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COMBINING THE REDUNDANCY CONCEPT AND VIBRATION CONTROL FOR ACTUATOR PLACEMENT IN ADAPTIVE STRUCTURES

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Abstract. Actuators can be used to control the dynamic behavior of vibrating structures. The placement of the actuators in the system has a significant impact on how well adaptive structures perform. The actuator placement for damping particular modes can be assessed using the fraction of modal strain energy. Recent research in structural engineering has demonstrated that the redundancy concept is a useful tool for assessing actuator placement for quasi-static structural behavior because it provides information about the distribution of the degree of statical indeterminacy in the structure. In order to combine fundamental measures from control theory and structural engineering, the similarities between the frequency response function and the redundancy matrix are pointed out. It is shown that, by using the redundancy concept and the fraction of modal strain energy as assessment criteria for actuator placement, adaptive structures can be optimally designed to withstand both static and dynamic loadings.

1 INTRODUCTION

Due to the impending challenges of climate change and resource scarcity, adaptive structures in the context of civil engineering have received significant attention in the last years. These light-weight structures are capable of effective adaptation to a variety of load cases due to the interaction of sensors, actuators and a control unit. Recent studies have demonstrated that adaptivity has a significant mass-saving potential by substituting embodied energy by operating energy when necessary [1].

The redundancy concept has proven to be an effective strategy for the placement of actuators because it offers beneficial insights into the load-bearing behavior of statically indeterminate structures, see for instance [2, 3]. However, the use of the redundancy matrix for actuator placement is limited to the static system responses of structures so far.

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While adaptive structures are a new field of research in civil engineering, the use of actuators for active damping of space structures is already common practice. The concepts developed in this area pay less attention to static load-bearing behavior and focus on the dynamic system response instead. Investigations on the placement of actuators and the necessary control laws can be found, for example, in [4, 5]. It has been discovered that the fraction of modal strain energy is a useful metric for assessing the actuator placement in order to damp specific modes [4].

In this contribution we bring together control theory and recent research in structural engineering to optimally design adaptive civil engineering structures to withstand both static and dynamic loadings. In Section 2 we first demonstrate how the open-loop frequency response function and the redundancy matrix are connected. The actuator placement for quasi-static and dynamic loadings is then introduced and combined in Section 3. Using numerical examples, in Section 4 the effects of different actuator arrangements on the static and dynamic system responses of truss structures are discussed. Section 5 concludes by summarizing the findings and providing an outlook for future research.

2 SIMILARITIES BETWEEN REDUNDANCY MATRIX AND FREQUENCY RESPONSE FUNCTION

2.1 Redundancy matrix in structural mechanics

The redundancy matrix for truss structures was first used in [6]. In addition to the application for actuator placement in adaptive structures, it can also be used for other use cases such as robust design or quantifying imperfection sensitivity. The redundancy matrix and its mathematical and mechanical properties are deduced in detail in [7]. We limit our discussion in this contribution to the key considerations for the placement of actuators in truss structures, and therefore make reference to [3].

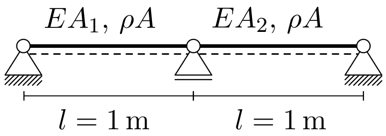
According to [7] the $n_\mathrm{e} \times n_\mathrm{e}$ redundancy matrix for a truss structure with n_d degrees of freedom and n_e elements can be derived from the constitutive equations and finally formulated by matrix operations:

$$\mathbf{R} = \mathbf{I} - \underbrace{\mathbf{A}\left(\mathbf{A}^\mathrm{T}\mathbf{C}\mathbf{A}\right)^{-1}\left(\mathbf{A}^\mathrm{T}\mathbf{C}\right)}_{\mathbf{A_0}}. \tag{1}$$

