

AUXETIC ENHANCEMENT OF THE SHUNTED PIEZOELECTRIC EFFECT FOR VIBRATION SUPPRESSION

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Abstract. Shunted piezoelectric patches connected to passive electric circuits can be attached to a host structure for effective attenuation of structural vibrations. The structure under consideration consists of a beam, piezoelectric patches and possibly an auxetic layer between them. In this study, a new technique is presented for enhancing the vibration suppression of structure by using an auxetic layer between the shunted piezoelectrics. First, a modification of the “current-flowing” shunt circuit for multimode vibration control is presented. Two finite element (FE) models are created to simulate the dynamic behavior of the beam. The frequency response of the compound system and the optimal values of the electric parameters are calculated by using two FE models and a comparison is provided. Finally, it is shown that the vibration reduction of the second and third eigenmodes can be enhanced, provided that an auxetic of considerable thickness is placed between the piezoelectrics and the structure.

Key words: Auxetic Material, Multimode Circuits, Current-flowing, Piezoelectrics, Shunt Circuits, Vibration Control.

1 INTRODUCTION

Passive vibration control of structures can be achieved by using shunted piezoelectric elements connected to passive electric circuits. The fundamental idea behind piezoelectric shunt damping is the transformation of mechanical energy into electrical energy by piezoelectric material, which is then dissipated by heating through a resistor circuit [1].

Shunt piezoelectric systems have been extensively used for vibration and noise control during the last several years. Also, shunt circuits have been used for the development of smart metamaterials in order to enhance their properties. In fact, resonant shunts are proved to be very efficient and stable for the reduction of vibration on smart piezoelectric structures, such as beams and plates, [2].

The tuning of a simplified “current-flowing” shunt circuit, [3] able to mitigate the vibration amplitude of multiple structural resonances is addressed in this work. This study considers the capacitances and the shunting branch inductors as new design variables along with the resistor-damping components.

Auxetic materials constitute a kind of smart metamaterial with a negative Poisson ratio. As opposed to conventional materials, auxetic materials expand laterally under tension and contract under the action of compressive forces. Microstructures, such as star-shaped ones [4] or chiral ones [5] lead to auxetic behavior.

The combined usage of the aforementioned techniques is presented here by adding an auxetic composite layer between the piezoelectric shunt patches. The reduction in the vibration depends on the thickness of the auxetic layer. The objective of the present study is to enhance the damping performance of the whole structure by adding suitably designed shunted networks and auxetic layer.

Therefore, the present work is divided into five parts. After the present introduction, Section 2 refers to the theoretical part of multi-mode shunt circuits. The following section is dedicated to the analytical reference on auxetic materials. The numerical results of the investigation are presented next, while the last section is devoted to the conclusions of the study.

2 MULTIPLE-MODE DAMPING SYSTEM

The process to improve the multimode vibration damping of composite beam structures is described in this section. The system under consideration consists of a cantilever beam structure with surface piezoelectric patches bonded on its top and bottom surfaces at a distance x_p ($x_p=0.5\text{mm}$) from the fixed end (Fig. 1). The top piezoelectric patch is connected to a passive shunt circuit while the bottom one can be used as sensor or actuator.

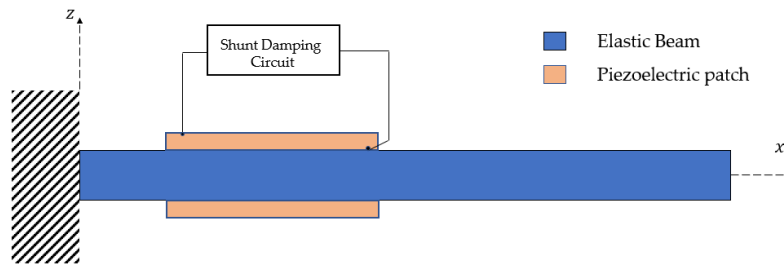


Figure 1: A Cantilever Beam with a piezoelectric patch connected to a shunt damping network.

