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APPLICATION OF THE EFMM TO FLUID-STRUCTURE COUPLED ANALYSIS AND ITS PARALLELIZATION METHOD

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Abstract. *There is a new fluid-structure coupled analysis method that combines a structure analysis method using Enriched Free Mesh Method (EFMM); a type of meshless analysis methods and a fluid analysis method using SUPG/PSPG stabilized FEM. in this study, solutions for problems during the parallelization of the abovementioned method are described. In EFMM, it is relatively easy to parallelize problems which do not require remeshing such as static analyses. Application of EFMM, on the other hand, becomes challenging on the problems needs remeshing due to issues in analysis algorithm of EFMM. Nevertheless, parallelization is essential in the fluid-structure coupled analysis. There fore, we propose a method for parallelization of EFMM in this study.*

1 INTRODUCTION

Recently, as the popularity of numerical analysis is rising along advancement in computer performance. Especially, Finite Element Method (FEM)[1] that is the most famous numerical analysis method is applying into a lot of fields.

Variety of different phenomena affecting each other is called coupled phenomenon in the numerical analysis field[2]. Majority of phenomena occurring in our bodies are coupled phenomena. However, only a few number s of coupled analysis have been performed, compared with single phenomenon cases.

In the management of coupled phenomena, various unsolved problems are left, compared to the cases of single phenomena comparison. The reason for this is that these problems are not able to be solved solely by improvement in calculators, but rather they have close relationships with their analysis algorithm.

The phenomenon that fluid and structure are affecting each other is called fluid-structure coupled phenomenon. Recently, fluid-structure coupled phenomenon is becoming a very important issue in a lot of field.

2 PURPOSES

The new fluid-structure coupled analysis method that is already proposed by us is using Enriched Free Mesh Method (EFMM)[3] and SUPG/PSPG stabilized FEM[4-7]. These methods are used linear elements. In short, nodes on boundary of fluid analysis field and structure analysis field can be consistent completely.

As a result, we were able to find solutions for the problems during considering coupling effects at the fluid-structure interface that is important in facing fluid-structure problems.

Moreover, when conducting numeric computations using a finite element method, generally highly accurate elements with intermediate nodes are used, but in our proposed method, only linear elements are used at the fields of fluid analysis and structure analysis to conduct mesh generation as described previously. Although highly accurate elements were not used, accuracy of the analyses of this method improved relative to the conventional analysis using only linear elements. This result indicates a possibility of reduction in the calculation resources and computation volume that are disadvantages during management of large-scale problems.

Thus, our proposed method is a very effective method. Because, the new fluid-structure coupled analysis method that is combining EFMM and FEM can resolve a lot of problems of coupled analysis.

However, we found a major problem in this proposed method when conducting parallelization that is essential for large-scale analysis in recent years. This is caused by analysis algorithm of EFMM. Additionally, in numerical analysis using EFMM, application of the conventional domain decomposition method based on elements is challenging. Moreover, due to the abovementioned problems, increase of communication volume between each processor that is a critical problem in parallelization occurs, resulting in reduction of analysis efficiency and parallelization efficiency.

Therefore, in the present study, to apply the fluid-structure coupled analysis method combining previously proposed EFMM and SUPG/PSPG stabilized FEM to parallelization analysis, procedures that solve problems during parallelization of the structural analysis method are proposed.

3 FLUID-STRUCTURE COUPLED ANALYSIS METHOD

In this chapter, we discuss a new fluid-structure coupled analysis method combining previously proposed EFMM and SUPG/PSPG stabilized FEM.

When conducting analysis considering fluid-structure interaction effects, it is desirable that node locations are consistent on the interface between fluid and structure domain as shown in Fig. 1. As a reason, if fluid-structure interface is inconsistent when considering the coupling effects of two different fields, it is necessary to interpolate the analysis results between the nodes. Algorithm of fluid-structure coupled analysis becomes complicated by interpolation process. Moreover, analysis accuracy will be decreased.

