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A NONLINEAR METAMATERIAL INDUCED BY NONLINEAR DAMPING EFFECT WITH INERTIA AMPLIFIERS

BAO ZHAO[*,†], HENRIK THOMSEN[*], XINGBO PU[**], BART VAN DAMME[***], ANDREA BERGAMINI[***], ELENI CHATZI[*], AND ANDREA COLOMBI[*]

[*] Department of Civil, Environmental, and Geomatic Engineering, ETH Zurich, Zurich, Switzerland
[**]Department of Civil, Chemical, Environmental, and Materials Engineering, University of Bologna, Bologna, Italy
[***]Laboratory for Acoustics/Noise Control, Empa Materials Science and Technology, Dubendorf, Switzerland
[†] Corresponding author: Bao Zhao
email: bao.zhao@ibk.baug.ethz.ch

Abstract. In this paper, we studied a nonlinear metamaterial based on the nonlinear damping effect. The proposed design combines a linear host cantilever beam and periodically distributed inertia amplifiers as nonlinear local resonators. Firstly, the geometric nonlinearity induced by the inertia amplifiers is studied to reveal the amplitude-dependent damping effect. Secondly, a modal analysis method is implemented to form a lumped parameter model for the nonlinear metamaterial. This model is further solved by a numerical harmonic balance method under periodic base excitation. Finally, the nonlinear energy transfer inside the proposed design is studied to investigate the nonlinear interaction between local resonators and different vibration modes. The theoretical results show that the bandgap is amplitude-dependent, broadened, and gradually degenerated due to the nonlinear damping effect. It also further leads to an efficient modal dissipation capacity of the host structure, which has significant potential in shock wave mitigation.

Key words: Metamaterial; Nonlinear damping; Nonlinear energy transfer

1 INTRODUCTION

In engineering applications such as civil infrastructures, industries, and vehicles, mechanical vibrations are pervasive. These vibrations have the potential to cause structural and operational failures as well as harm human bodies. As a result, the research communities and industries have focused heavily on vibration attenuation. There are three commonly used methods of vibration control: damping tuning, stiffness tuning, and vibration isolation by auxiliary attachments. These methods can be categorized as passive, semi-passive (active), and active [1]. However, passive and semi-passive (active) methods are particularly well-suited for mitigating vibrations in host structures without the need for complex control systems. With respect to the quantity of auxiliary attachments and their dynamic properties, these methods can be classified into four different catalogs:

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1. Single linear attachment, e.g., tuned mass damper [2] and piezoelectric shunting [3];
2. Single nonlinear attachment, e.g., nonlinear energy sink [4] and nonlinear damping [5];
3. Multiple linear attachments, e.g., multiple tuned mass dampers [6], and linear locally resonant metamaterials [7,8];
4. Multiple nonlinear attachments, e.g., nonlinear metamaterials [9];

The tuned mass damper (TMD) [2] is a widely used passive vibration control technique in structural dynamics. It consists of a mass-spring-damper system that is attached to the host structure to be controlled. TMD works by reducing the amplitude of the structure's vibrations through the mechanical damping effect. On the other hand, the piezoelectric shunting technique [3] uses a piezoelectric material attached to the structure. The material converts the mechanical energy into electrical energy, which is dissipated through a resistive shunt. The electrical shunting behaves as an electrically induced damping effect, reducing the host structure's vibration amplitude [9]. Compared with the linear vibration control methods, nonlinear energy sink (NES) [4] is a passive vibration control technique that utilizes nonlinear effects. The NES device is connected to the structure and is designed to transfer the mechanical energy from the host structure into the nonlinear attachment. Most of the nonlinear effects are amplitude-dependent. Thus, NES is considered to be applied with large vibration amplitudes. Take the example of nonlinear damping [5], and it is particularly suitable for shock wave attenuation with large velocity impact, which results in significant nonlinear damping or drag force to further dissipate the mechanical energy compared with a linear damped system.

Besides the previous single attachment methods for vibration mitigation, multiple TMDs and linear locally resonant metamaterials have also been studied as passive vibration control techniques that utilize multiple linear attachments to control the vibration of structures. Multiple TMDs [6] work by distributing the mass-spring-damper system over the host structure, which reduces the vibration amplitude corresponding to different modal frequencies. With a similar idea of adding multiple attachments but in a periodic manner, linear metamaterials [7,8] are engineered structures that utilize the local resonant effect with periodically distributed local resonators. They work by absorbing or reflecting the propagation wave in a specific frequency range called bandgap. Compared with single attachments, multiple TMDs, and linear locally resonant metamaterials have a broader attenuation range. However, their bandgaps decided by the underneath linear structures are usually narrowed when compared with the well-known nonlinear energy sinks.