From finite element analysis, the equations of motion and charge equilibrium of the system can be written as follows:

$$\begin{aligned} M_u \ddot{d}(t) + K_u d(t) + \Theta v(t) &= F_m(t) \\ -\Theta^T d(t) + C_p v(t) &= q(t) \end{aligned} \quad (1)$$

where $d(t)$ and $v(t)$ are the global vector of mechanical and electrical degrees of freedom (DoF). Also, M_u, K_u are the global mass and stiffness matrices and Θ is the global electromechanical coupling matrix. C_p is the global capacitance matrix, $F_m(t)$ is the global vector of mechanical forces and $q(t)$ is the global vector of electric charge. Equation (1)₂ can be expressed in terms of the electric charge vector as

$$v(t) = C_p^{-1}(q(t) + \Theta^T d(t)) \quad (2)$$

By connecting a shunt circuit to the upper piezoelectric patch, the effects of shunt damping must be introduced in equation (1). In this work, the “current flowing” shunt circuit is used for damping simultaneously the two vibration modes of the piezoelectric system (Fig. 2). The following equations were presented by Behrens et al. [3] for an appropriate selection of the shunt parameters

$$\hat{L}_i = \frac{1}{\omega_i^2 C_i}, \quad \tilde{L}_i = \frac{1}{\omega_i^2 C_p} \quad i=1,2 \quad (3)$$

where ω_i^2 are the i – th target resonance frequency of structure and C_i is an arbitrary capacitance. It is possible that the shunt circuit shown in Fig. 2(a) to be replaced by an equivalent or modified shunt circuit, [1], (Fig. 2(b)) with total inductor of each branch given by $L_i = \hat{L}_i + \tilde{L}_i$.

Applying Kirchhoff's circuit laws to the simplified shunt circuit of Fig. 2(b) and taking into account the fact that the piezoelectric voltage and shunt voltages must be equal ($v = V_{sh}$), the equations that describe the behavior of the compound system are obtained as,

$$\begin{aligned} M_u \ddot{d} + K_o d + \Theta C_p^{-1} q &= F_m \\ \dot{q} &= \dot{q}_1 + \dot{q}_2 \\ L_1 \ddot{q}_1 + R_1 \dot{q}_1 + \frac{1}{C_1} q_1 &= -V_{sh} \\ L_2 \ddot{q}_2 + R_2 \dot{q}_2 + \frac{1}{C_2} q_2 &= -V_{sh} \\ V_{sh} &= C_p^{-1} q + C_p^{-1} \Theta^T d \end{aligned} \quad (4)$$

where $K_o = K_u + \Theta C_p^{-1} \Theta^T$.

After the presentation of the mathematical part, some information related to practical implementation within each software is provided. Especially, a MATLAB code has been developed to implement the FE model as well as to present the state space model of the piezoelectric shunt damping system into the frequency domain. This 1D model uses two-node super-convergent beam elements. For a detailed description, the reader is referred to [1].

Then, a 3D finite element model of a cantilever beam structure containing a shunted PZT patch bonded to its surface has been developed using a commercial finite element modeling (FEM) software (COMSOL).