Since linear tetrahedral elements are used in both EFMM and SUPG/PSPG stabilized FEM for their analysis, integrity at fields of fluid analysis and structure analysis can be obtained. In addition, these analysis methods have features that the accuracy of the analysis will be improved relative to the conventional analysis solely using linear elements.

As an example of application of this method on the biological field, we present the simulation results of capillary blood flow-derived erythrocyte deformation and changes in flow at the fluid field that accompanies the deformation. Fig 3 is an analysis model that is used in this numerical example.

In this analysis, membrane of red blood cell is using as a structure analysis model. On the other hand, capillary is using as a fluid analysis model. Fig. 4 and Fig. 5 are analysis result of this numerical example. Fig. 4 shows a changing in the blood blow in the capillary by change shape of membrane of red blood cell. Fig. 5 shows a changing shape of membrane of red blood cell by blood flow.

In blood vessels with smaller diameter than that of erythrocytes such as capillaries, erythrocytes deform themselves with taking forms of parachutes as they travel[8][9]. In this numerical analysis, this phenomenon is well computed.

Especially in the results of the structure analysis, the highest pressure was observed at the center of erythrocytes on the upstream side. The pressure makes large depression at the center of the erythrocyte membrane on the upstream side, taking forms of parachutes as with erythrocytes observed in our body.

From these analysis result, we can obtain fine analysis result computed by our proposed method that is combining EFMM and SUPG/PSPG stabilized FEM compared with phenomenon that is observed in our body.

4 PARALLEL EFMM

Our new fluid-structure coupled analysis method can be possible to obtain fine analysis result.

Although the coupled analysis using combination of EFMM and SUGP/PSPG stabilized FEM showed excellent performance when conduction single process without parallelization, several problems are found once parallelization is attempted.

The greatest source of the problems during parallelization is found in the analysis algorithm of EFMM. The analysis algorithm of EFMM is described in the next section.

4.1 Fundamental concept of EFMM

EFMM is based on FMM (Free Mesh Method)[10][11], which is one of mesh-less method. The most important feature of FMM is that it requires only the coordinate data of each node in analysis domains as the input information. Based on the given coordinate data of nodes, a lo-

cal elements cluster is created at each node. To produce such a local elements cluster, there are a variety of methods including the diagonal comparison method, the packaging method, and the method for developing a Delaunay triangle[12][13] for each central node based on the planar relative relation between a Voronoi polygon and a Delaunay triangle.

Here, the node located at the center of a local elements cluster is called the central node, while nodes located at the edge of the elements cluster are called the satellite nodes(see Fig. 6).

In EFMM analysis, stiffness matrix is calculated by different way with conventional FEM and FMM using this local elements cluster.

In the next section, formulation of EFMM will be described.

4.2 Formulation of EFMM

In EFMM, the displacement and the strain fields are respectively, assumed in independent locations, and these two fields are linked with the Hellinger-Reissner principle[14](see Fig. 7). The principle, in which displacement u and strain ε are assumed to be independent variables, is given by

$$\Pi(\varepsilon,u)=\int_{\Omega}\{\varepsilon\}^{T}[D]\{\partial u\}d\Omega-\frac{1}{2}\int_{\Omega}\{\varepsilon\}^{T}[D]\{\varepsilon\}d\Omega-\int_{\Omega}\{u\}^{T}\{b\}d\Omega-\int_{S_{\sigma}}\{u\}^{T}\{\tilde{t}\}dS \tag{1}$$

with

$$\{\partial u\}=[B]\{\overline{u}\},\ \{u\}=[N^{u}]\{\overline{u}\},\ \{\varepsilon\}=[N^{\varepsilon}]\{\overline{\varepsilon}\} \tag{2}$$

