applications of these two different performances with Reynolds number of 3040 and 10 000, which are examples from the current literature, yield satisfactory results.*

1 INTRODUCTION

The aerodynamic performance of any object flying near the ground is affected by its presence. The physics of the problem necessitates a mathematical modeling with the image vortices to satisfy the tangency boundary condition at the ground. The steady state lift coefficient formula is provided by [1] in series form based on the Keldish-Lavrentiev expansion of the Kernel function as described in [2]. In this series expression of the lift, taking the first term only gives additional lift near the ground, however, adding more terms generate reduction in the lift for the cases where the mean distance to the ground is very small.

For the pitching plunging, simple harmonic motion is considered with a phase difference. The unsteady lift during the pitch and plunge is calculated with the concept of reduced circulation based on the integration of the amplitude of the bound vortex sheet strength over the chord. The unsteady boundary layer edge velocity is also calculated from the bound vortex sheet strength and its image. This edge velocity is utilized as the boundary condition for the numerical solution [3] of the unsteady boundary layer equations to obtain the drag force which opposes to the propulsive force.

Here, the interaction of the airfoil with the ground is studied for two different cases. The first case considered is the restrained case where the mean distance to the ground is kept constant in calculating the lift and the propulsion. The second case considers the airfoil to be free to ascend or descend as the result of interacting with the ground. The path of the airfoil, here, is calculated with Runge-Kutta method for the integration of the system of second order ordinary differential equations. At each time step first the aerodynamics is handled to obtain the forces, and these forces are used to determine the trajectory of the motion. The detailed description of the computational procedure is provided in the following sections.

2 FORMULATIONS

The lifting and the propulsive forces of a pitching plunging airfoil near the ground are formulated using the bound and the wake vortex sheets and their images as shown in Fig.1.

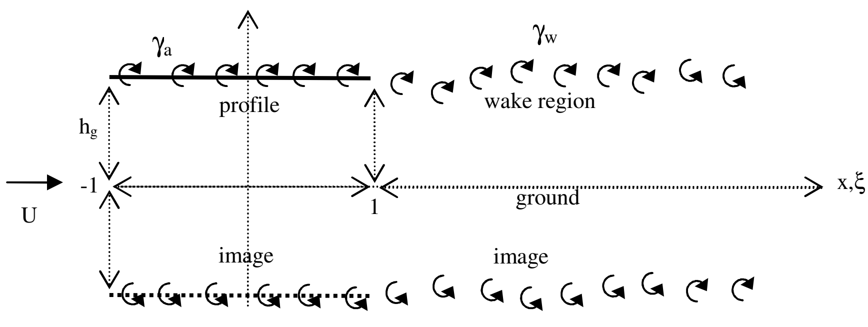

Fig1. Bound and the wake vorticies and their images

The flapping motion is assumed as the simple harmonic pitch and plunge combined in the following form

$$h(t) = h' \cos(\omega t)$$
$$\alpha(t) = \overline{\alpha} + \overline{\alpha}\cos(\omega t + \varphi) \qquad \text{(1-a,b)}$$

In the presence of ground, the distance between the ground and the wing is going to change because of plunge motion as

$$h'_g(t) = h'_o - h'\cos(\omega t) \tag{2}$$

The relation between the downwash w_a and the equation of the motion of the body reads as

$$w_a(x,t) = \frac{\partial z_a}{\partial x'} + U\frac{\partial z_a}{\partial x'} \tag{3}$$

where, $z_a(x,t) = -h(t) - \alpha(t)x'$ for a pitching-plunging thin airfoil which is pitching about its mid chord. For simple harmonic motion, the amplitude of the vortex sheet strength $\bar{\gamma}_a$, with the effect of the image of the bound and the wake vortices according to the Biot-Savart law [4] induces the following downwash amplitude

$$\overline{w}_a(x) - \frac{ik\overline{\Omega}}{2\pi}\int_1^\infty e^{-ik\xi}\left(\frac{1}{x-\xi} - \frac{x-\xi}{(x-\xi)^2+4h_g^2}\right)d\xi = -\frac{1}{2\pi}\int_{-1}^1 \bar{\gamma}_a(\xi,t)\left(\frac{1}{x-\xi} - \frac{x-\xi}{(x-\xi)^2+4h_g^2}\right)d\xi \tag{4}$$