Herein, $\mathbf{I}$ is the $n_\mathrm{e} \times n_\mathrm{e}$ identity matrix, $\mathbf{A}$ the $n_\mathrm{e} \times n_\mathrm{d}$ equilibrium matrix, which provides information about the topology of the structure, and $\mathbf{C}$ the $n_\mathrm{e} \times n_\mathrm{e}$ diagonal stiffness matrix with the element stiffnesses $\frac{EA_i}{L_i}$ on the main diagonal entries. The $n_\mathrm{e} \times n_\mathrm{e}$ total self-strain matrix $\mathbf{A_0}$ is introduced for simplification. The redundancy matrix describes the relation between the n_e initial elongations $\mathbf{\Delta l_0}$ and the n_e negative elastic elongations $-\mathbf{\Delta l}_\mathrm{el}$

$$\mathbf{\Delta l}_\mathrm{el} = -\mathbf{R}\mathbf{\Delta l_0}. \tag{2}$$

Initial elongations can be caused, for example, by actuator displacements, temperature loads or manufacturing imperfections. The elastic element elongations $\mathbf{\Delta l}_\mathrm{el}$ can directly be transferred to normal forces $\mathbf{N}$ with the material law $\mathbf{N} = \mathbf{C}\mathbf{\Delta l}_\mathrm{el}$. The initial elongations and the elastic elongations sum up to the total elongations $\mathbf{\Delta l} = \mathbf{\Delta l_0} + \mathbf{\Delta l}_\mathrm{el}$. While the redundancy matrix



Material data

$0 < EA_1 < EA_2$

$\rho A = \text{const.} = 1\,\text{kN/m}$

Input matrices

$$\mathbf{A} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \frac{EA_1}{l} & 0 \\ 0 & \frac{EA_2}{l} \end{bmatrix}$$

Figure 1: Example structure with variable element stiffnesses EA_i.

maps initial elongations onto the negative elastic elongations, the total self-strain matrix $\mathbf{A_0}$ provides the relation between initial and total elongations

$$\mathbf{\Delta l} = \mathbf{A_0}\mathbf{\Delta l_0}. \tag{3}$$

The trace of the redundancy matrix corresponds to the degree of statical indeterminacy of the structure $n_\text{s} = n_\text{e} - n_\text{d}$. The main diagonal entries r_{ii} therefore provide information about the spatial distribution of the statical indeterminacy and describe the constraint of the surrounding structure on the element i. Initial elongations do not result in elastic elongations in statically determinate substructures, so their redundancy is equal to zero. The maximum value of r_{ii} would be one, where initial elongations do not result in total elongations but lead to elastic elongations only.

The redundancy matrix for the example structure in Fig. 1 can be calculated using Eq. (1) and depends on the ratio of the element stiffnesses EA_2/EA_1

$$\mathbf{R} = \begin{bmatrix} 1 - \frac{EA_1}{EA_1+EA_2} & \frac{EA_2}{EA_1+EA_2} \\ \frac{EA_1}{EA_1+EA_2} & 1 - \frac{EA_2}{EA_1+EA_2} \end{bmatrix}. \tag{4}$$

With an increasing stiffness ratio EA_2/EA_1 the redundancy of element 1 increases and that of element 2 decreases, while the degree of statical indeterminacy $n_\text{s} = \text{tr}(\mathbf{R})$ remains equal to one.

2.2 Frequency response function in vibration control

The frequency response function is now derived, following the ideas from [4] and transferring it to the matrix notation from Section 2.1 in parallel. Starting point is the modal expansion by solving the eigenvalue problem

$$(\mathbf{K} - \omega_n^2\mathbf{M})\boldsymbol{\phi}_n = \mathbf{0} \tag{5}$$

with $\mathbf{K} = \mathbf{A}^\text{T}\mathbf{C}\mathbf{A}$ being the stiffness matrix, $\mathbf{M}$ the mass matrix, ω_n the n-th circular frequency and $\boldsymbol{\phi}_n$ the n-th eigenmode. The stiffness matrix and the mass matrix can both be diagonalized using the eigenmodes. The modal mass and the modal stiffness result in

$$\boldsymbol{\phi}_n^\text{T}\mathbf{M}\boldsymbol{\phi}_n = M_n \tag{6}$$

$$\boldsymbol{\phi}_n^\text{T}\mathbf{K}\boldsymbol{\phi}_n = K_n = \omega_n^2 M_n. \tag{7}$$