Where $\{\overline{u}\}$ represents the nodal displacement, $\{\overline{\varepsilon}\}$ the unknown parameter of strain, $\{b\}$ the body force, $\{\tilde{t}\}$ the surface force on the boundary S_{σ}, and Ω the analysis domain. $[N_{\varepsilon}]$ is an arbitrary function that determines the strain of a local region, which can be is assumed as

$$[N^{\varepsilon}]=\begin{bmatrix} p^{T} & 0 & 0 & 0 & 0 & 0 \\ 0 & p^{T} & 0 & 0 & 0 & 0 \\ 0 & 0 & p^{T} & 0 & 0 & 0 \\ 0 & 0 & 0 & p^{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & p^{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & p^{T} \end{bmatrix} \tag{3}$$

where

$$p^{T}=\begin{bmatrix}1 & x & y & z\end{bmatrix} \tag{4}$$

The stationary condition of Eq.(1) is expressed by

$$\int_{\Omega}\delta\{\varepsilon\}^{T}[D]\left([B]\{\overline{u}\}-[N^{\varepsilon}]\{\overline{\varepsilon}\}\right)d\Omega=0 \tag{5}$$

and

$$\int_{\Omega} \delta\{u\}^T [B]^T [D][N^{\varepsilon}]\{\bar{\varepsilon}\} d\Omega - \int_{\Omega} \delta\{u\}^T \{b\} d\Omega - \int_{S_\sigma} \delta\{u\}^T \{\tilde{t}\} dS = 0 \tag{6}$$

These equations are written as follows,

$$\begin{bmatrix} -A & C \\ C^T & 0 \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon} \\ \bar{u} \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \tag{7}$$

where

$$\begin{cases} A = \int_{\Omega} [N^{\varepsilon}]^T [D][N^{\varepsilon}] d\Omega \\ C = \int_{\Omega} [N^{\varepsilon}]^T [D][B] d\Omega \\ f_1 = 0 \\ f_2 = \int_{\Omega} [N^u]^T \{b\} d\Omega + \int_{\Gamma} [N^u]^T \{\tilde{t}\} d\Gamma \end{cases} \tag{8}$$

Next, Eq. (7) is condensed to the following equation:

$$C^T (A^{-1} C \bar{u}) = f_2 \tag{9}$$

Finally, the local enriched stiffness matrix is derived as

$$[k_{HR}] = C^T A^{-1} C \tag{10}$$

Above enriched stiffness matrix is expected to give more accurate solutions than the usual FEM or FMM with the linear displacement base.

This is a formulation of EFMM.

4.3 Treatment for Parallel EFMM

Not every single case using EFMM faces difficulty in parallelization. Specifically, it is possible to conduct parallelization process without a major problem in the cases of static analyses that do not require mesh regeneration in general. In that case, the parallelization CG method was solved in high speed by conducting the local element cluster-by-local element cluster method based on the conventional element-by-element method for EFMM that carries on the process by each local element.

On the other hand, in the cases of adaptive problems that require mesh regeneration, parallelization process becomes difficult due to the feature of analysis algorithm in EFMM.

The intended problems are fluid-structure coupled problems. In a majority of the fluid-structure coupled problems, the mesh form changes over time because of the interaction between fluid and structure. In addition, due to increase in the number of analysis fields, the number of mesh for the analysis also rises substantially. Thus, implementation of parallelization is absolutely essential.

The reason for difficulty in application of parallel EFMM on adaptive analysis is that all nodes in the entire analysis field create a local elements cluster around the nodes (see Fig.6). Generating stiffness matrices by each local element cluster and adding the stiffness matrices generated by each node and the entire stiffness matrix, it is possible to obtain the stiffness matrix of the entire analysis field.

Since stiffness matrices are generated by each node, about 100% of the parallelization efficiency can be achieved in generation of the local stiffness matrix. Fig. 8 indicates changes in the parallelization efficiency when parallelization process was added to generation process of the stiffness matrix by EFMM for certain analysis model.

As can be seen, since this is the analysis model based on nodes, it is possible to obtain high levels of parallelization efficiency in the cases of application of EFMM on parallelization analysis.