where, $\overline{\Omega} = \overline{\Gamma}_a e^{ik}/b = e^{ik}\int_{-1}^1 \bar{\gamma}_a(x)dx$, is the amplitude of the reduced circulation which contributes to the second term of the integral as the ground effect and b is the half-chord. The integral equation above is inverted with the approach of Keldysh and Lavrentiev, as described in [2], as follows. The kernel of the integral at the right hand is expanded into the Taylor series given below

$$K(x-\xi) = \frac{1}{x-\xi} - \frac{x-\xi}{(x-\xi)^2+4h^2} = \frac{1}{x-\xi} + h^{-1}\sum_{n=0}^\infty K_n\left(\frac{x-\xi}{h}\right)^n \;, \quad K_n = (-1)^{(n+1)/2}2^{-(n+1)},\; n = 1,3,5.... \tag{5}$$

Note that, the same function also acts as the coefficient of the exponent in the second tem of left hand side of (4), which will be expanded into power series as given in (5).
Then the bound vortex sheet strength with two terms becomes

$$\bar{\gamma}_a(x) = \sum_{n=1}^\infty h_g^{-n}\bar{\gamma}_n \cong \bar{\gamma}_o + \bar{\gamma}_2/h_g^2 \tag{5}$$

For $\bar{\gamma}_2$, in terms of $\bar{\gamma}_o$ which is the out of ground effect, we have for the first term in the series as described in [5]. After the inversion of the integral (4) for pitching and plunging motion the full expression for the reduced circulation amplitude becomes

$$\frac{\overline{\Omega}}{U} = \frac{-2\pi\left[\overline{\alpha} + ik(\overline{h} + \overline{\alpha}/2)\right] - \frac{\pi}{2}\left[ik\overline{h} + (1 - \frac{3ik}{4})\overline{\alpha}\right]/h_g^2}{ik\left[C_1(k)\right] + ik\left\{\frac{3}{16}\left(\frac{1}{2k^2} - \frac{1}{4ik}\right)e^{-ik} + \frac{1}{4}\left[\left(-\frac{1}{k^2} + \frac{3}{2ik}\right)e^{-ik} + \frac{3}{2}C_1(k) - C_2(k)\right]\right\}/h_g^2} \tag{6}$$

First terms of the numerator and the denominator of (6) gives the reduced circulation without the presence of ground as hg goes to infinity.

2.1 Lift and propulsive forces

The integration of unsteady lifting pressure coefficient gives the total of circulatory and the non-circulatory lift as follows [5]

$$\begin{aligned}\overline{C}_l &= 2\pi C(k)\left[\overline{\alpha} + ik(\overline{h} + \overline{\alpha}/2)\right] - \pi k^2(\overline{h} + \overline{\alpha}) + \frac{\pi}{2}\left[ik\overline{h} + (1 - \frac{3ik}{4})\overline{\alpha}\right] / h_g^2 \\ &+ ik\frac{\overline{\Omega}}{U}\left\{\frac{3}{16}(\frac{1}{2k^2} - \frac{1}{4ik})e^{-ik} + \frac{1}{4}\left[(-\frac{1}{k^2} + \frac{3}{2ik})e^{-ik} + \frac{3}{2}C_1(k) - C_2(k)\right]\right\} / h_g^2 \\ &- k^2\frac{\overline{\Omega}}{U}\left\{\frac{5}{16}(\frac{1}{k^2} - \frac{1}{2ik})e^{-ik} + \frac{5}{4}(\frac{e^{-ik}}{ik} + C_1(k)) - 2(\frac{e^{-ik}}{k^2} + C_2(k)\right\} / h_g^2\end{aligned} \tag{7}$$

In (7), the second and the last terms represent the contributions of the apparent mass to the lift. The steady state value of the lift coefficient, for k=0, becomes $C_l = 2\pi\overline{\alpha}(1 + 0.25 / h_g^2)$ as expected [1].

