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EMBANKMENT DAM STABILITY ANALYSIS USING FEM

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Abstract. *This paper presents stability analysis of embankment dam with the surrounding heterogeneous rock mass using finite element method. In order to perform stability analysis of the dam and surrounding rock mass, several elastic-plastic material models for soil are customized and implemented in program package PAK. A 3D FE model of the embankment dam and the surrounding rock mass containing various material distributions according to their real distribution was made. The model includes a wider area around the dam in order to minimize the influence of the boundary conditions. The initial material parameters were determined using the identification of material parameters on the basis of the material from the dam body. Analysed dam is equipped with dam crest displacement transducers, as well as with transducers for the total and pore pressure in the clay core. Certain deviations have been noticed while comparing the measured values of these quantities with the results of the simulation. Since the analysed dam has been in operation for a long time, these deviations from the initial values of the parameters are caused due to the changes of the mechanical properties of materials during the dam operation. These changes are caused by several factors: the settlement of the dam and foundation, flushing of the material in the body of the dam and foundation, load changes, etc. In order to take into account the change of mechanical properties in materials and achieve the results of numerical simulation to describe the behaviour of the dam as close to the real behaviour, the calibration of the material parameters is carried out. Calibration of material parameters was performed using the measured displacements of the dam crest, as well as pore and total pressures in the clay core. Using the calibration we obtain new material parameters which give results of the numerical simulation that are closer to the behaviour of the real dam. In this manner we manage the dam safety and we can predict its future behaviour.*

1 INTRODUCTION

Rock-fill dams belong to the group of embankment dams, which due to their providence, adaptability and fewer demands concerning the fundament strength, represent the most frequently used type of barrier constructions. Comprehension of mechanical behaviour of dam material is essential for the design and analysis of its stability. Nowadays, finite element method is commonly used for dam stability analysis. In order to accurately reproduce mechanical behaviour of the dam using this method in stability analysis, key factor is selection of an appropriate constitutive model. Adopted constitutive model is to efficiently reproduce material behaviour of the dam being influenced by different load types. After adopting the appropriate constitutive model, it is necessary to determine its parameters. These data are often obtained by analysing sample material in the laboratory according to which material characteristics necessary for stability analysis are calculated. This is valid both for stability calculation in the construction phase and for stability calculation during dam exploitation.

As the dam, which is the subject of the paper, has been used for a long time, mechanical characteristics of dam material changed as the consequence of material consolidation and stress redistribution in the dam body. Additionally, small material particles are flushed due to the fluid flow through the dam body resulting in inhomogeneity in the dam material. Due to these reasons, adopted material parameters determined in the laboratory represent only initial parameters for strength calculations. Thus, it is necessary to conduct calibration of the dam model to assimilate numerical simulation results to the measured values.

In order to follow the changes in the dam body, transducers of stress and pore pressure are installed during the dam construction. The dam is also equipped by benchmarks for measuring crest dam displacement being conducted periodically.

After calibration of the dam model by using measured values on the real dam, calculations of object safety factors are conducted using the method of shear strength reduction. Shear strength of all materials in the dam body is gradually reduced until the failure or until the stability loss in numerical calculations.

2 CONSTITUTIVE LAWS FOR SOIL

Two constitutive models are used in the analysis of Prvonek dam stability. Generalized Hoek-Brown constitutive model [1] is used for mechanical behaviour simulation of upstream and downstream slope of the rockfill dam and surrounding rock mass whereas Mohr-Coulomb constitutive model [2] is used for mechanical behaviour simulation of clay core and sand filters. These constitutive models are adopted according to laboratory tests of materials the dam is consisted of. Both constitutive models are implemented in the program PAK [3]. Theoretical bases of the used constitutive models are presented further in the paper.

2.1 Mohr-Coulomb constitutive model

Mohr-Coulomb represents elastic-plastic constitutive model whose failure surface is in the space of principal stresses defined by hexagonal pyramid whose height coincides with the hydrostatic axis (Figure 1). This surface divides the principal stress space into the domain of purely elastic and purely plastic strains.