When the structure is vibrating according to mode n, the strain energy in the whole structure is given as

$$U^n_\text{struct} = \frac{1}{2}\omega_n^2 M_n = \frac{1}{2}\left(\boldsymbol{\phi}_n^\text{T}\mathbf{A}^\text{T}\mathbf{C}\mathbf{A}\boldsymbol{\phi}_n\right), \tag{8}$$

while the strain energy in a single element i can be calculated as

$$U_i^n = \frac{1}{2} \left(\mathbf{A}_i \boldsymbol{\phi}_n\right) \left(\boldsymbol{\phi}_n^{\mathrm{T}} \mathbf{A}_i^{\mathrm{T}}\right) C_{ii}, \tag{9}$$

where $\mathbf{A}_i$ is the i-th row of the equilibrium matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_{n_\mathrm{e}} \end{bmatrix}. \tag{10}$$

By replacing $\mathbf{A}_i$ by $\mathbf{A}$ and $\mathbf{C}_{ii}$ by $\mathbf{C}$ the distribution of the strain energy in the structure is obtained in matrix form. The normalization by dividing by the strain energy in the whole structure U_{struct}^n leads to the distributed fraction of modal strain energy

$$\boldsymbol{\nu}^n = \left(\mathbf{A}\boldsymbol{\phi}_n\right) \left(\boldsymbol{\phi}_n^{\mathrm{T}} \mathbf{A}^{\mathrm{T}} \mathbf{C} \mathbf{A} \boldsymbol{\phi}_n\right)^{-1} \left(\boldsymbol{\phi}_n^{\mathrm{T}} \mathbf{A}^{\mathrm{T}} \mathbf{C}\right). \tag{11}$$

The relation between the initial elongation, e. g. from a actuator extension, and the elastic elongation in one distinct element i, which can be obtained from a measured normal force, is defined by the open-loop frequency transfer function given in [4]:

$$\Delta l_{\mathrm{el},i} = \left[\sum_{n=1}^{n_\mathrm{d}} \frac{\nu_{ii}^n}{1 - \frac{\omega^2}{\omega_n^2}} - 1 \right] \Delta l_{0,i}. \tag{12}$$

To give the relation between the initial and elastic elongations in all elements the distributed fraction of modal strain energy from Eq. (11) is used

$$\boldsymbol{\Delta}\mathbf{l}_{\mathrm{el}} = \underbrace{\left[\sum_{n=1}^{n_\mathrm{d}} \frac{1}{1 - \frac{\omega^2}{\omega_n^2}} \boldsymbol{\nu}^n - \mathbf{I} \right]}_{\mathbf{G}(\omega)} \boldsymbol{\Delta}\mathbf{l}_0. \tag{13}$$

The frequency response function $\mathbf{G}(\omega)$ now gives a description with potential active elements in all members in terms of actuation. Therefore, it provides an insight into the dynamic load-bearing behavior – as well as the potential ways to influence it, which will be covered in Section 3.2. A remarkable relationship exists between the redundancy matrix and the frequency response function: The frequency response function is equal to the negative redundancy matrix for $\omega = 0$, which represents a quasi-static load-bearing behavior:

$$\mathbf{G}(\omega = 0) = \sum_{n=1}^{n_\mathrm{d}} \boldsymbol{\nu}^n - \mathbf{I} = \mathbf{A_0} - \mathbf{I} = -\mathbf{R}. \tag{14}$$

Therefore, it is possible to interpret the redundancy matrix as a special case of the frequency response function. Additionally, both formulations can be applied to the placement of actuators with various actuation goals.

3 ACTUATOR PLACEMENT IN ADAPTIVE STRUCTURES

3.1 Manipulation of forces and displacements

For actuator placement the adaptation goal is crucial, so we distinguish between manipulating forces and displacements. It seems reasonable to place actuators to manipulate displacements in statically determinate parts of the structure because no normal forces and actuator forces are generated in this case. In order to influence normal forces, constraint must be activated. Therefore, in this case actuators are placed in statically indeterminate parts of the structure.

To quantify these intuitive evaluations, the redundancy matrix offers useful information, see [3]. The image of the redundancy matrix $\mathrm{im}(\mathbf{R})$ contains the n_s independent self-stress states without any displacements. The nullspace of the redundancy matrix $\ker(\mathbf{R})$ spans the n_d-dimensional space of compatible elongations without any normal forces. For the small example structure in Fig. 1 with fixed element stiffnesses $EA_2 = 2EA_1$ the eigenvalues and eigenvectors are

$$\lambda_1 = 0 \quad \text{with} \quad \boldsymbol{\varphi}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \tag{15}$$

$$\lambda_2 = 1 \quad \text{with} \quad \boldsymbol{\varphi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}. \tag{16}$$

There are no normal forces produced if element 1 experiences an initial elongation of 1 and element 2 undergoes an initial elongation of -1. In contrast to this, if element 1 undergoes an initial elongation of 2 and element 2 an initial elongation of 1, there are no displacements but only normal forces.