The propulsive force due to flapping is generated by the leading edge suction, which is given in terms of the vortex sheet strength evaluated at the leading edge, as follows [6]

$$S = -(\pi\rho\overline{P}^2 + \alpha L), \quad \overline{P} = \lim_{x' \to -b}\left[\overline{\gamma}_a(x')\frac{\sqrt{b + x'}}{2}\right] \tag{8-a,b}$$

In order to calculate P in (8-b), sum of all vortex sheet strengths at the leading edge is obtained as $(\overline{\gamma}_a = U/2\ \overline{C}_{pa})_{LE}$, hence from (8-b) we have

$$\overline{P} = \sqrt{2b}\left\{U\left[C(k)\overline{\alpha} + ikC(k)(\overline{h} + \overline{\alpha}/2) - ik\overline{\alpha}/2\right] + \frac{U}{4h_g^2}\left[\overline{\alpha} + ik(\overline{h} + \overline{\alpha})\right] + \frac{ik\overline{\Omega}}{4\pi h_g^2}\left[(-\frac{3}{4k^2} + \frac{7}{8ik})e^{-ik} + C_1(k) - C_2(k)\right]\right\} \tag{9}$$

We have to note here that the apparent mass term makes no contribution to the leading edge suction force because of the limiting process described in (8-b).

2.2 Drag

The drag force, here, is evaluated from the unsteady boundary layer equations in velocity (u,v) and vorticity, ω, formulation expressed in body attached coordinates x-y [3]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

and

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\frac{\partial^2 \omega}{\partial y^2} \tag{11}$$

wherein, (10-11) are the continuity and the vorticity transport equations, respectively. In addition, the definition of vorticity in the boundary layer reads as

$$\omega = -\frac{\partial u}{\partial y} \tag{12}$$

The skin friction is obtained from the surface vorticity value as follows

$$c_f = -\frac{2}{\mathrm{Re}}\omega_o \tag{13}$$

2.3 Numerical formulation

The boundary layer equations given above are discritized with finite differencing. Accordingly, (11) is discritized on a Cartesian grid with finite steps of Δt in time, Δx and Δy in space. Equation (12) is integrated with the trapezoidal rule to give the edge velocity as follows

$$U_e\big|_i = -\int_0^{\delta} \omega dy\big|_i = -\left[\omega_{i,o}/2 + \sum_{j=1}^{J-1} \omega_{i,j}\right]\Delta y \tag{14}$$

Discretizing (11) in time and x direction with forward differencing and y direction with central differencing, and combaining with (14), we obtain the following matrix equation at each time level in the following manner

$$\begin{bmatrix} 1/2 & 1 & 1 & . & . & . & 1 \\ a_2 & d_2 & c_2 & . & . & . & \\ . & a_3 & d_3 & c_3 & . & . & \\ . & . & . & . & . & . & \\ . & . & . & & a_J & d_J & \end{bmatrix}_i \begin{pmatrix} \omega_{i,o} \\ \omega_{i,1} \\ \omega_{i,2} \\ . \\ \omega_{i,J-1} \end{pmatrix}^n = \begin{pmatrix} -U_e/\Delta y \\ b_2 \\ b_3 \\ . \\ b_J \end{pmatrix}_i^n \tag{15}$$

here, the coefficient matrix is given in terms of the known quantities of velocity and the vorticity. The velocity field at the discrete locations are calculated from the vorticity definition for u and the continuity equation for v as follows:

$$u_{i,j} = -\int_0^{y_j} \omega dy\big|_i = -\left[\sum_{m=1}^{j-1} (\omega_{i,m-1} + \omega_{i,m})/2\right]\Delta y\ ,\ j=2,...,J \tag{16}$$

and

$$v_{i,j}^n = v_{i,j-1}^n - \frac{\Delta y}{2\Delta x}(u_{i,j} + u_{i,j-1} - u_{i-1,j} - u_{i-1,j-1})^n,\ \ j = 2, J \tag{17}$$

3 APPLICATIONS, RESULTS AND DISCUSSIONS

The effect of the ground on a thin airfoil, here, is going to be demonstrated with several applications. First, the steady state case is studied. Afterwards, two different unsteady cases are going to be demonstrated as to determining the ground effect on an airfoil where the mean distance to the ground kept constant with i) only plunging, and with ii) pitching and plunging. Finally, the unrestrained plunging airfoil is free to translate horizontally and vertically with the propulsive and lift force change due to the presence of the ground.