Nevertheless, since EFMM is a special analysis method based on nodes, the coordinate data of nodes outside of the analysis field depending on the local stiffness matrix generated by nodes in the analysis field are required in addition to coordinate date of nodes in its assigned field when conducting element based field separation. As a result, when coordinate values of nodes changes with analysis such as adaptive analysis, coordinate values of the moved nodes should be transmitted between each field along with the changes. This causes a large amount of the communication cost since this communication was required in each analysis step.

Therefore, in this study, domain decomposition based on elements is conducted on EFMM (see Fig. 9) and using nodes solely in the field, the local elements cluster is generated to develop a stiffness matrix.

This method eliminates data communication of coordinate values at each node even after remeshing. On the other hand, although this is an analysis method based on nodes, local elements clusters are generated in that identical nodes serve as central nodes in multiple fields. The local elements clusters having an identical node as a central node in multiple fields. The local elements clusters having an identical node as a central node in multiple fields can cover the domain integration, but static condensing that is essential for stiffness matrix process is not performed precisely. Resultantly, an inaccurate stiffness matrix is generated, causing the reduction of analysis accuracy.

Therefore, we performed evaluation regarding the relationship between analysis accuracy of EFMM with the field process based on elements and the number of the parallelization processor. Cantilever model like a Fig. 10 is used in this numerical example. And, example of domain decomposition is shown in Fig. 11. Since the entire stiffness matrix obtained by this method becomes an asymmetric matrix rather than a symmetric matrix that is obtained in original EFMM analysis, a solver that can manage asymmetric matrices such as GPBi-CG method will be needed.

In Fig. 12, x-axis means number of processors, y-axis means normalized displacement. As can be seen in Fig.11, analysis accuracy decreases in inverse proportion to the number of parallelization processors. When field separation is performed, inaccurate local elements clusters are generated near the boundary. The number of the inaccurate local elements clusters increases as the number of analysis separation rises because of the increase of boundaries between each field. Resultantly, inaccurate element stiffness matrices were largely computed, lowering the analysis accuracy.

Nevertheless, based on the tendency of the analysis results, it is suggested that the difference can be miniscule if the number of nodes in the entire analysis field are high. Relative to the enormous increase of the communication cost at the time of accurate parallelization of EFMM based on nodes, this method showed a great benefit.

5 CONCLUSIONS

- In this study, solutions for various problems during the parallelization of fluid-structure coupled analysis using EFMM are proposed.
- The proposed method in this study showed decreasing tendency in the analysis results since this method obtains a different type of a stiffness matrix from the one supposed to be obtained originally. Especially as the number of domain decomposition increases, the abovementioned tendency became prominent from the examples of the numerical analysis. However, it was also revealed that by preparing a large enough number of nodes for analysis, the numerical errors can be minute scales. This method is considered to have a great benefit in terms of the communication cost from accurate parallelization of EFMM.
- When conducting domain decomposition, generation of inaccurate local elements clusters should be minimized as much as possible. To parallelize EFMM, a new domain decomposition method based on nodes should be developed since it is an analysis method based on nodes originally. On the other hand, since a conventional domain decomposition method based on elements can be applied in this method, the communication volume at the analysis as well as inaccurate local elements clusters can be minimized.
- Here are research topics for future studies.

  -Detailed evaluation of parallelization efficiency.

  -Evaluation of analysis accuracy when using a large number of processors.

  -Implementation of the large-scale parallelization fluid-structure coupled analysis combining EFMM applying this method and SUPG/PSPG stabilized FEM.

6 FIGURES

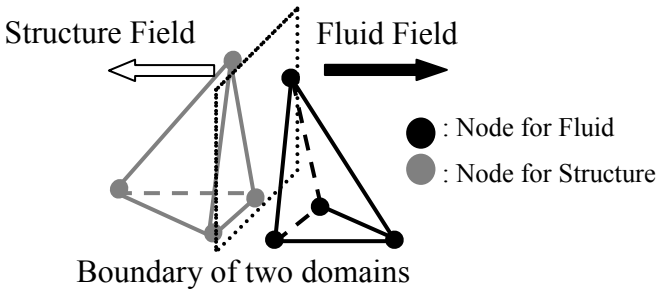

Fig. 1 Boundary of two types of analysis field retaining element formation consistent to each other