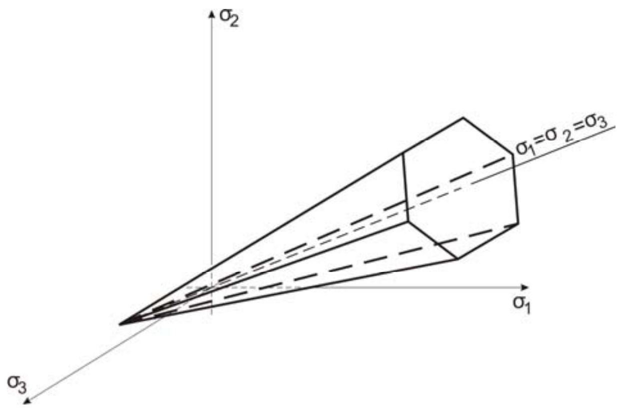

Figure 1 Mohr-Coulomb failure surface

Failure surface equation of this constitutive model is the function of the stress level and is defined by the following form:

$$f = \frac{I_1}{3}\sin\phi + \sqrt{{}^tJ_{2D}}\left(\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\sin\phi\right) - c\cos\phi \tag{1}$$

whereas the surface of plastic potential is defined as:

$$g = \frac{I_1}{3}\sin\psi + \sqrt{{}^tJ_{2D}}\left(\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\sin\psi\right) \tag{2}$$

In the equations (1) and (2), values I_1 and J_{2D} represent first stress invariant and second deviatoric stress invariant, respectively. Values ϕ, ψ and c represent material constants whereas value θ represents Lode's angle. This angle can be calculated using second and third deviatoric stress invariants in the following form:

$$\theta = \frac{1}{3} arcsin\left(-\frac{3\sqrt{3}}{2}\frac{J_{3D}}{J_{2D}^{3/2}}\right) \qquad -\frac{\pi}{6} \le \theta \le \frac{\pi}{6} \tag{3}$$

2.2 Generalized Hoek-Brown constitutive model

Generalized Hoek-Brown constitutive model [1] represents the elastic-plastic model whose failure surface, in the space of principal stresses, is defined by hexagonal pyramid of hyperbolic form, whose height coincides with the hydrostatic axis (Figure 2). This surface also divides the principal stress space into the domain of purely elastic and purely plastic strains.

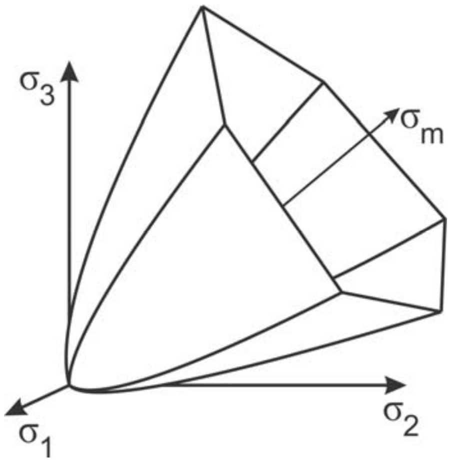

Figure 2 Failure surface of generalized Hoek-Brown constitutive model

Failure surface equation of generalized Hoek-Brown constitutive model is the function of stress level and is presented as:

$$f = \frac{I_1}{3} m_b \sigma_C^{\left(\frac{1}{a}-1\right)} - s\sigma_C^{\frac{1}{a}} + 2^{\frac{1}{a}} \left(\sqrt{J_{2D}} \cos\theta\right)^{\frac{1}{a}} + m_b \sqrt{J_{2D}} \sigma_C^{\left(\frac{1}{a}-1\right)} \left(\cos\theta - \frac{1}{\sqrt{3}} \sin\theta\right) \tag{4}$$

whereas the surface of plastic potential is formulated as:

$$g = \frac{I_1}{3} m_{bdil} \sigma_C^{\left(\frac{1}{a}-1\right)} - s\sigma_C^{\frac{1}{a}} + 2^{\frac{1}{a}} \left(\sqrt{J_{2D}} \cos\theta\right)^{\frac{1}{a}} + m_{bdil} \sqrt{J_{2D}} \sigma_C^{\left(\frac{1}{a}-1\right)} \left(\cos\theta - \frac{1}{\sqrt{3}} \sin\theta\right) \tag{5}$$

Values I_1 and J_{2D} represent first stress invariant and second deviatoric stress invariant, whereas m , σ_C, s and a are material constants. Lode's angle θ is calculated using the equation(3).