3.1 Steady case

The lift change near the ground under the angle of attack α is given with three terms involving h as [1]

$$C_l = 2\pi\alpha(1 + \frac{1}{4h^2} - \frac{3}{32h^4} - \frac{1}{512h^6}) \tag{18}$$

The second term in (18) has a positive effect on the lift, whereas the last two terms reduces the lift for values of h less than a critical value which makes all three terms to add to zero. This critical value $h_{cr} \approx 0.62$. Hence, two different steady cases are considered. The distance to the ground h<0.62 and h>0.62. Figure 2 shows the first steady case where the airfoil descends because h being less than h_{cr}. Here, y is the vertical distance and v is the vertical velocity with respect to reduced time s=Ut/b.

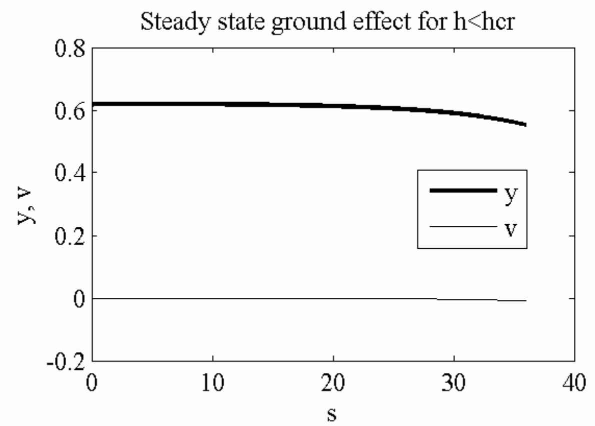

Fig2. Descending airfoil due the ground effect

Shown in Fig.3 is the second case where the ground effect is positive on the lift.

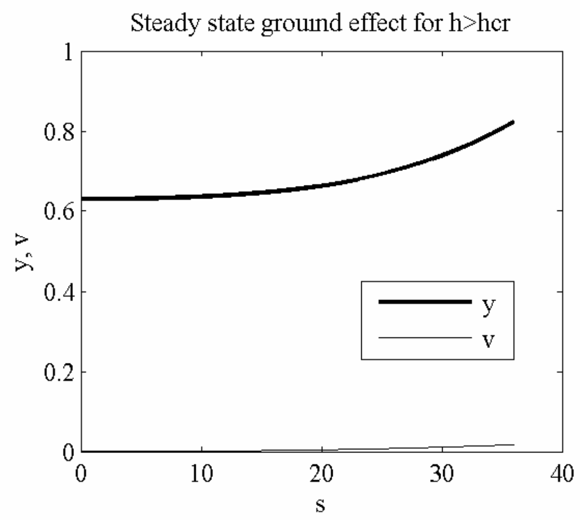

Fig3. Ascending airfoil due the ground effect

3.2 Unsteady restrained case

The mean distance to the ground for this case is kept constant. The solution procedure, here, is as follows:

i) at a given time level n, the lift and the propulsive force are evaluated with (7) and (8-a),
ii) the drag is found with integrating (13) over the top and bottom surfaces,
iii) the net propulsive force is obtained as the difference between the propulsive force and the drag.

Shown in Fig.4 are the variations of the force coefficients for the heaving-plunging airfoil. Here, the reduced frequency value k=ωb/U=0.5 is taken for the airfoil plunging near the ground with $h'_g(t) = 1.1 - 0.4\cos(ks)$ being the variable distance to the ground. The averaged net propulsive force coefficients, obtained with integration of $C_s=(S+D)/(\rho U^2 b)$ over a period of time from Fig.4, are C_T=-0.0431 with the ground effect and C_T=-0.0206 without the ground effect which yields more than 100% increase in trust.

