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DYNAMIC BEHAVIOUR OF LAMINATED PLATES SUBJECTED TO THERMOMECHANICAL LOADS

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Abstract. *The engineering structures may be subjected to the dynamic loads such as blast. Therefore, in the design of such structures, the effect of the blast loading should be taken into account. There are two possible effects of the blast: sudden pressure and the temperature rise. In this study, a closed form solution is presented for the thermomechanical transient analysis of the simply supported laminated composite plates subjected to blast loading. In-plane stiffness and inertia effects are considered in the formulation of the problem and transverse shear stresses are ignored. The geometric nonlinearity effects are taken into account by using the von Karman large deflection theory of thin plates. Approximate solution functions are assumed for the space domain and substituted into the equations of motion. The Galerkin method is used to obtain the nonlinear differential equations in the time domain. The finite difference method is applied to solve the system of coupled nonlinear equations. The displacement-time and strain-time histories are obtained for critical cases and compared the isothermal condition. The method presented here can be used for the dynamic response analysis of laminated plates in preliminary design.*

1 INTRODUCTION

The composite plate structures are used in many engineering applications such as aircraft structures, automobiles, space vehicles, wind turbines and a wide range of defense industry to build the lightweight components and vehicles. Moreover, predicting the dynamic response of plates subjected to time dependent loads is very important for the more reliable design process. For instance, the engineering structures may be subjected to the blast load. Therefore, in the design of such structures, the effect of the blast loading should be taken into account. There are two possible effects of the blast: sudden pressure and the temperature rise. In wind turbines, the blades may be subjected to the wind and thermal effects at the same time.

There are several studies found on both linear and nonlinear analysis of isotropic and laminated composite flat plates subjected to air blast loading [1-7]. Susler et al. investigated nonlinear dynamic response of tapered laminated composite plate [8] and tapered sandwich plate [9] along the thickness subjected to blast load. All these previous studies were considered in the absence of temperature effects. There are also previous studies about transient analysis of laminated composite plates subjected to thermomechanical loads [10-12].

In this study, a closed form solution is presented for the thermomechanical transient analysis of the simply supported laminated composite plates subjected to blast loading. In-plane stiffness and inertia effects are considered in the formulation of the problem and transverse shear stresses are ignored. The geometric nonlinearity effects are taken into account by using the von Karman large deflection theory of thin plates. Approximate solution functions are assumed for the space domain and substituted into the equations of motion. The Galerkin method is used to obtain the nonlinear differential equations in the time domain. The finite difference method is applied to solve the system of coupled nonlinear equations. The displacement-time and strain-time histories are obtained for some critical cases and compared with the results of isothermal blast loaded conditions. The method presented here can be used for the dynamic response analysis of laminated plates subjected to thermomechanical loads in preliminary design.

2 FORMULATION OF THE PROBLEM

In this section, a mathematical model is presented for the simply supported laminated composite plate subjected to combined loading of the air blast and temperature rise. The rectangular plate with the length a and the width b is depicted in Figure 1.

The strain-displacement relations for the von Kármán plate theory is defined for the isothermal case in Equation (1). If there is a temperature increment from a reference state ($\Delta T= T-T_{ref}$) which will affect the plate, thermal strains will be occurred and this leads to a different form of stress-strain relations shown in Equation (2).

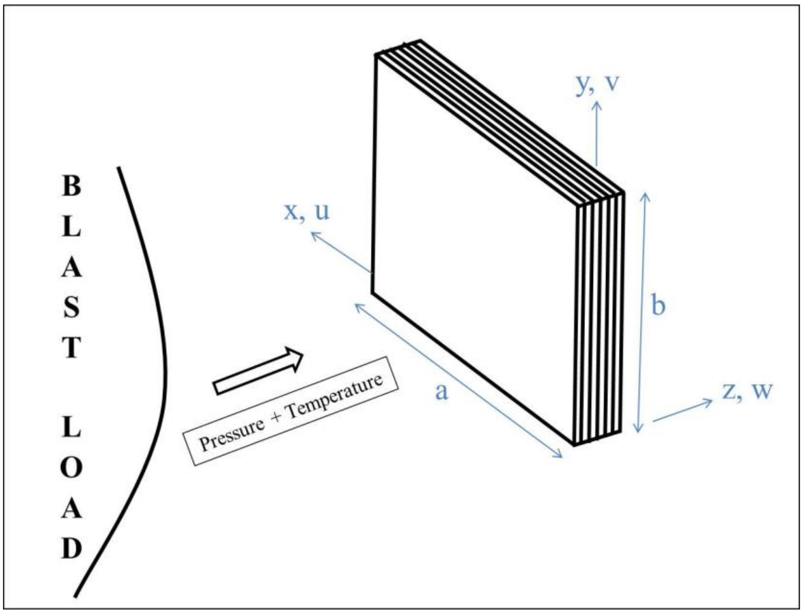

Figure 1: The schematic view of the plate subjected to the blast load.