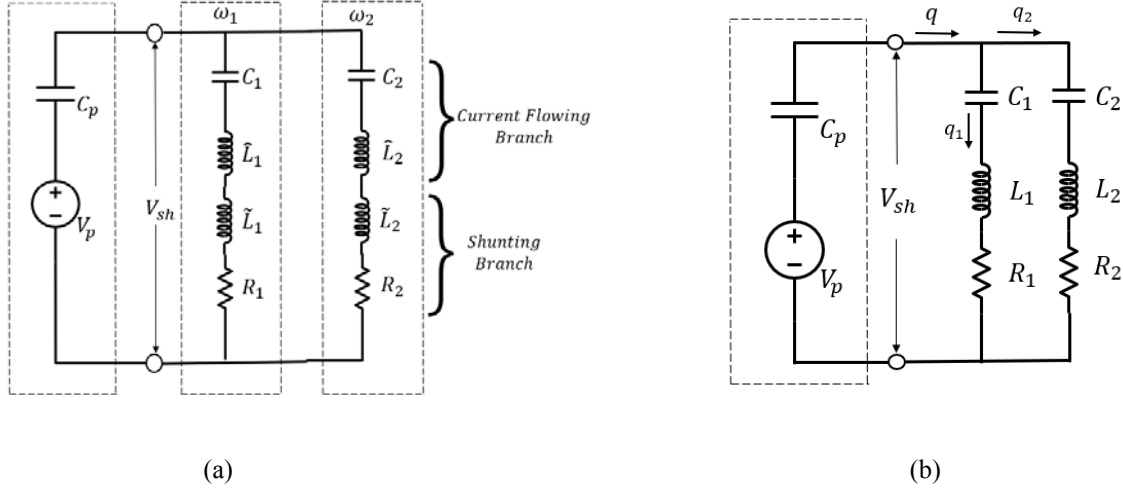


Figure 2: (a) A single-shunt circuit for two-mode current-flowing control. (b) Simplified shunt circuit.

3 AUXETIC MATERIALS AND THEIR PROPERTIES

Auxetic materials are one of the most studied branches of mechanical metamaterials while they exhibit counterintuitive behavior during deformation. Specifically, under uniaxial compression (tension), conventional materials expand (contract) in the directions which are orthogonal to the applied load. In contrast, auxetic materials become wider when stretched and thinner when compressed. Thus, they have the exactly opposite behavior in comparison to the conventional materials. These materials have negative Poisson's ratio, a material property which provides information about the modification of the length in the perpendicular to the loading direction. The conventional and auxetic behavior is schematically depicted in Fig. 3.

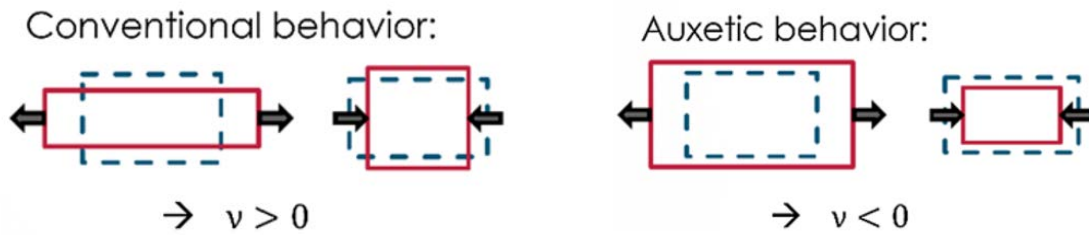


Figure 3. Conventional versus auxetic materials, [6].

Particularly, a thin layer with in-plane negative Poisson's ratio expands in all directions if pulled towards one of them. Let us recall that the upper side of a three-dimensional beam in

bending expands in the longitudinal direction of the beam and shrinks in the perpendicular direction, along the thickness of the beam. Therefore, an isotropic piezoelectric patch is not used effectively since the positive Poisson coefficient of the host beam structure prevents its expansion in both directions. An auxetic layer of considerable thickness between the host structure and the piezoelectric patch transforms the deformation from perpendicular to the loading direction. The idea of using an auxetic layer is quite general, it is based on the homogenized properties of the layer and does not include the details defining the kind and other quantities of the microstructure that lead to the auxetic behavior.

The approach of utilizing an auxetic to change a tensile-compressive input at the top of a bending beam into a tensile-tensile signal at the piezoelectric patch may be more widely applicable. The different stress distribution, [7], is due to the ability of the auxetic layer to transform a tension-compression signal from the surface of the host beam to a tension-tension one on the surface of the piezoelectric patch. It must be noted that the usage of 3D modeling is necessary for the study of this effect, which is invisible in 2D models. For more information, the reader is referred to the paper [7].

4 ANALYSIS RESULTS AND DISCUSSION

This section presents numerical results for multimode vibration damping of the cantilever composite beam, see Fig 1. Thomas et al. [8] and Marakakis K. [9] previously studied this beam for single-mode shunt damping. Here the same structure is used to study the simultaneous vibration damping of the second and third eigenfrequency. For this reason, a “current-flowing” multi-mode circuit is used, as referred in section 2. Specifically, simulation results for multimode vibration suppression using the developed FE models are provided. Then, an auxetic enhancement layer is added and its influence on the response of the structure is investigated.