3 METHOD OF SAFETY EVALUATION

3.1 Overview of shear strength reduction method

In the stability analysis of the Prvonek dam, shear strength reduction (SSR) method [4] was used. Shear strength reduction method represents the use of finite element method for calculation of construction safety factor with a gradual decrease of shear material strength until reached the state of boundary equilibrium. In other words, material shear strength is reduced until there is a convergence of numerical solutions. The maximum value of the strength reduction factor, which the stability condition is satisfied for, represents the safety factor S of the structure [5]. The procedure of the shear strength reduction method for both constitutive models is described in this paper.

3.2 SSR using Mohr-Coulomb material model

In the case of using Mohr-Coulomb model, shear stress during failure is described by equation:

$$\tau = c + \sigma \tan\phi \tag{6}$$

Reduced shear strength for Mohr-Coulomb material model can be presented in the equation:

$$\frac{\tau}{F}=\frac{c}{F}+\sigma\frac{\tan\phi}{F} \tag{7}$$

where τ is shear stress during failure, σ is corresponding normal stress, c and ϕ are material constants (cohesion and internal friction angle), whereas F is represents shear strength reduction factor. Shear strength reduction of this model can be schematically presented as in (Figure 3).

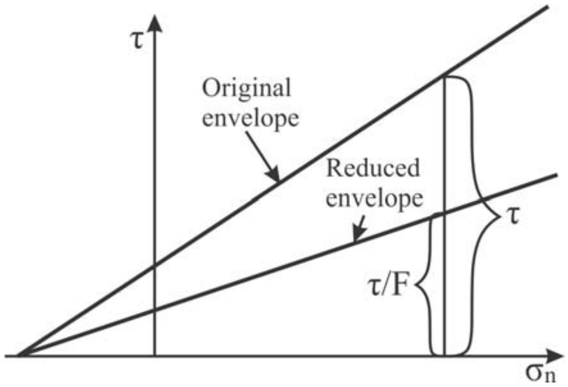

Figure 3 Shear strength reduction for Mohr-Coulomb model

In order to determine safety factor using SSR method, it is necessary to determine maximum value of shear strength reduction factor F with satisfied stability condition. Numerically, this indicates the increase of shear strength reduction factor until there is a convergence of numerical calculation within the given tolerance. In the case of exceeded stability, shear strength of material is exceeded as well which is manifested through divergence in numerical calculation.

Equation (7) indicates that it is necessary to determine maximum value of shear strength reduction F with satisfied stability condition or to calculate the following:

$$c_r=\frac{c}{F} \qquad \phi_r=\arctan\left(\frac{\tan\phi}{F}\right) \tag{8}$$

According to the equation (8), it is concluded that shear strength reduction in Mohr-Coulomb model is based on reduction of material constants c and ϕ. Described procedure is implemented in the program PAK and is used in stability analysis of Prvonek dam.

3.3 SSR using generalized Hoek-Brown material model

As it was previously mentioned, shear strength defined using generalized Hoek-Brown failure surface has nonlinear character [6], so reduced shear strength cannot be calculated using simple reduction of material parameters as in the previous case.

Shear strength reduction of constitutive model with nonlinear failure surface consists of the following steps:

- Reduction of failure envelope by shear strength factor F ,
- Determination of material parameters corresponding to the reduced envelope using method for error minimization,
- Use of calculated parameters describing reduced failure curve in the repeated FEM analysis.