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 + z\kappa_x = \frac{\partial u^0}{\partial x} + \frac{1}{2}\left(\frac{\partial w^0}{\partial x}\right)^2 - z\frac{\partial^2 w^0}{\partial x^2} \\
\varepsilon_y &= \varepsilon_y^0 + z\kappa_y = \frac{\partial v^0}{\partial y} + \frac{1}{2}\left(\frac{\partial w^0}{\partial y}\right)^2 - z\frac{\partial^2 w^0}{\partial y^2} \\
\varepsilon_{xy} &= \varepsilon_{xy}^0 + z\kappa_{xy} = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \frac{\partial w^0}{\partial x}\frac{\partial w^0}{\partial y} - 2z\frac{\partial^2 w^0}{\partial x \partial y}
\end{aligned} \tag{1}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \varepsilon_{xy} - 2\alpha_{xy}\Delta T \end{Bmatrix} \tag{2}$$

In the constitutive equations, α_x, α_y and α_{xy} are transformed thermal coefficients of expansion and are expressed in Equation (3) by using longitudinal (α_1) and transverse (α_2) thermal coefficients of expansion of a lamina. α_1 and α_2 are defined as engineering constants of required material.

$$\begin{aligned}
\alpha_x &= \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta \\
\alpha_y &= \alpha_1 \sin^2\theta + \alpha_2 \cos^2\theta \\
2\alpha_{xy} &= 2(\alpha_1 - \alpha_2)\sin\theta\cos\theta
\end{aligned} \tag{3}$$

The stresses have linear variation through the thickness of each layer. If it is assumed that the temperature increment varies linearly consistently with the mechanical strains, ΔT will be expressed as [13]:

$$\Delta T = T_0(x, y, t) + zT_1(x, y, t) \tag{4}$$

The membrane strains and curvatures for a nonisothermal problem will be transformed into Equation (5). According to the constitutive relations of a laminated composite plate and using

Equations (4) and (5), the force and moment resultants in a compact form for the nonisothermal case are written in Equation (6).

$$\left\{\varepsilon^0\right\}=\begin{Bmatrix}\varepsilon_x^0-\alpha_x T_0(x,y,t)\\ \varepsilon_y^0-\alpha_y T_0(x,y,t)\\ \varepsilon_{xy}^0-2\alpha_{xy}T_0(x,y,t)\end{Bmatrix}\quad,\quad \left\{\kappa\right\}=\begin{Bmatrix}\kappa_x-\alpha_x T_1(x,y,t)\\ \kappa_y-\alpha_y T_1(x,y,t)\\ \kappa_{xy}-2\alpha_{xy}T_1(x,y,t)\end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix}N\\ M\end{Bmatrix}=\begin{bmatrix}A & B\\ B & D\end{bmatrix}\begin{Bmatrix}\varepsilon_0\\ \kappa\end{Bmatrix}-\begin{Bmatrix}\left\{N^T\right\}\\ \left\{M^T\right\}\end{Bmatrix} \tag{6}$$

where the thermal force and moment resultants are given in Equation (7) [13]. A_{ij}, B_{ij} and D_{ij} are the extensional, coupling and bending stiffness matrices.

$$\left\{N^T\right\}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\left[\bar{Q}\right]^{(k)}\left\{\bar{\alpha}\right\}^{(k)}\Delta T dz\quad,\quad \left\{M^T\right\}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\left[\bar{Q}\right]^{(k)}\left\{\bar{\alpha}\right\}^{(k)}\Delta T z dz \tag{7}$$

If an assumption is made that all layers have the same engineering constants and the same orientation, the thermal force and moment resultants have the same stiffness matrices as the mechanical resultants. Equation (8) will have the following open form and thermal force and moment resultants will be described as Equation (9). The superscript $(\ )^T$ indicates thermal expressions.