4.1 Simulation results and comparison

The host beam is made of aluminum, and the piezoceramics are PIC151 and are placed 0.5 mm from the fixed end. The material and geometric properties of the structure are given in [1]. In this study, the second and third mode of the cantilever beam are damped using the multimode resonant shunt shown in Fig. 1. The reason for only addressing the second and third modes is that, due to the position of the piezoelectric patch, these modes have greater authority than other structural modes. For the determination of the frequency response, the described FE models are used, and the harmonic force is applied at the tip of the beam. The system response is determined at the same position.

The finite element formulation which is used by 1D home-made program is based on the super-convergent FE approach developed by Foutsitzi et al [10]. For information about the mesh element quality of the reduced-order 1D FE beam model, the mesh consists of 41 one-dimensional beam elements. Similarly, the mesh of multiphysics 3D model consists of 528 quadrilateral elements.

Using the 1D finite element model, the optimal values of the resistance R , the inductance L and capacitance C of a multi-mode shunt circuit, are obtained in [1] by the particle swarm

optimization technique. Then, they are given in table 1. Also, the corresponding values for the multiphysics 3D model obtained by a parametric study are given in the same table.

Table 1: Optimal shunt element values based on a 1D and a 3D FEM model.

MATLAB	FEM 3D
$C_2 = 1.00\text{E-}06$ F	$C_2 = 1.00\text{E-}06$ F
$R_2 = 1400$ Ohm	$R_2 = 1800$ Ohm
$L_{2\text{tot}} = 7.68$ H	$L_{2\text{tot}} = 11.30$ H
$C_3 = 8.30\text{E-}09$ F	$C_3 = 8.30\text{E-}09$ F
$R_3 = 2230$ Ohm	$R_3 = 6500$ Ohm
$L_{3\text{tot}} = 5.06$ H	$L_{3\text{tot}} = 7.5$ H

The damping effect of the shunt circuit around the second eigenfrequency, from both models, is depicted in Figure 4. It can be observed that the two models provide similar damping performance.

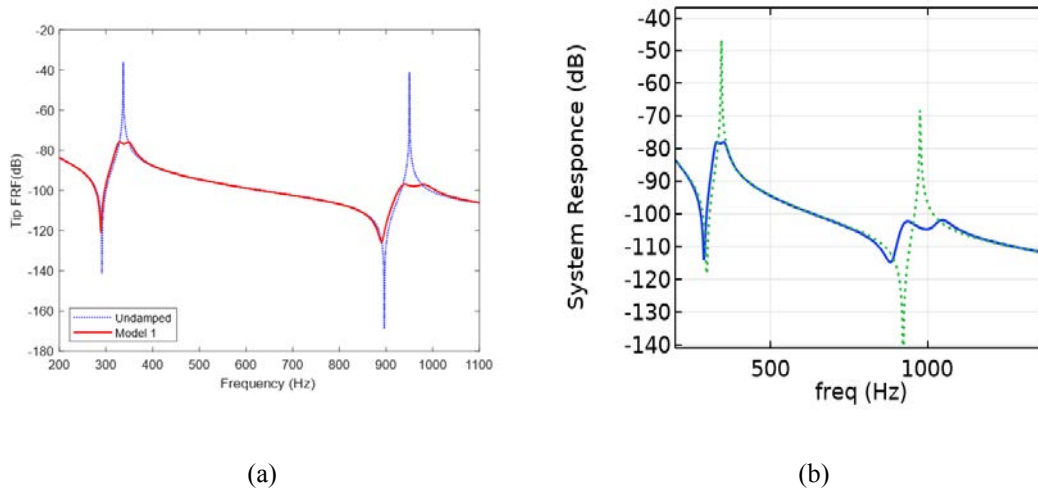


Figure 4: Simulated frequency response of the model. (a) Open Circuit (blue line) and Shunt Circuit (red line), MATLAB 1D Environment (b) Open Circuit (green line) and Shunt Circuit (blue line), Comsol 3D Environment.