In order to manipulate forces, actuators should be placed in components with non-zero redundancy and in such a way that the space of self-stress states is completely spanned. For this purpose exactly n_s actuators are required. In order to manipulate displacements, the actuators should be placed in elements with low redundancy instead. The kinematic equation can be used to convert the total elongation states in the columns of matrix $\mathbf{A_0}$ into nodal displacements [3]. Each column i of the resulting matrix contains the nodal displacements for elongations of element i and every row n gives the displacements of the n-th degree of freedom due to elongations in all elements. For every displacement that is to be controlled independently, one actuator is necessary [8].

3.2 Active damping using the fraction of modal strain energy

In order to actively damp certain modes, the use of the fraction of modal strain energy for actuator placement is suggested in [4]. When the structure is moving according to mode n, an element i with a high fraction of modal strain energy ν_{ii}^n receives a lot of strain energy. It is reasonable to place actuators in those elements to damp a certain mode n, since there is less constraint the actuator has to actuate against. When the actuator is, for example, controlled by an integrated force feedback, see [9], energy can be taken from the structure and the system is damped as a consequence.

Direct derivation of the connection to the actuator placement according to the redundancy concept from Section 3.1 is possible. The matrices $\boldsymbol{\nu}^n$ are the projections of the total self-strain

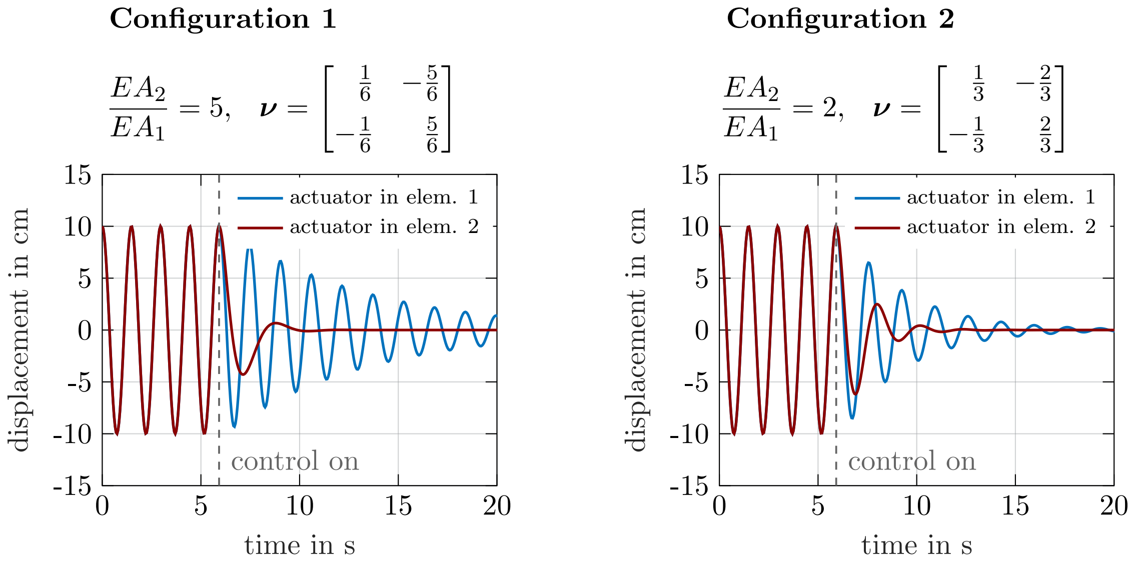


Figure 2: Active damping of structure from Fig. 1 for two different stiffness ratios EA_2/EA_1.

matrix $\mathbf{A_0}$ in the directions of the eigenmodes $\boldsymbol{\phi}_n$. Therefore, the columns of $\boldsymbol{\nu}^n$ are states of compatible elongations in the direction of the eigenmode $\boldsymbol{\phi}_n$. The damping of vibrations according to eigenmodes is a manipulation of displacements. In contrast to Section 3.1, actuator placement is not evaluated in order to control certain nodal displacements but certain modes, which are also displacement states. For every mode that is to be controlled independently, one actuator is necessary, analogous to the manipulation of displacements from Section 3.1.