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{Bmatrix}=\begin{pmatrix}A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16}\\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26}\\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66}\\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16}\\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26}\\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66}\end{pmatrix}\begin{Bmatrix}\varepsilon_x^0-\alpha_x T_0(x,y,t)\\ \varepsilon_y^0-\alpha_y T_0(x,y,t)\\ \varepsilon_{xy}^0-2\alpha_{xy}T_0(x,y,t)\\ \kappa_x-\alpha_x T_1(x,y,t)\\ \kappa_y-\alpha_y T_1(x,y,t)\\ \kappa_{xy}-2\alpha_{xy}T_1(x,y,t)\end{Bmatrix} \tag{8}$$

$$\begin{Bmatrix}N_x^T\\ N_y^T\\ N_{xy}^T\\ M_x^T\\ M_y^T\\ M_{xy}^T\end{Bmatrix}=\begin{pmatrix}A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16}\\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26}\\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66}\\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16}\\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26}\\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66}\end{pmatrix}\begin{Bmatrix}\alpha_x T_0(x,y,t)\\ \alpha_y T_0(x,y,t)\\ 2\alpha_{xy}T_0(x,y,t)\\ \alpha_x T_1(x,y,t)\\ \alpha_y T_1(x,y,t)\\ 2\alpha_{xy}T_1(x,y,t)\end{Bmatrix} \tag{9}$$

Using the force and moment resultants with the strain-displacement relations in the virtual work and applying the variational principles, the nonlinear and nonisothermal dynamic equations of a laminated composite plate in a compact form can be obtained in terms of mid-plane displacements in Equations (10). L_{ij} and N_i denote linear and nonlinear operators.

$$\begin{aligned}
&L_{11}u^0 + L_{12}v^0 + L_{13}w^0 + N_1(w^0) + \bar{m}\ddot{u}^0 - \left(q_x + q_x{}^T\right) = 0 \\
&L_{21}u^0 + L_{22}v^0 + L_{23}w^0 + N_2(w^0) + \bar{m}\ddot{v}^0 - \left(q_y + q_y{}^T\right) = 0 \\
&L_{31}u^0 + L_{32}v^0 + \left(L_{33} + L_{33}{}^T\right)w^0 + N_3(u^0, v^0, w^0) + \bar{m}\ddot{w}^0 - \left(q_z + q_z{}^T\right) = 0
\end{aligned} \tag{10}$$

The boundary conditions for the simply supported plates are given in Equation (11) and Equation (12). Besides, initial conditions are shown in Equation (13).

$$\begin{aligned}
&u^0(0,y,t) = u^0(a,y,t) = u^0(x,0,t) = u^0(x,b,t) = 0 \\
&v^0(0,y,t) = v^0(a,y,t) = v^0(x,0,t) = v^0(x,b,t) = 0 \\
&w^0(0,y,t) = w^0(a,y,t) = w^0(x,0,t) = w^0(x,b,t) = 0
\end{aligned} \tag{11}$$

$$\begin{aligned}
&M_x = 0 \text{ at x = 0, a} \\
&M_y = 0 \text{ at y = 0, b}
\end{aligned} \tag{12}$$

$$\begin{aligned}
&u^0(x,y,0) = 0 \quad , \quad v^0(x,y,0) = 0 \quad , \quad w^0(x,y,0) = 0 \\
&\dot{u}^0(x,y,0) = 0 \quad , \quad \dot{v}^0(x,y,0) = 0 \quad , \quad \dot{w}^0(x,y,0) = 0
\end{aligned} \tag{13}$$

The blast load is expanded in Fourier series and only the first term is chosen. It is assumed to be varying exponentially in time and Friedlander decay function is used to express the air blast load as shown in Equation (14). p_m is the peak pressure, t_p is positive phase duration, and φ is a waveform parameter.

$$p(x,y,t) = p_m\left(1 - \frac{t}{t_p}\right)e^{-\frac{\varphi t}{t_p}} \tag{14}$$

$T_0(x,y,t)$ and $T_1(x,y,t)$ in Equation (4) are defined in Equation (15) as variables along the length a and the width b. Equation (4) and Equation (15) are also time dependent functions with the assumption of Friedlander function as the blast load.

$$\begin{aligned}
&T_0(x,y,t) = T^0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}\left\{\left(1 - \frac{t}{t_p}\right)e^{-\frac{\varphi t}{t_p}}\right\} \\
&T_1(x,y,t) = T^1 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}\left\{\left(1 - \frac{t}{t_p}\right)e^{-\frac{\varphi t}{t_p}}\right\}
\end{aligned} \tag{15}$$