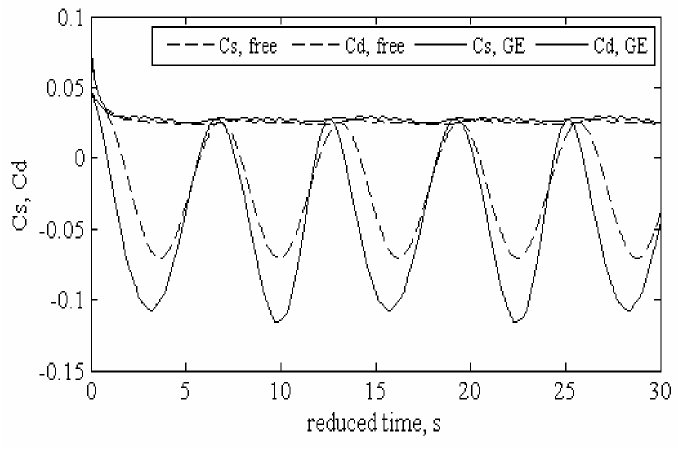

Fig4. Propulsive force coefficient variation with time for k=0.5 and Re=10 000

For the pitching plunging airfoil the following motion is imposed similar to that given in [7]

$$h_g = 1.1 - 0.3\cos(ks)$$
$$\alpha = -10.4^o \cos(ks + 79.9^o)$$

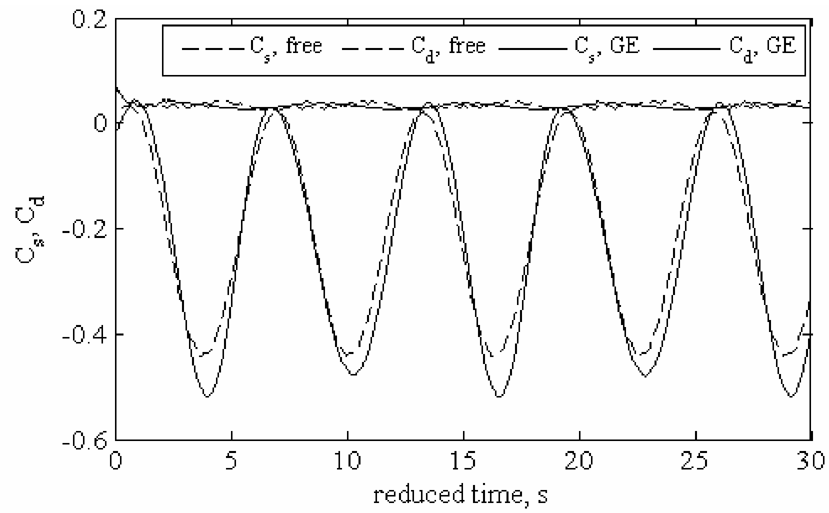

Fig5. Propulsive force coefficient variation for pitch and plunge with time for k=0.5 and Re=10 000

It has been already demonstrated that adding pitch to the plunge with a phase difference increases the propulsive force coefficients [7]. The averaged net propulsive force coefficients for this case are found to be -0.244 with the ground effect and -0.166 without the presence of the ground which causes almost 50% increase.

3.3 Unsteady unrestrained case

Unsteady effect is studied with the parameters of an ornithopter sudy given in [8] for a plunging airfoil with h=y(s)+0.61cos(ks), y(0)=1.1. Here, k=0.5 and the Re=3040 so kh=0.3>0.2 criteria is satisfied. Shown in Fig.6 are the vertical y, and the horizontal x, displacement of the airfoil versus the reduced frequency s where at each time step the position of the airfoil is determined using the Runge-Kutta method for the following system of ODE written in non-dimensional coordinates

$$\begin{aligned} \frac{d^2x}{ds^2} &= C_s(s)\frac{b}{U^2}g, \quad x(0)=0 \\ \frac{d^2y}{ds^2} &= \Delta C_l(s)\frac{b}{U^2}g, \quad y(0)=1.1 \end{aligned} \qquad (19\text{-a,b})$$

During calculation of horizontal displacement, only the contribution coming from the acceleration caused by the thrust is considered. Here, g is the acceleration due to gravity.