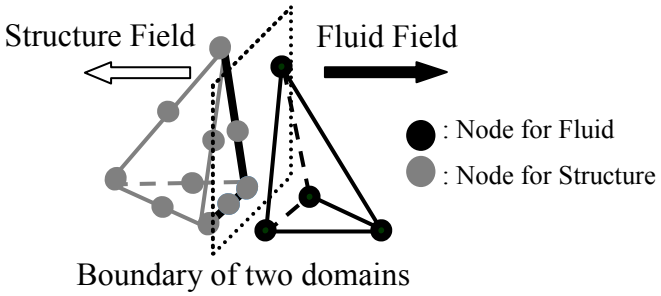

Fig. 2 Boundary of two types of analysis field retaining elements formation inconsistent to each other incompatible boundary

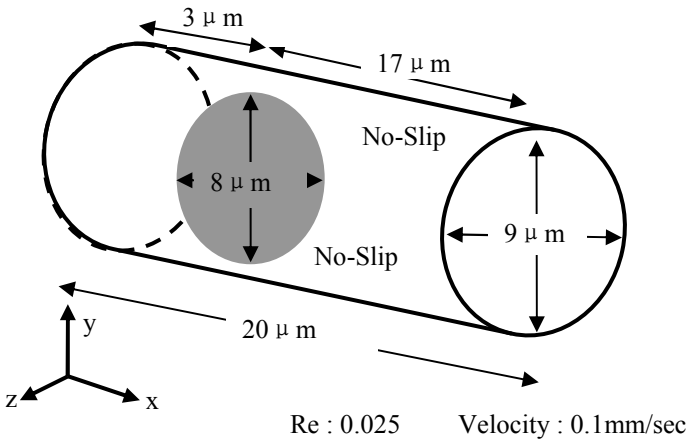

Fig. 3 Analysis model

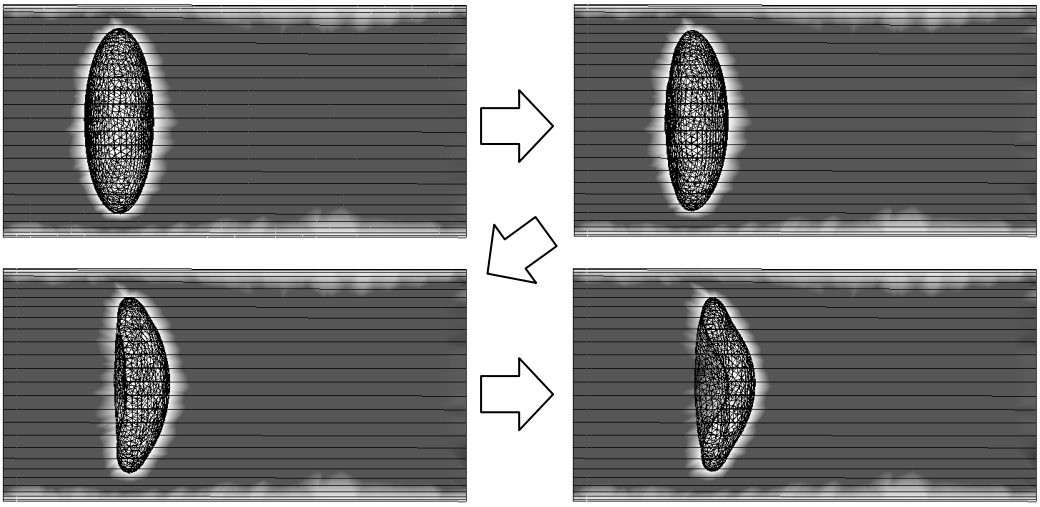

Fig. 4 Fluid analysis result

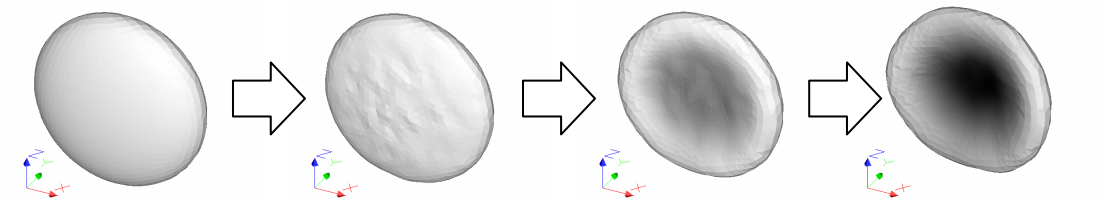