In Fig. 2, the potential damping with integrated force feedback (IFF) is assessed for two configurations of the small example presented in Section 2.1. Those differ in the distribution but not the overall stiffness of the structure. In both configurations an actuator in element 2, which is the element with a higher fraction of modal strain energy, performs better than an actuator in element 1. While the damping with an actuator in element 2 is almost equally good for both configurations, a significantly better performance of an actuator in element 1 is achieved due to the higher fraction of modal strain energy of $\boldsymbol{\nu}_{11} = 1/3$.

4 ACTUATOR PLACEMENT FOR ADAPTIVE HIGH RISE BUILDING

Finally, we will apply our findings to an example structure that has already been thoroughly investigated in earlier research [1, 3] with regard to actuator placement and mass minimization through adaptivity. Fig. 3 shows the example structure with the material parameters and the distribution of redundancy in the structure. Three sections make up the example structure. The degree of static indeterminacy for the lower part of the structure, which has members 1 to 10, is equal to two, while the degree of static indeterminacy for the upper part, which has members 14 to 19, is equal to one. Elements 11 to 13 connect the two statically indeterminate parts of the structure in a statically determinate way. As a result, both the degree of statical indeterminacy for the entire structure and the trace of the redundancy matrix are equal to three. For the sake of simplicity, this study will only consider a structure with equal cross-sections in all elements.

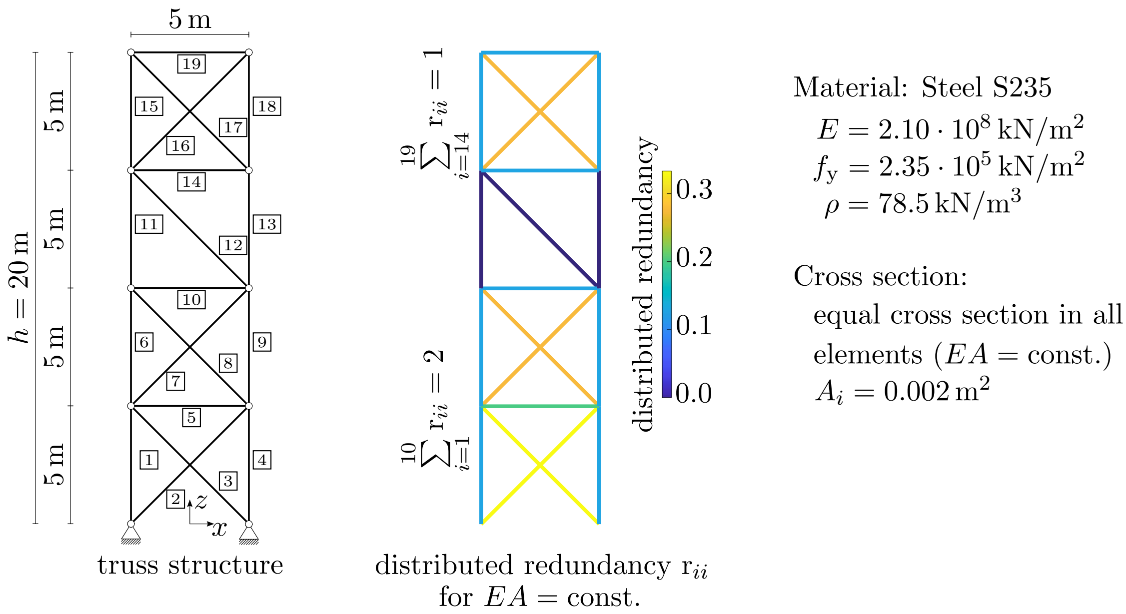


Figure 3: Exemplary truss structure and its redundancy distribution.