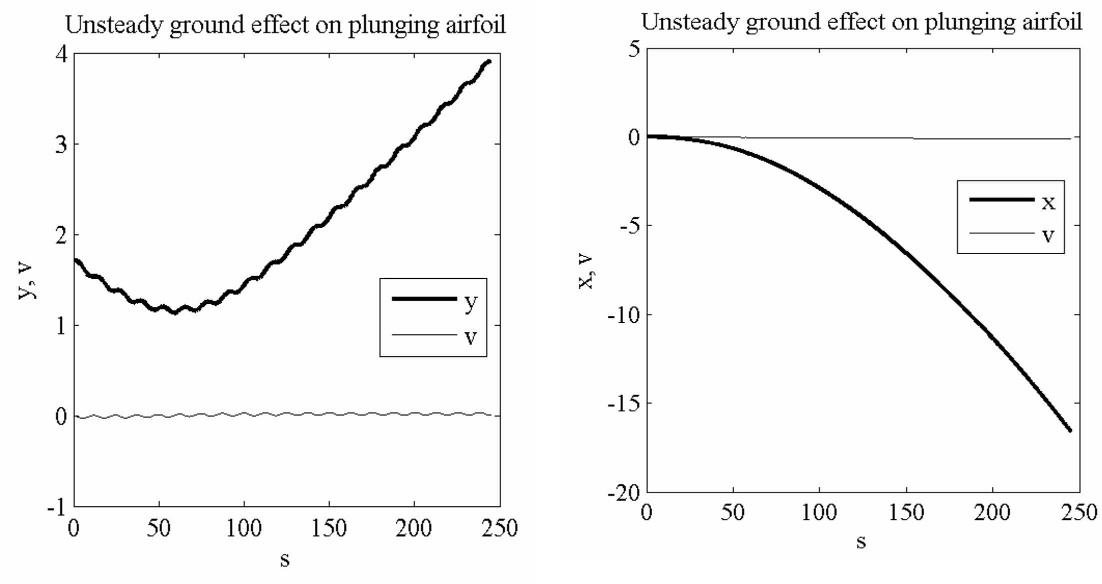

Fig6. Left, vertical y, and right horizontal displacement x and their rates v under the unsteady ground effect

Figure 7 shows the unrestrained actual path of the airfoil under the influence of the ground which is obtained by solving (19-a,b) with variable force coefficients determined simultaneously using the solution procedure described in **section 3.2.** Here, the horizontal distance covered includes the contribution of the free stream together with the acceleration due to thrust.

The computational time does not exceed a few seconds for more than a 12 000 time steps for the boundary layer solutions coupled with the path determination through solution of system of ODEs all performed with a MATLAB [9] code.

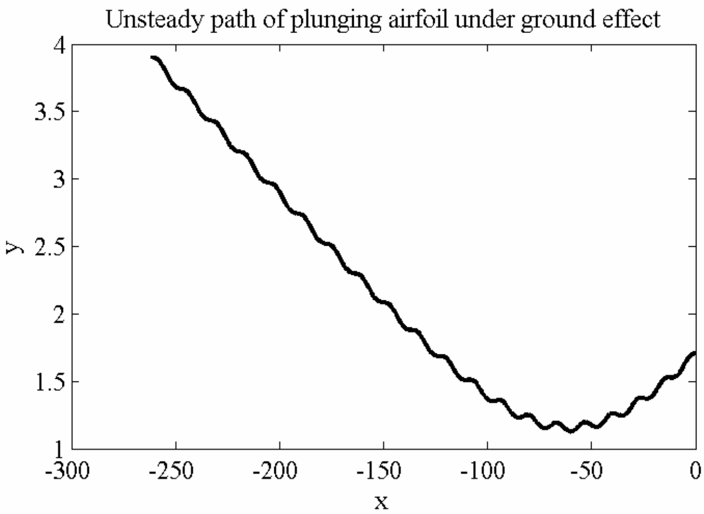

Fig.7 Trajectory of motion under the unsteady ground effect

4 CONCLUSIONS

The effect of the presence of ground on the airfoils, with emphasis on flapping, is analyzed with vortex sheets and their images coupled with unsteady boundary layer for predicting the viscous drag.

For the steady case, there is a critical value of the distance to the ground below that value the airfoil undergoes a descending motion rather than an ascending one.

The flapping airfoil, for the motion where the mean distance to the ground kept constant, produces considerably more thrust when pitch added to the plunge with a phase difference, as given in literature.

The unrestrained airfoil with plunging only, gains small altitude because of flapping under the influence of ground.

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