Relations for shear stress are defined using the principal stresses in the following form [1]:

$$\tau = \left(\sigma_1 - \sigma_3\right) \frac{\sqrt{1 + a m_b \left(m_b \dfrac{\sigma_3}{\sigma_{ci}} + s \right)^{a-1}}}{2 + a m_b \left(m_b \dfrac{\sigma_3}{\sigma_{ci}} + s \right)^{a-1}} \tag{9}$$

Reducing the shear stress equation (9) using shear strength reduction factor F , reduced failure envelope is obtained as (Figure 4):

$$\tau^{red} = \frac{\tau_f}{F} = \left(\sigma_1 - \sigma_3\right) \frac{\sqrt{1 + a^{red} m_b^{red} \left(m_b^{red} \dfrac{\sigma_3}{\sigma_{ci}^{red}} + s^{red} \right)^{a^{red}-1}}}{2 + a^{red} m_b^{red} \left(m_b^{red} \dfrac{\sigma_3}{\sigma_{ci}^{red}} + s^{red} \right)^{a^{red}-1}} \tag{10}$$

where σ_{ci}^{red} , m_b^{red} , s^{red} and a^{red} represent material parameters of reduced failure envelope and they are to be determined.

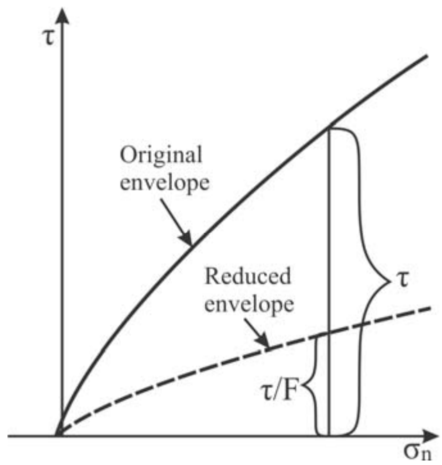

Figure 4 Shear strength reduction for Hoek-Brown model

New material parameters in the equation (10) are not possible to be determined directly, as in the previous case, so it is necessary to use some error minimization methods (fitting):

$$\varepsilon\left(\sigma_n\right)^2 = \left(\tau^{apr} - \tau^{red}\right)^2 \tag{11}$$

Values τ^{apr} and τ^{red} represent approximated and reduced values of shear stress, respectively. Parameters calculated in this manner are used in the repeated calculation of object stability.

4 CALIBRATION OF CONSTITUTIVE LAW PARAMETERS

As it is previously mentioned, parameter determining of adopted constitutive models is essential for accuracy of numerical simulation in embankment dams. Samples of all geotechnical environments of the dam body are analysed in the laboratory. These analyses obtained the data necessary to define failure surface of the corresponding constitutive models [7].

Result of shear test with estimated failure surface for one geotechnical environment is presented in Figure 5. Failure surface of adopted constitutive model is obtained by fitting the experimental results and failure surface with applying the following condition:

$$\varepsilon = \left(\tau^* - \tau_f\right)^2 \quad \rightarrow \quad \varepsilon_{\min} \tag{12}$$

In the equation (12) value τ^* represents shear stress during failure obtained through analysis, whereas τ_f is calculated shear stress for the same value of the normal stress. Value ε represents estimation error which is necessary to be minimized.

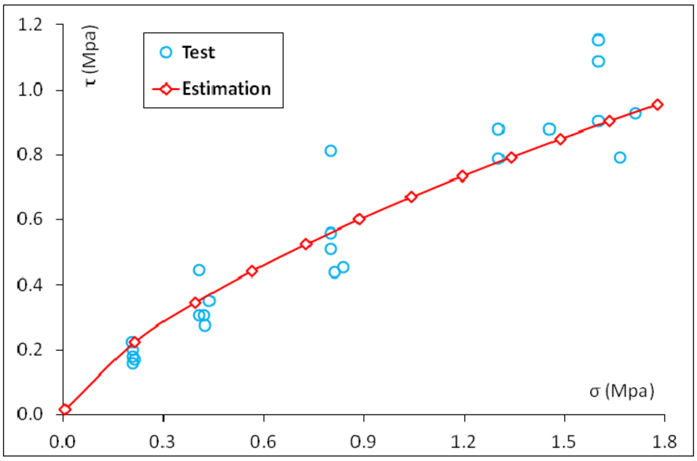

Figure 5 Results of material parameter identification

Determining of material constants represents the input for dam stability calculation. However, besides determining of material constants there is a certain deviation in numerical simulation results from the results of measuring on the real dam. This deviation is caused by dam material inhomogeneity as the consequence of different settlement s as well as of dam material degradation throughout time. This leads to redistribution of the stress state in the dam body so the initial parameters are to be modified and the dam model calibration is to be conducted [8]. Stresses and/or displacements can be used as model calibration values (multicriteria optimization).