Approximate solution functions are chosen crucially by considering the results of static large deformation analysis of laminated composite and shown in Equation (16). Only the first term of the series for in-plane displacements and out-of-plane displacements for the simply supported plate is accounted.

$$\begin{aligned}
&u^0 = U_{11}(t)y^2(y-b)^2 \sin\frac{2\pi x}{a} \\
&v^0 = V_{11}(t)x^2(x-a)^2 \sin\frac{2\pi y}{b} \\
&w^0 = W_{11}(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}
\end{aligned} \tag{16}$$

The time dependent nonlinear differential equations are obtained by applying the Galerkin method to the equations of motion given in Equation (10). The nonlinear-coupled equations of motion are then solved by using the finite difference method. Finally, the equations of motion are reduced into a form that can be easily solved by one of the methods for solution of linear equation systems such as Gauss elimination method or LU decomposition.

3 NUMERICAL RESULTS

The dynamic behaviour of thermomechanically affected simply supported composite plate is obtained using a closed form solution by writing a FORTRAN program. The pressure distribution because of the blast loading is assumed to be uniform on the plate, and the parameters shown in Equation (14) are p_m=29 kN/m^2, t_p=0.0018 s, α=0.35. The constants of temperature increment (T^0 and T^1) in Equation (15) are selected as 40 °C, 80 °C and 120 °C to understand the effect of temperature variation on the dynamic response. Only square shaped plates are considered in this paper and the dimensions are taken as a=b=220 mm. The plates are made of six layers of laminae, which are assumed to be behaving linearly and elastically. The layers are perfectly bonded. The composite material type is AS/3501 and the fiber orientation angle is chosen as 0° for all layers. The material properties of AS/3501 unidirectional carbon epoxy are given in Table 1.

	AS/3501 Carbon Epoxy
E_1 (GPa)	137.895
E_2 (GPa)	8.963
ν_{12}	0.3
G_{12} (GPa)	7.102
ρ (kg/m3)	1590
α_1 (1/°C)	1.8×10^{-6}
α_2 (1/°C)	54×10^{-6}
Thickness (mm)	0.12954

Table 1: Material properties [13].

The displacement-time histories of the selected points obtained by using closed form solution are shown in Figure 2 and compared with the isothermal blast loaded plate. The thermal load increases the displacement amplitude as expected. The comparative strain-time histories at the middle, top and bottom surfaces for the center of the plate are shown in Figure 3a-c, respectively. Figure 4 shows the displacement-time and strain-time histories at the middle surface for the center of the plate subjected to only time dependent thermal forces. The effect of temperature increment is also shown in Figure 4 by using three different cases.

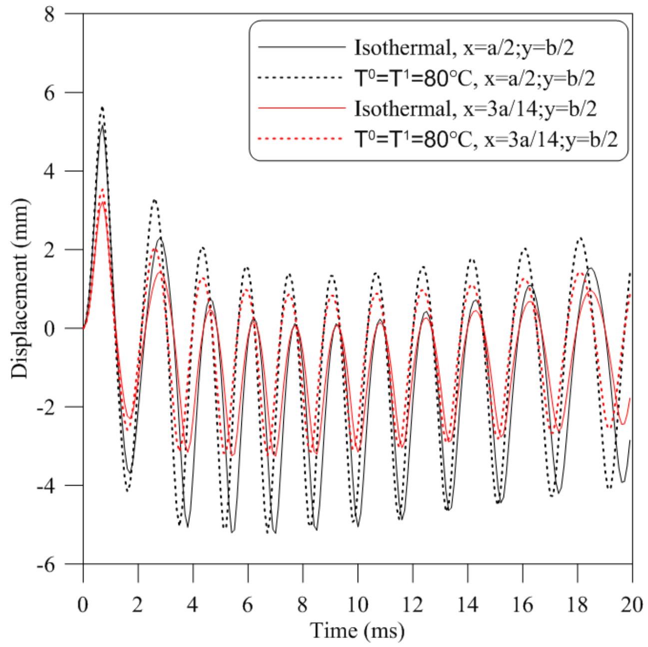

Figure 2: The displacement-time histories of the selected points for nonisothermal and isothermal blast loaded case.