4.2 Investigation of auxetic materials and results

In the last part of this section a method for enhancing the vibration suppression is proposed. The structure follows the 3D model of Paragraph 4.1 and an auxetic composite layer is placed between the upper PZT patch and the host structure. More details about numerical values of structure and mechanical identities can found in [7]. The whole structure is showed in the next figure (Fig. 5).

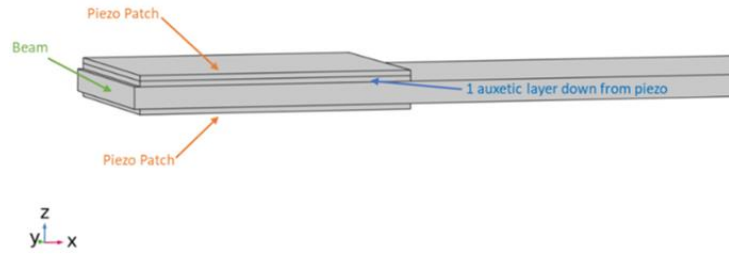


Figure 5: A Cantilever Beam with piezoelectric patches and auxetic layer.

Furthermore, the impact of thickness of the auxetic in vibration suppression is investigated. According to [7], the ideal thickness is $hA = 0.5\text{mm}$. Now considering the ideal thickness of the auxetic layer a “current-flowing” shunt piezoelectric circuit has been added. A classical, with positive Poisson's ratio and an auxetic layer have been studied for comparison reasons. In each case the material is placed at the same position. After computational experiments, the optimal values for the auxetic layer have been calculated and they are given in Table 2. Using the same values for the circuit parameters, the system response of a conventional material is investigated. The total FRF graph around the second and third eigenfrequency is presented in the Figure 6 and 7. Results demonstrate that the addition of an auxetic layer improves the efficiency of the shunt system. In contrary, a classical layer does not give an efficient frequency response.

Table 2: Optimal values of “current-flowing” circuit.

	Auxetic Material
2nd Eigenmode	$C_2 = 1.00\text{E-}06 \text{ F}$ $R_2 = 1500 \text{ Ohm}$ $L_{2\text{tot}} = 10.7 \text{ H}$
3rd Eigenmode	$C_3 = 8.30\text{E-}09 \text{ F}$ $R_3 = 6800 \text{ Ohm}$ $L_{3\text{tot}} = 7 \text{ H}$

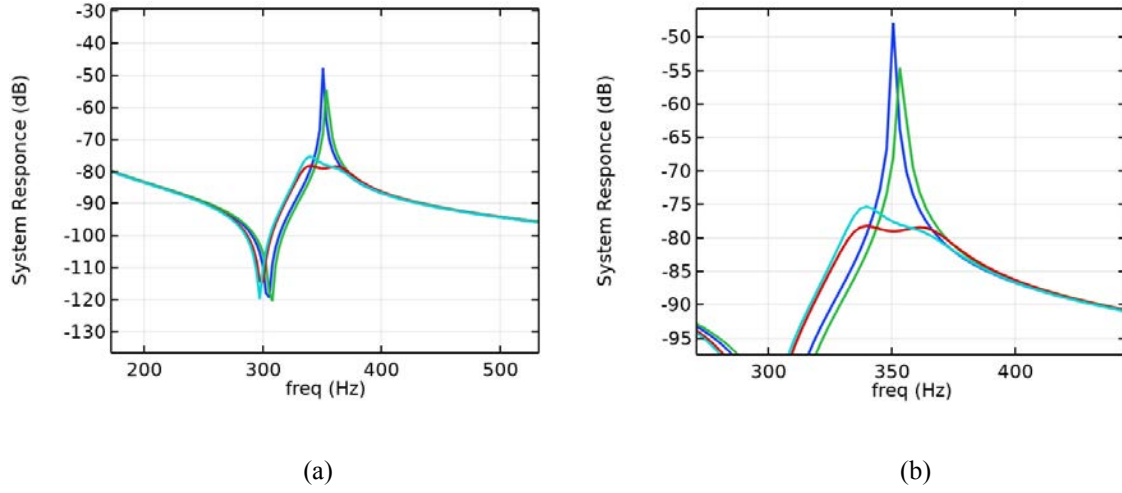


Figure 6: Simulated frequency response of the model around the second eigenfrequency - Open circuit curve for classical layer (blue line), Open circuit curve for auxetic (green line), Clas. Shunt circuit curve $R_2=1500\Omega$, $L_2=10.7H$, $R_3=6800\Omega$, $L_3=7H$ (turquoise line) and Aux. Shunt circuit curve $R_2=1500\Omega$, $L_2=10.7H$, $R_3=6800\Omega$, $L_3=7H$ (red line). (a) Full Image (b) Enlarged Image.