5 STABILITY ANALYSIS OF THE DAM

5.1 Main features of the dam

Construction of analysed Prvonek dam is a rock fill embankment with a inclined central clay core and filter zones (Figure 6). On the spot of the dam, valley is of asymmetrical ravine type with the slope on the left riverside of 45°, and on the right of about 35°. Upstream the dam, the valley is widened which morphologically makes the barrier spot more suitable for accumulation formation.

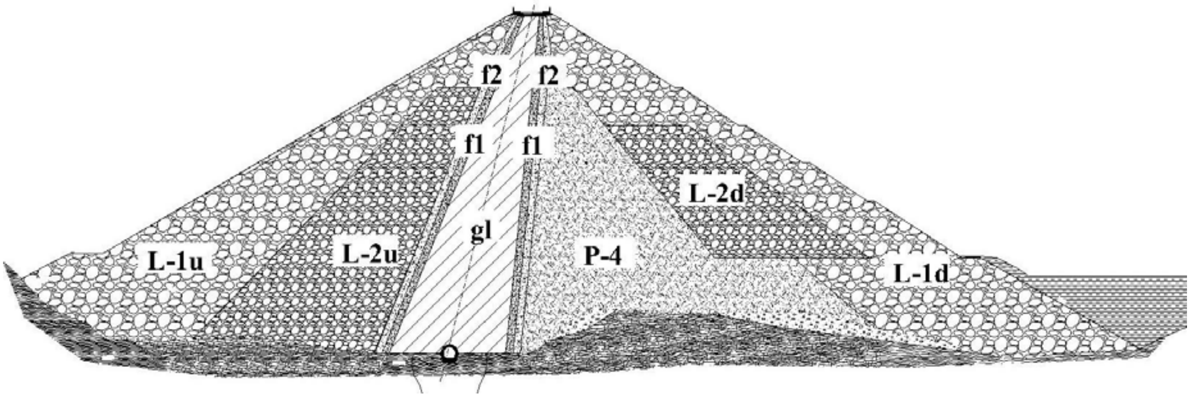

Figure 6 Cross-section of the dam

Terrain of barrier spot in the surrounding of barrier construction is modelled with the dimensions in the base, 498 x 1000 meters, from elevation of 370 to 630 metres. Model boundaries include a wider area around the object with the aim of setting the real boundary conditions as well as all proper quasi-homogenous zones. 3D 8-node finite elements with incompatible modes are used for modelling. A 3D mesh of finite elements is also developed with the total of 94986 nodes and 87128 elements. For dam stability analysis program PAK [3] is used. Finite element mesh of the dam model with surrounding rock mass is presented in Figure 7.

Natural boundary conditions are used in the analysis: horizontal node displacement is restricted on the model side boundaries, vertically on the boundary plane whereas displacements are restricted in all the directions on the lower model boundary.

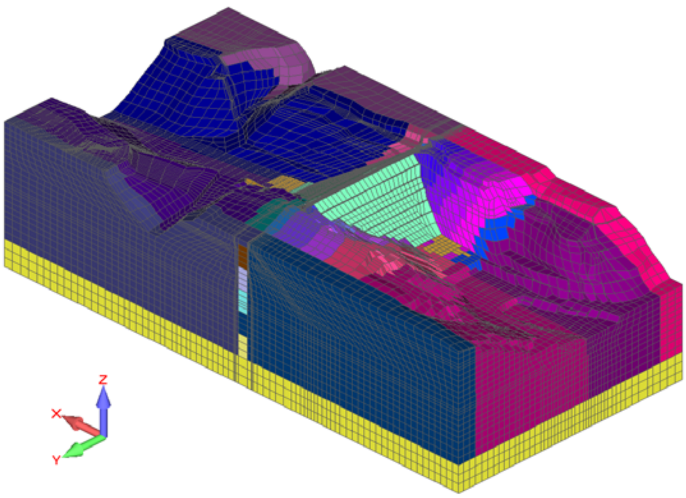

Figure 7 FE model of the dam and environment

Criteria for dam stability analysis are considered from the aspect of upstream and downstream slope stability. Minimal safety factors [9] are presented in Table 1.