Therefore, the pursuits to introduce nonlinearities into metamaterials have combined the advantages of these two research areas and raised novel concepts in the sense of structural dynamics, such as broadband vibration attenuation [9]. We propose a nonlinear metamaterial with inertia amplifiers as local resonators with a nonlinear damping effect based on this idea. The structure of this paper is listed as follows: In addition to the Introduction in Section 1, Section 2 reviews the inertia amplifier and its nonlinear damping effect; Section 3 utilizes the modal analysis with numerical harmonic balance method in which the nonlinear transmissibility is discussed; Section 4 concludes this work.

2 NONLINEAR DAMPING EFFECT FROM INERTIA AMPLIFIERS

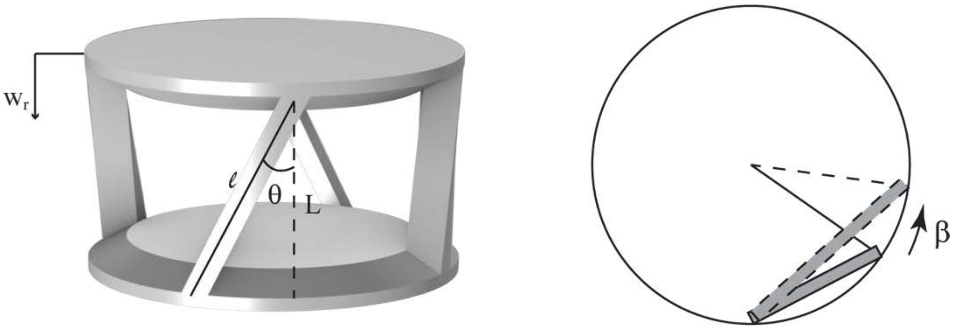

Figure 1: Schematic of an inertia amplifier. The translational displacement leads to a disk rotation β.

Prior to delving into the analysis of the nonlinear metamaterial, let us first revisit the nonlinear dynamics of a single rotational inertia amplifier proposed by Van Damme et al. [5] depicted in Fig.1. This amplifier comprises two identical disks, each with a mass of M, positioned at the top and bottom, and connected by beams tilted at an angle θ. The right diagram shows the projected length of a connected beam before and after deformation. The rotational angle of the top disk is thus determined as follows:

$$\beta = \frac{\sqrt{\ell^2 - (L - \quad)^2} - \sqrt{\ell^2 - L^2}}{R} \tag{1}$$

where R is the radius of the disk. The angular velocity reads:

$$\dot{\beta} = \frac{L - }{\sqrt{\ell^2 - (L - \quad)^2}} \frac{\dot{}}{R} \tag{2}$$

where $l = L/\cos\theta$. By Lagrangian principle with its kinetic and potential energy, its equation of motion can be given as:

$$F = m\ddot{w} \; + c|\dot{w} \;|\dot{w} \; + kw \tag{3}$$

where F represents the external force. Assuming that θ is not too small, the equivalent mass m and damping coefficient c can be determined with the 0-th order term of the Taylor expansion of Eq.2.

A nonlinear damping effect arises as a result of the geometric nonlinear coupling between w and β when expressing the viscous damping c. This type of nonlinear damping is absolute and results in an amplitude-dependent damping ratio, as well as the potential for amplitude-dependent frequency response. Fig.2 shows the frequency response functions (FRFs) of a case study of the above-mentioned inertia amplifier with different base excitation forces. It can be seen that with the increase of excitation level, the nonlinear damping effect becomes more prominent with the decrease of the resonant peaks and fatter phase transitions. These characteristics can be employed in the creation of amplitude-dependent bandgaps in nonlinear metamaterials.