Fig. 5 Structure analysis result

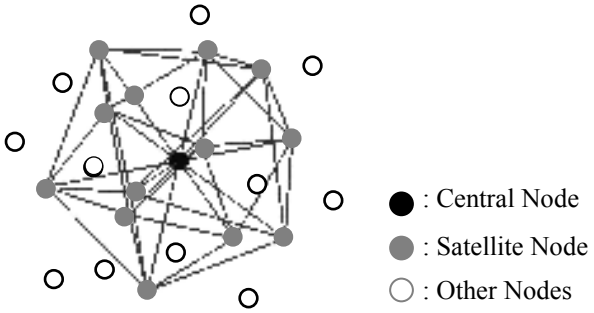

Fig. 6 Fundamental concept of local elements cluster

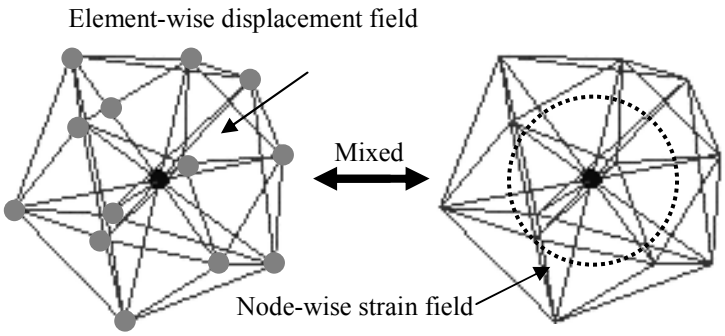

Fig. 7 Two types of field postulated in local elements cluster

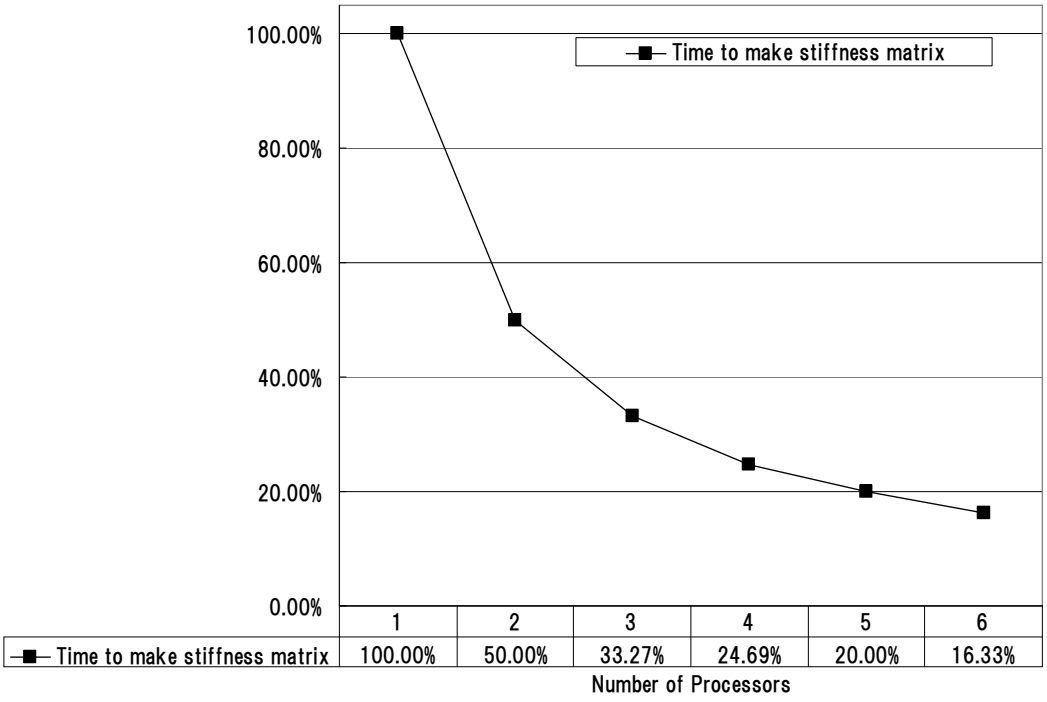

Fig. 8 Relationship between number of processors and time to make stiffness matrix