Analysis condition	Required minimum safety factor	Slope
End-of-Construction	1.3	Upstream and Downstream
Steady seepage	1.5	Downstream
Maximum surcharge poo	1.4	Downstream
Rapid drawdown	1.1-1.3	Upstream

Table 1 Required factors of safety

5.2 Model calibration

In order to consider water flow through the dam, filtration analysis was previously conducted. According to these calculations, filtration forces and pore pressures in the model are obtained. In order to determine the accurate distribution of these loads, it is also necessary to conduct the calibration of filtration model parameters (Figure 8). Thus, effective instead of total stresses are used in the strength calculation:

$$\sigma_{ij}' = \sigma_{ij} + \delta_{ij} p \tag{13}$$

where: σ_{ij}' - effective stresses, σ_{ij} - total stresses, p - pore pressures.

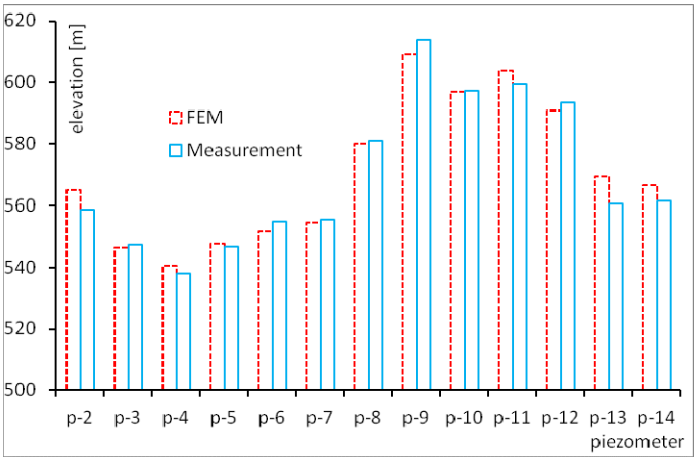

Figure 8 Pore pressures

After the use of such obtained material parameters, calculations are repeated and dam crest displacements of measured values are compared to the values obtained in numerical calculation (Figure 9 and Figure 10).

According to the analysis of measured displacement values, it is concluded that a year after instalment of measuring devices there were no significant dam crest displacements whereas all the measuring was conducted with slight changes of water level in accumulation. Measuring conducted two years from instalment of measuring devices show a significant displacement increase in horizontal direction (downstream), with a slight change of water level in accumulation, which is caused by dam material creep.

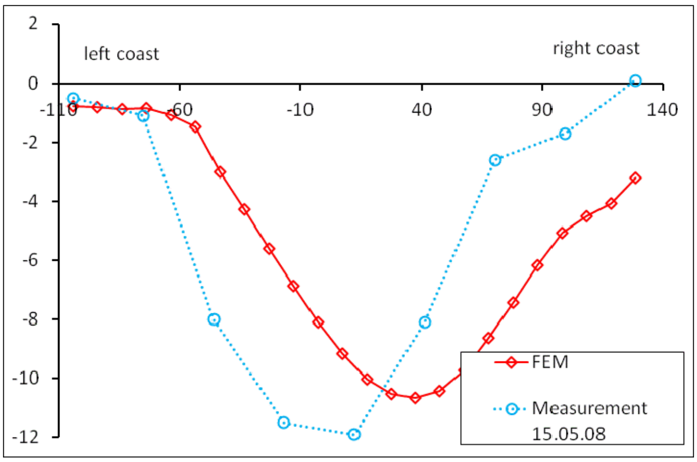

Figure 9 Horizontal displacement of the dam crest – downstream

Since used material models do not consider the creep effect, calibration of material constants is conducted according to the measured values of displacement in the first year after instalment of measuring devices.