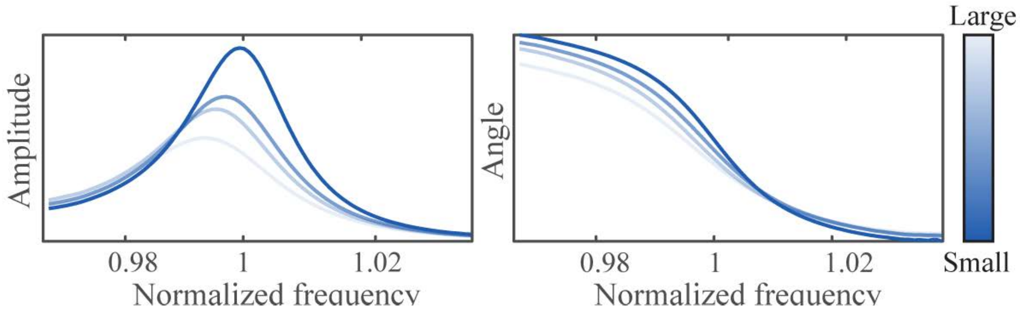

Figure 2: Frequency response function of the inertia amplifier. The left and right views correspond to the amplitude and phase angle of the FRFs.

3 NONLINEAR FREQUENCY RESPONSE

Due to boundary reflections under low-frequency vibrations, the infinite long beam assumption with nonlinear dispersion relationships investigated, often fails. Furthermore, nonlinear local resonators may interact with modal frequencies induced by boundary conditions. Consequently, in this section, we analyze the frequency response of the proposed nonlinear metamaterial with a finite length using modal analysis. We also take into account higher harmonics using the harmonic balance method.

The schematic of the nonlinear metamaterials is shown in Fig.3, which combines a linear host cantilever beam with total S inertia amplifiers as nonlinear local resonators. By combing the equations of the host beam with the Euler-Bernoulli beam and the nonlinear local resonators with nonlinear damping effect, the governing equation of the proposed nonlinear metamaterial can be formulated as:

$$\begin{cases} \boldsymbol{\mathcal{L}}w(x) - \sum_{j=1}^{S} \left(k_j \quad_{rj} + f_{NL,j}(x) \right) \delta(x - x_j) = -m\ddot{w}_b \\ m_j \ddot{\ }_{rj} + k_j \quad_{rj} + f_{NL,j}(x) = -m_j \left(\ddot{w}_b + \partial_{tt} \quad (x_j) \right) \end{cases} \tag{4}$$

where $\boldsymbol{\mathcal{L}}$ is a linear operator denoting the differentiation operations for the Euler-Bernoulli beam equation. δ is the Dirac function defines the positions x_j on the host beam where the reaction forces of nonlinear local resonators applied. $\ddot{w}_b$ denotes the base excitation acceleration on the clamped side of the host beam. f_{NL} is the nonlinear damping force taking the form as shown in Eq.3. To solve this equation of the nonlinear metamaterial, we assume the host beam is a linear beam with well separated modes, and the nonlinear modal interactions are neglected. The nonlinear damping effect from local resonators have also been taken into account. The ansatz of solutions, displacement w of the host beam, take the form of modal superpositions with Fourier expanded modal weighs:

$$(x, t) = \sum_{k=-H}^{K=H} \hat{\eta}_r(k) e^{ik\omega t} \sum_{r=1}^{N} \varphi_r\,(x) \tag{5}$$

where H and N are the truncation numbers of the Fourier series and mode shapes. By substituting Eq.5 into Eq.4 and applying boundary conditions, the governing equations can be transformed into a matrix form as:

$$\mathbf{M\ddot{x}} + \mathbf{D\dot{x}} + \mathbf{Kx} + \mathbf{F_{NL}}(\mathbf{x}) = \mathbf{F_{ex}} \tag{6}$$

where $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$, $\mathbf{F_{nl}}$, and $\mathbf{F_{ex}}$ denote the matrix form of the mass, damping, stiffness, nonlinear internal forces, and external forces matrixes, respectively. And the solution matrix $\mathbf{x}$ is given as:

$$\mathbf{x} = [\eta_1\ \eta_2\ \cdots \eta_N\ w_{r1}\ w_{r2}\ \cdots\ w_{rS}]^T \tag{7}$$

Therefore, the solution of the nonlinear metamaterial can be solved with Eq. 6. Using a numerical harmonic balance with the Newton-Raphson method [10], we investigate a practical finite length case of the proposed nonlinear metamaterial under harmonic excitations. The transmissibilities between the free tip and clamped base of the host beam are shown in Fig.3. It can be seen that the bandgap has been broadened due to the nonlinear damping effect, which agrees well with the dispersion analysis. It also leads to the degeneration of bandgap with the increase of excitation level. This bandgap broadening effect features the first capacity of the nonlinear damping-induced metamaterial.