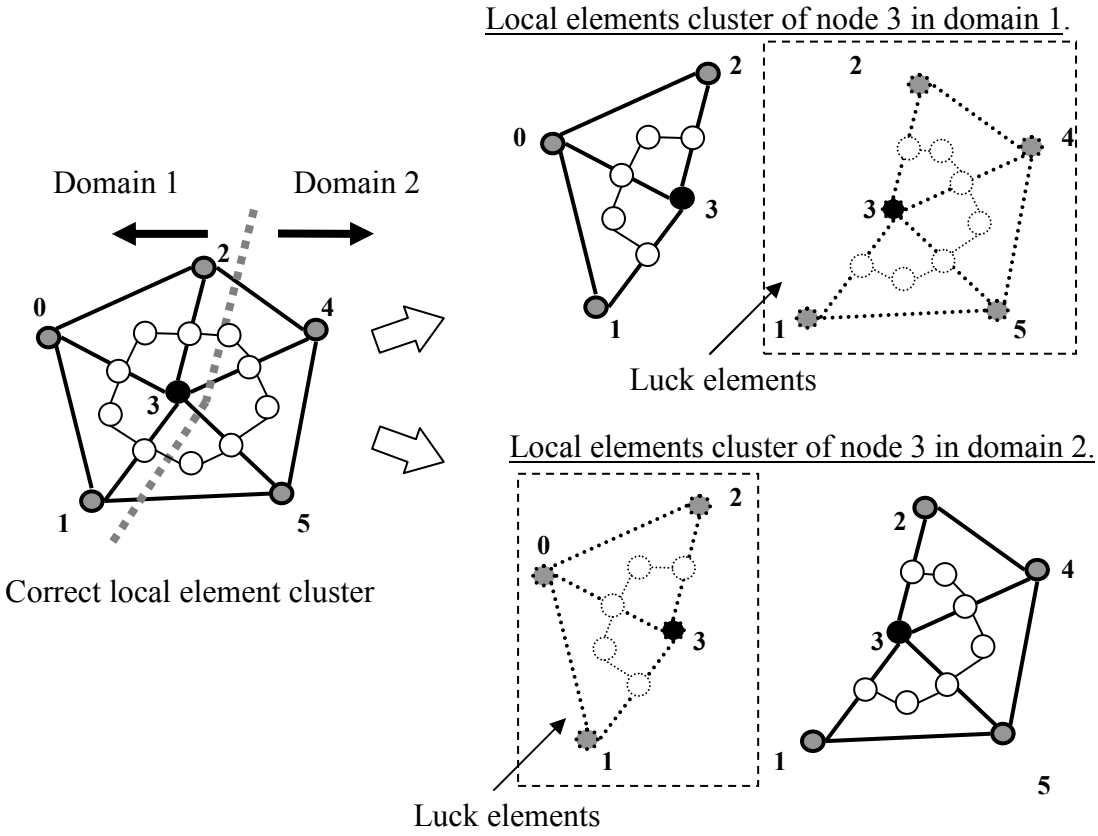

Fig. 9 Example of domain decomposition

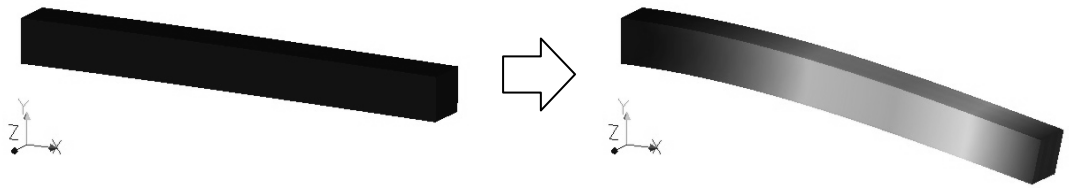

Fig. 10 Cantilever model

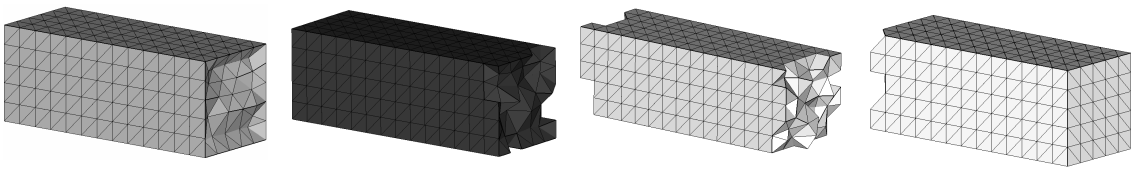