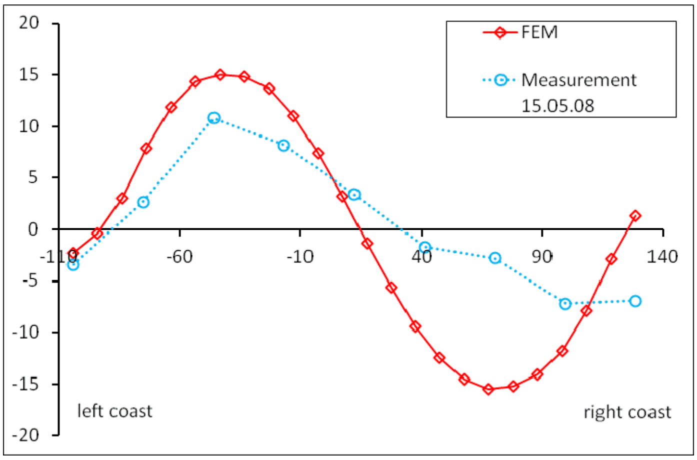

Figure 10 Horizontal displacement of the dam crest - transverse

Analysis of obtained displacements indicates that the displacement character obtained in FEM simulation corresponds to the character of measured displacements.

5.3 Analysis results

Stress-strain analyses were conducted using material parameters adopted after the calibration. Following stress-strain analyses were conducted [9]:

- Stability calculation during steady-state flow on normal water level.
- Stability calculation during steady-state flow on maximum water level.
- Analysis of rapid drawdown. Simulation of rapid drawdown was conducted from the elevation level of 614.7 meters to the level of 580.00 meters in 4.5-day period.
- Analysis of earthquake effect. After setting of initial conditions, quasi-static analysis was done with given horizontal acceleration. The influence of accumulation on the dam was considered by giving hydrodynamic pressure according to Zangar distribution [10].

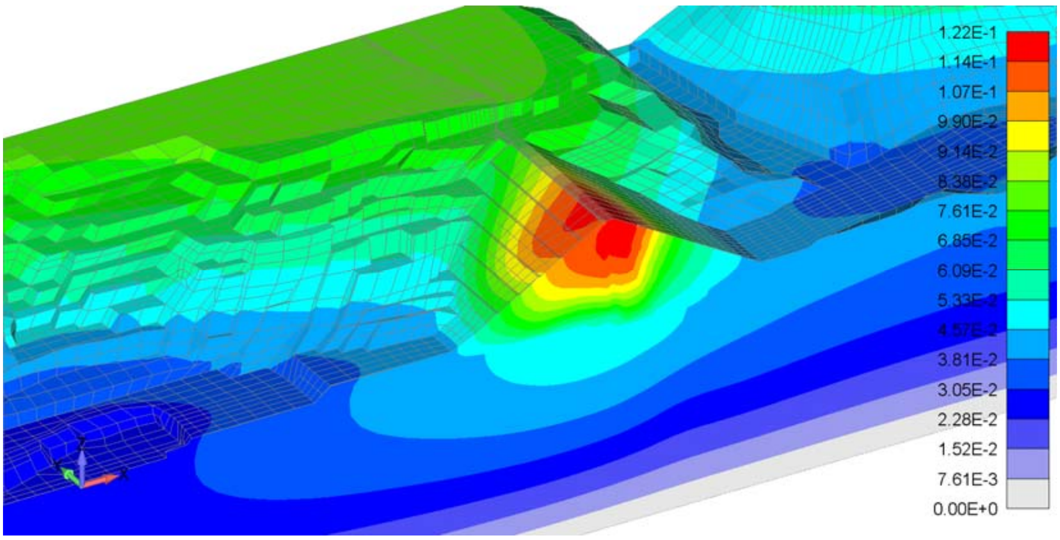

Figure 11 Total displacement distribution (y= 0.0 m)

Total displacement distribution for the case of stationary flow on the normal water level is shown in Figure 11. Maximum displacement (78.5mm) was observed in the core zone of the dam.