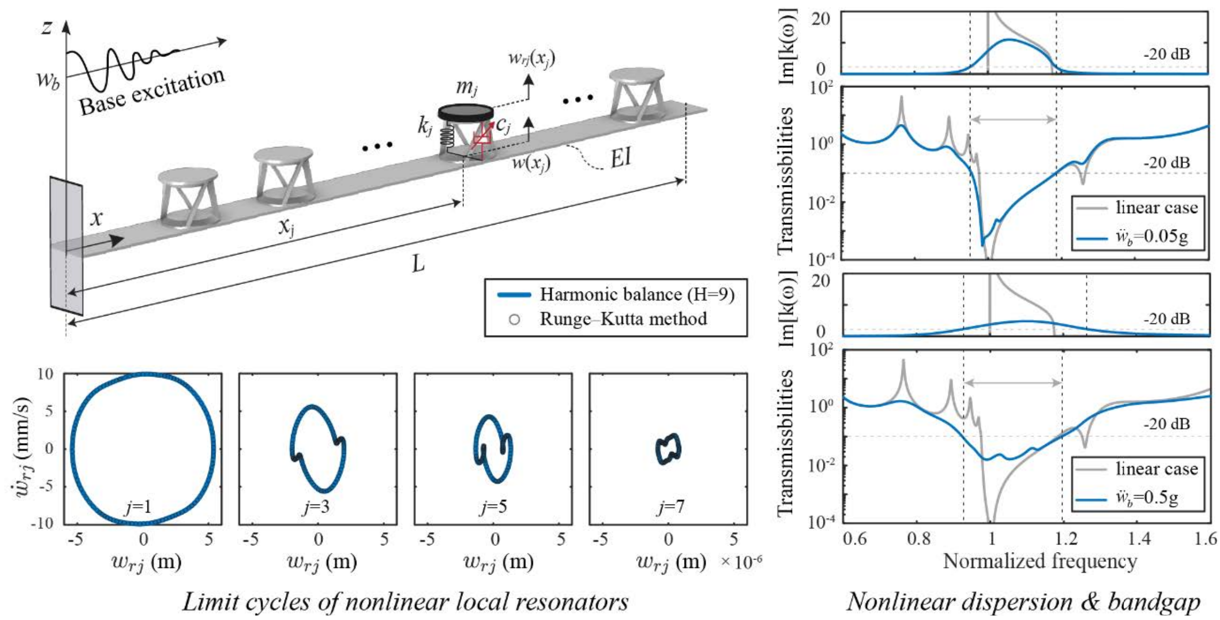

Figure 3: The schematic of the nonlinear metamaterial with results from nonlinear local resonators (limit cycles) and the host beam (nonlinear dispersion and bandgap).

In addition to the bandgap range, the modal frequency peaks have also been attenuated due to the nonlinear interactions with local resonators, which is verified by the increase of equivalent damping at those modal frequencies. With the wave propagation from the left to the right side of the host beam, the limit cycles of local resonators indicate more nonlinearity and harmonic generations. This nonlinear interaction between the host beam and local resonators features the second capacity of the nonlinear metamaterial–efficient modal dissipation capacity, which could be potentially used in shock wave mitigation [11].

4 CONCLUSIONS

In conclusion, we present a nonlinear metamaterial based on the nonlinear damping effect. The proposed design combines a linear host cantilever beam and periodically distributed inertia amplifiers as nonlinear local resonators. The geometric nonlinearity of the inertia amplifiers is studied to reveal the amplitude-dependent damping effect. A modal analysis method is implemented to form a lumped parameter model for the nonlinear metamaterial, which is further solved by a numerical harmonic balance method under periodic base excitations. The results show that the bandgap of the proposed design is amplitude-dependent, broadened, and gradually degenerated. The nonlinear damping effect also leads to an efficient modal dissipative capacity of the host structure for shock wave mitigations. The proposed theoretical modal and application with the nonlinear damping-induced metamaterial take advantage of the local resonant metamaterials and nonlinear vibration absorbers, which opens new opportunities for broadband vibration attenuations.

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