Fig. 11 Example of domain decomposition

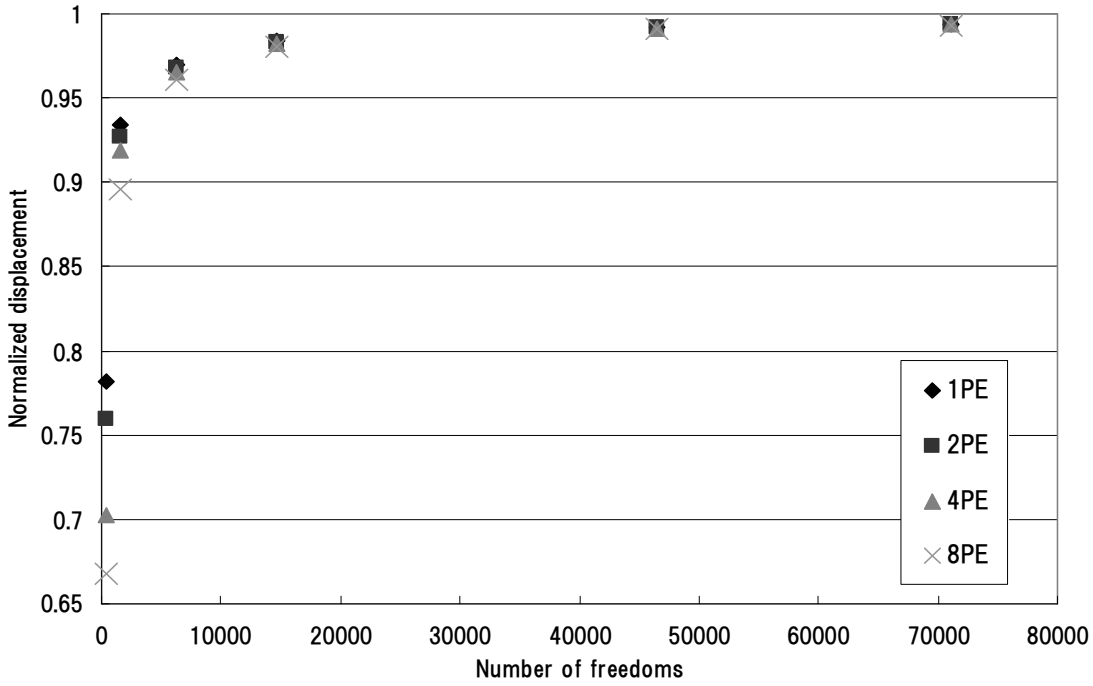

Fig. 12 Relationship between analysis accuracy and number of processors

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