Plastic strain distribution for the case of stationary flow on the normal water level (613.56m) is shown in Figure 12. Maximum value of plastic strain appears in the core and filter zone closer to upstream dam slope.

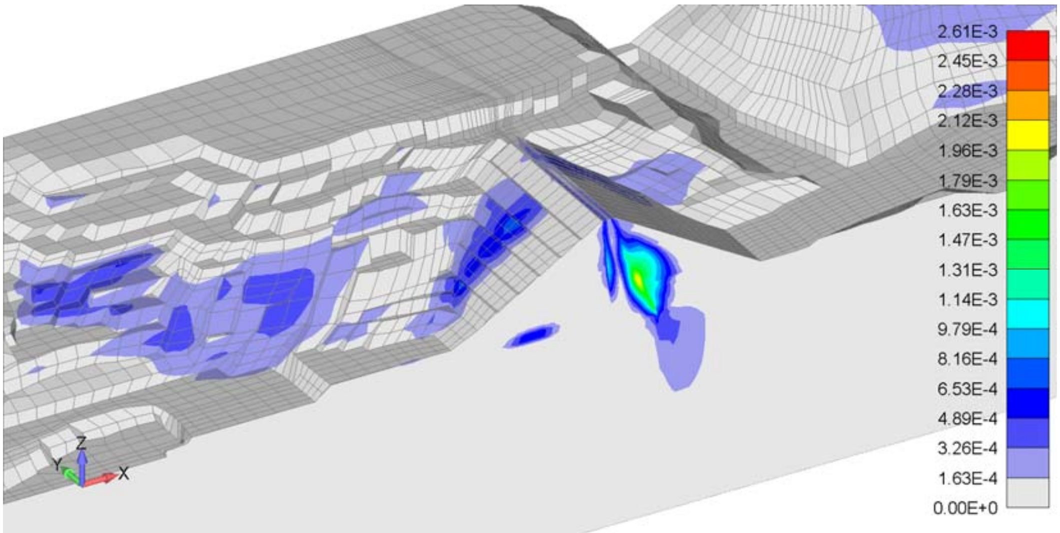

Figure 12 Equivalent plastic strain distribution (y=0.0 m)

After the analysis, calculations of dam safety factor were conducted using the shear strength reduction method of all the materials in the dam body according to the previously described procedure. Obtained safety factors for all analysed cases are greater than demanded values of safety factors presented in Table 1.

6 CONCLUSIONS

Stress-strain analysis of the Prvonek dam, which represents the central object of water supply system in city of Vranje, was conducted. A 3D model, based on finite element method was formed for stress-strain analysis. During numerical model forming, the analysis of results of geophysical and geomechanical researches related to engineering features of rock mass was done. Parameterization of abstracted quasi-homogenous zones in terms of parameters for stress-strain analysis was conducted.

Stress-strain processes were analysed using elastic-plastic model in order to obtain distribution of pore pressures in numerical model. After the obtained loads, construction stability was calculated.

In numerical analysis of dam stability generalized Hoek-Brown and Mohr-Coulomb constitutive models were used. All material samples embedded in the dam body were analysed to obtain the initial material parameters of adopted constitutive models. However, besides this material constant determining, there are certain discordances between FEM simulation and the results of measuring on the dam. Deviations are caused by material inhomogeneity of the real object (consolidation, flushing) during the time. In order to increase accuracy of stress-strain calculations, after determining material parameters, dam model calibration was done using multicriteria optimization.

Measuring markers were set on the real dam in order to implement model calibration on displacements. Two years of measuring after instalment of measuring devices proved significant increase of displacement in horizontal direction with a slight change of water level in accumulation. This was caused by creep dam material. As available constitutive models do not consider the creep effect, calibration of material constants was conducted according to the measured displacement values in the first year after the instalment of measuring devices.

After the stress-strain analysis for all defined cases of loads, global dam safety factor was obtained. This was conducted using the method of shear strength reduction factor. Obtained safety factors for all analysed cases were higher than minimum approved values for this type of dam so it is concluded that the analysed Prvonek dam is stable.

Acknowledgements

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