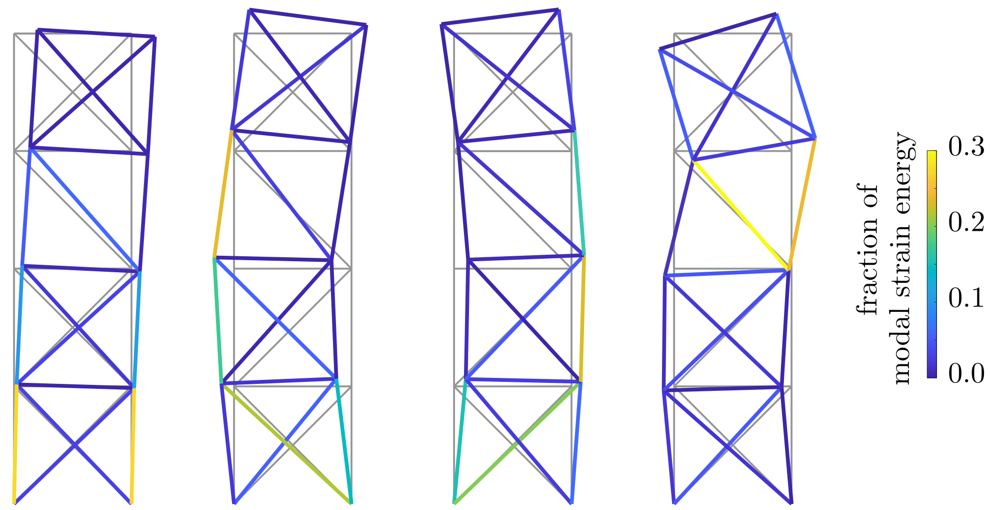
We concentrate on the interaction of actuator placement for force manipulation of quasi-static behavior and damping in case of dynamics and do not take mass minimization into account.

The system's first four eigenmodes are depicted in Fig. 4 along with the corresponding natural frequencies. The distributed fraction of modal strain energy in each mode is illustrated by the color coding. The highest fractions of modal strain energy appear in different elements, depending on the mode considered. While elements 1 and 4 would be ideal as active elements for the first mode, their damping potential is lower in modes 2 and 3, and it is close to zero in mode 4. The structural part that is statically determinate contains almost all of the modal strain energy in mode 4. This means that an optimal placement for damping mode 4 would be in statically determinate elements with redundancy equal to zero, which is a typical feature of displacement adaptation.

Now, it is possible to place actuators using both the redundancy distribution shown in Fig. 3 for the static case and the distribution of modal strain energy shown in Fig. 4 for the dynamic case. However, the combination of these criteria is not straightforward because the goals obviously oppose each other. Already the actuator placement for a few modes according to the fractions of modal strain energy contains contradictions and the actuator placement for displacement adaptation and force adaptation are generally also rather opposed to each other. Therefore, three different actuator sets are formed, each optimized for only one of the tasks, in order to check how much their overall performance differs.

Set 1 is optimized for force adaptation. In order to achieve this, the actuators are positioned in elements 2, 7 and 16 to completely span the image of the redundancy matrix, see Fig. 5. The active elements have high redundancy distributions r_{ii}.

The first step in placing actuators to damp particular modes is to choose all relevant modes for the structural behavior. For the following considerations we want to focus on modes 1 to 3



Mode 1 $\omega_1 = 1.83\,\frac{\text{rad}}{\text{s}}$ Mode 2 $\omega_2 = 8.18\,\frac{\text{rad}}{\text{s}}$ Mode 3 $\omega_3 = 8.61\,\frac{\text{rad}}{\text{s}}$ Mode 4 $\omega_4 = 16.32\,\frac{\text{rad}}{\text{s}}$

Figure 4: Lowest four eigenmodes for example structure and related distributed fractions of modal strain energy.

and damp them using as many actuators as in set 1. In [9], a method is proposed in which one actuator is placed such that the sum of weighted fractions of modal strain energy in the active element over the modes is maximized. The weighting depends on the application and is based on the engineer's knowledge and experience. For our example it will be one for modes 1 to 3 and zero for all other modes. This results in the composition of set 2 with actuators in elements 1, 4 and 9.

We want to add another variant to ensure that each of the chosen modes is adequately taken into account. Each actuator should be able to control one mode and is therefore placed in the element with the highest value for the fraction of modal strain energy for the respective mode. Element 4 is chosen for mode 1, element 11 for mode 2, and element 9 for mode 3.