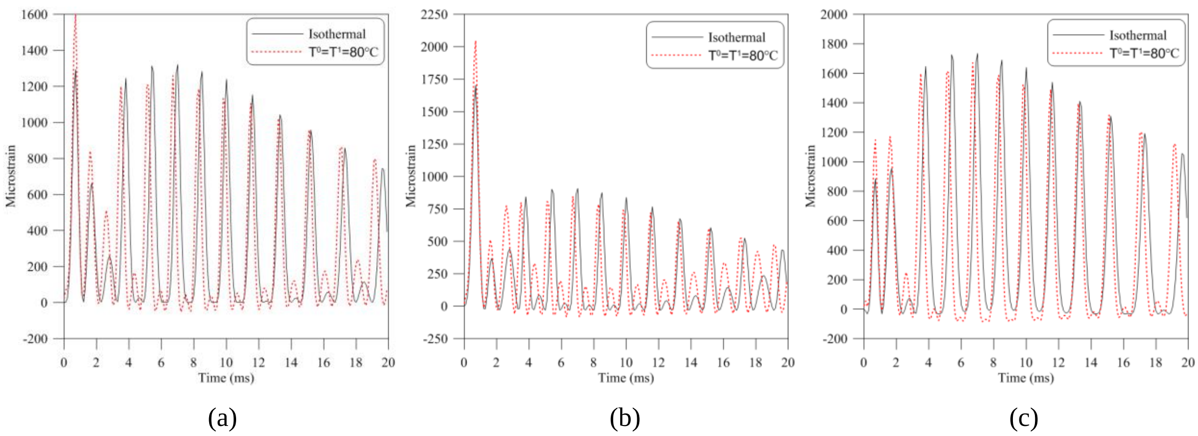

Figure 3: The strain-time histories for the center of the plate for nonisothermal and isothermal blast loaded case.

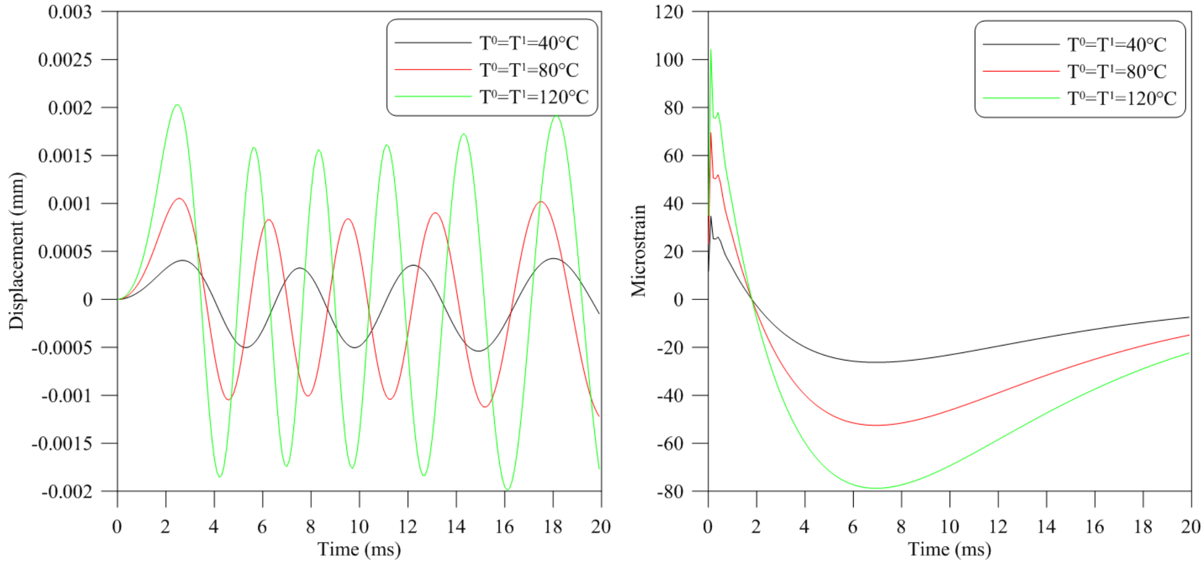

Figure 4: The displacement-time and strain-time histories for the center of the plate subjected to only time dependent thermal effects.

4 CONCLUSION

In this study, the dynamic response of thermomechanically loaded laminated composite plate is investigated theoretically. The displacement-time and strain-time histories are obtained for selected points. The strain-time histories are obtained at the mentioned points on the top, middle and bottom surfaces of the plate. The temperature rise during the blast or shock loading increases the displacement amplitude and strain. Therefore it is important to consider the temperature changes during the blast or shock loading in the design of structures which are subjected to thermal effects.

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