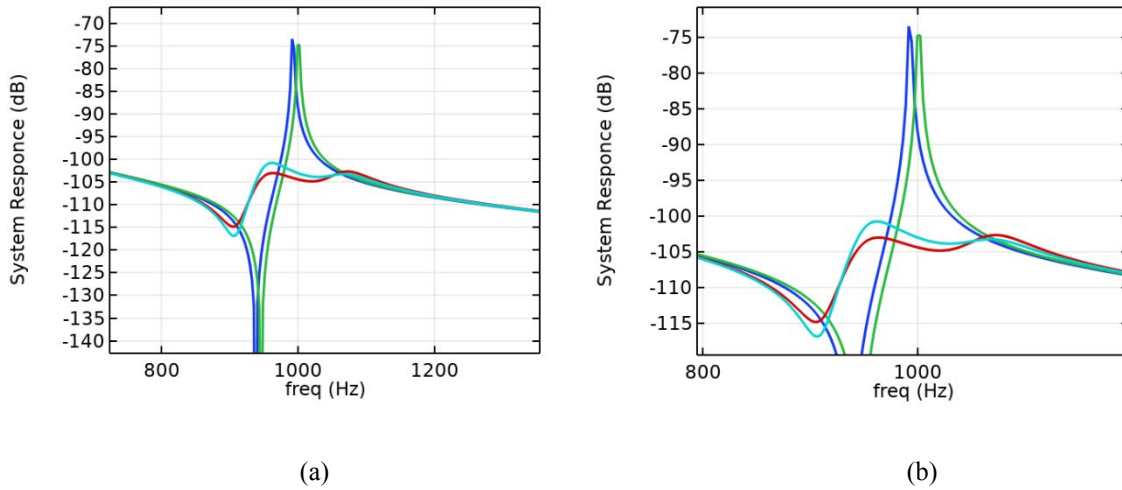


Figure 7: Simulated frequency response of the model around the third eigenfrequency - Open circuit curve for classical layer (blue line), Open circuit curve for auxetic (green line), Clas. Shunt circuit curve $R_2=1500\Omega$, $L_2=10.7H$, $R_3=6800\Omega$, $L_3=7H$ (turquoise line) and Aux. Shunt circuit curve $R_2=1500\Omega$, $L_2=10.7H$, $R_3=6800\Omega$, $L_3=7H$ (red line). (a) Full Image (b) Enlarged Image.

5 CONCLUSIONS

In this work, the finite element formulation for modelling the coupled electromechanical behavior of an elastic structure equipped with piezoelectric patches has been used. Two different computational models have been created, a one-dimensional homemade FE model

and a three-dimensional, based on multiphysics general purpose software. The main topic of this study was the presentation of a passive control strategy for effective damping multiple vibration modes of beam structures, using a “current-flowing” shunt circuit and an auxetic layer. A two-mode shunt circuit was designed and simulated for a cantilever piezo laminated beam. The results underly the importance of optimization for the fine-tuning of the shunt piezoelectric systems. These still contribute to the enhancement of piezoelectric shunt damping. Also, from the above analysis it is shown that an auxetic layer enhances the shunt damping effect. A homogenized auxetic layer with sufficient thickness, such as [7], is required but it is a first investigation and is necessary further research for finding the best results.

Finally, it should be noted that the system studied here is an extension actuation piezoelectric system according to [11]. Investigation of other piezoelectric actuators will be considered in the future. Furthermore, reduced order [12] or 2.5D models could be developed in order to enhance the 2D model and make it able to take into account the influence of the auxetic layer.

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