The ability to control the forces in the structure varies between the three different sets. Set 1 was chosen based on this criterion, making it the best set in this regard. The other two sets are less suited for force adaptation because they cannot activate the third self-stress state as there are no actuators in the upper part of the structure. The redundancy ratios of the active elements are also not as high as those of set 1. Larger actuator forces are therefore required for achieving the same self-stress states.

To explore the influence of adaptivity on the dynamic behavior, the sets are numerically tested toward their ability to damp various modes. The actuators are controlled by an integrated force feedback (IFF), see for instance [4, 9]. The structure experiences initial displacements in the direction of a certain eigenmode and is then damped by the actuators to return to its static equilibrium. For each mode, a so-called optimal set is used as a reference solution. The optimal set is obtained by placing the three actuators in the elements with highest fractions of modal strain energy for this particular mode. For mode 1, set 2 already represents the optimal set.

In Fig. 6 the largest x- or y-displacement of a node in the initial displacement is chosen and

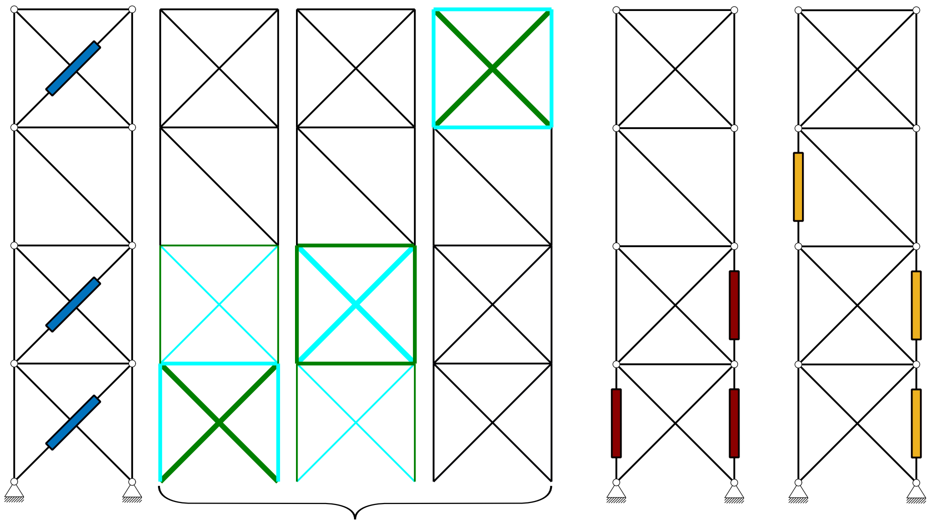


Figure 5: Actuator placement: set 1 {2, 7, 16} (optimized for force manipulation) with space of self-stress states, set 2 {1, 4, 9}, set 3 {4, 9, 11} (optimized to control modes 1 to 3).

then plotted over time.

As expected, the results for modes 1 to 3 show that set 1 exhibits the worst behavior and the smallest damping can be achieved. Modes 1 and 3 are slightly better damped with set 2 compared to set 3. Set 3, on the other hand, is more suitable for damping mode 2. The better of the two configurations is in all three modes as good or almost as good for damping as the optimal set. It is hardly possible to generalize which of the two methods for damping particular modes will produce better outcomes. Both methods behave comparably well here. Additionally, it can be assumed that the distribution of the fractions of modal strain energy over the modes and elements has a significant impact on the damping performance.

We also consider mode 4, which was not taken into account in the actuator placement. Here, sets 2 and 3 hardly have any damping effect. Set 1 performs slightly better, but is also far from being the optimal set. Fig. 4 shows that the strain energy for mode 4 is concentrated almost exclusively in elements 12 and 13. Damping is hardly possible through actuators in other elements. This demonstrates once more the significance of carefully choosing the modes to be damped and how the distribution of the modal strain energy over the elements varies between the modes.

5 CONCLUSIONS

First, the formal relationship between the redundancy matrix from structural mechanics and the frequency response function from control theory was established. Both can be used to place actuators for various adaptation goals, which have to be defined for effective actuator placement. For the quasi-static case, a distinction has already been made between the manipulation of forces and displacements. Therefore, the redundancy concept can be used for actuator placement.

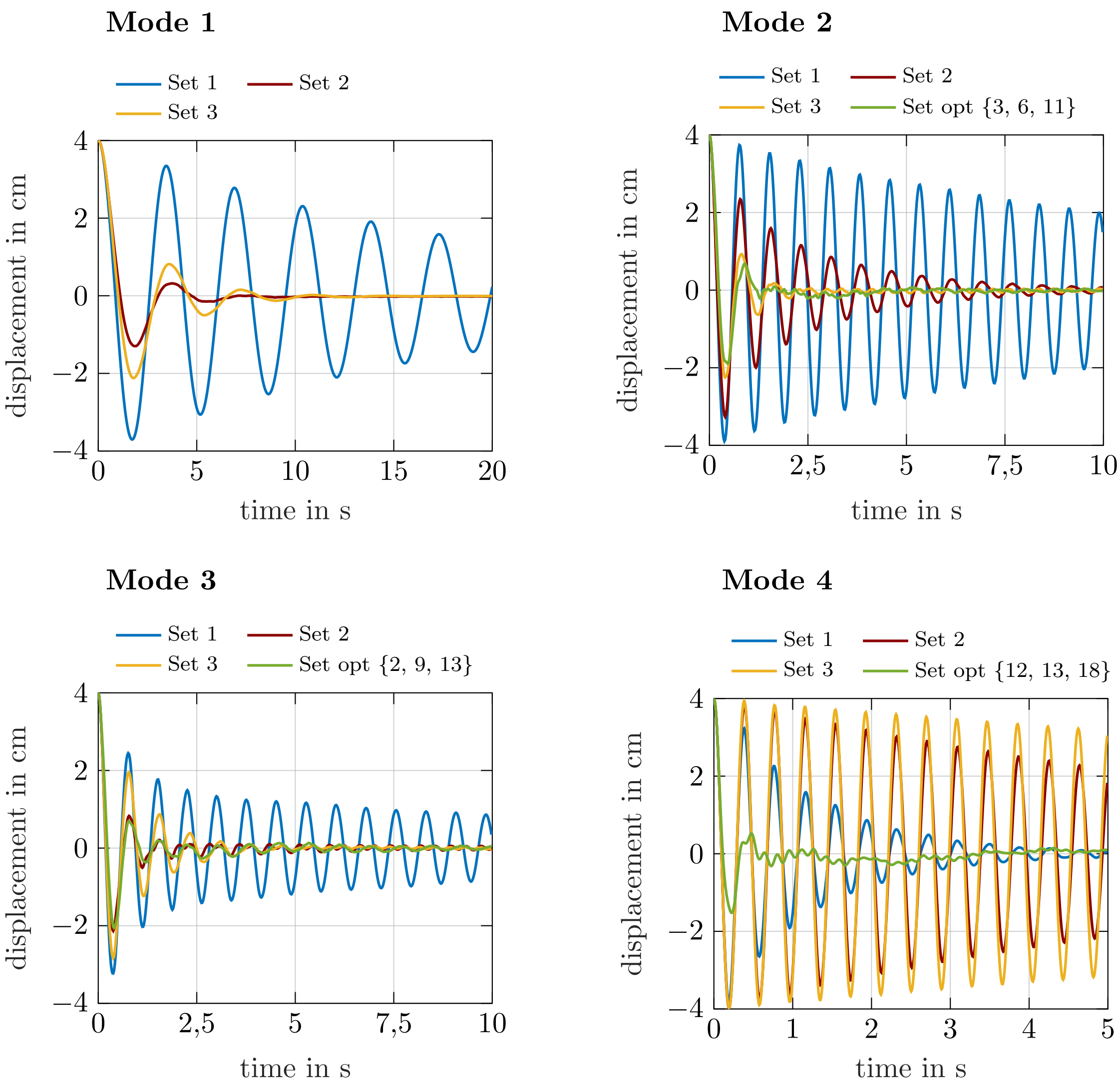


Figure 6: Damping of the first four eigenmodes with different actuator sets.

On the other hand, the distributed fraction of modal strain energy, which comes from a modal expansion of the frequency response function, serves as a quality indicator for actuator placement in order to damp certain modes. It has been shown that the damping of certain modes is a form of displacement adaptation.

Since the different adaptation goals often conflict, engineering knowledge and experience is particularly important in formulating adaptation goals and selecting the modes to be damped. Therefore, future research should also look into how a structure can be built to be suitable for various adaptation goals with as few actuators as possible.

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