Available online at www.eccomasproceedia.org
Eccomas Proceedia SMART (2023) 735-743

ECCOMAS Proceedia

X ECCOMAS Thematic Conference on Smart Structures and Materials
SMART 2023
D.A. Saravanos, A. Benjeddou, N. Chrysochoidis and T. Theodosiou (Eds)

DETECTION AND IDENTIFICATION OF STRUCTURAL FAILURE USING THE REDUNDANCY MATRIX

Tamara Prokosch*, **Jonas Stiefelmaier**†, **Cristina Tarìn**† **and Manfred Bischoff***

*University of Stuttgart
Institute for Structural Mechanics
Pfaffenwaldring 7, 70550 Stuttgart, Germany
e-mail: prokosch@ibb.uni-stuttgart.de

† University of Stuttgart
Institute for System Dynamics
Waldburgstraße 17/19, 70563 Stuttgart, Germany

Abstract. Structural failure in civil engineering can have catastrophic consequences such as property damage, economic losses and even loss of life. Therefore, early detection and identification are crucial. In addition to that, new technologies like adaptive structures demand for high safety as a prerequisite to gain acceptance. This contribution investigates the feasibility of using the redundancy matrix as a tool for detecting and identifying structural failure in adaptive structures via residual generation. Results provide prerequisites for detectability and suggest methodologies for the detection of structural failure by means of the redundancy matrix.

Key words: Failure Diagnosis, Redundancies, Residual Generation, Adaptive Structures

1 INTRODUCTION

Adaptive civil engineering structures are the topic of recent research and promise to be effective in saving material and resources in order to deploy more sustainable structures. They do so by adapting to external influences and therefore changing the response of the structure. Depending on the aim of the adaptation, immediate results are the reduction of deflections or redistribution of inner forces. As a consequence, deflection limits and stress limits can be met with use of less material. To understand and predict the response of adaptive structures, so-called redundancy matrices have already proven to be a useful tool [1]. They give insight into the load-bearing behavior and possible alternative load paths through redundant parts of the structure. The transition from adaptive structures being a theoretical research topic to building practice has already begun. In 2021 the construction of an adaptive high-rise tower in Stuttgart was finished as part of a research project, and this demonstrator is currently being tested [2]. For this transition, one of the most important aspects still to be covered by research is the topic of safety. Adaptive structures need to be proven safe not only to comply with building regulations, but also to gain widespread acceptance in the construction industry and society. With

Peer-review under responsibility of the organizing committee of SMART 2023.
doi: 10.7712/150123.9827.444431

the availability of sensors and a control unit in adaptive structures, it is evident to investigate the possibilities of structural health monitoring. In addition to that, quantification of structural redundancies is helpful for the purposeful integration of actuators in corresponding safety concepts.

The idea of adaptive structures has been around since the 1960s and one of the first conceptualizations of an adaptive structure can be found in Soong and Manolis [3]. The authors present the basic idea of structures adapting to changing environmental loads and provide a problem formulation for their model. In the following years, there has been a variety of works giving a good overview of the topic. An important contribution can be found in Utku [4]. The author provides a fundamental understanding of adaptive structures, their design principles, and their application in various engineering fields. Domke [5] presents a detailed exploration of active civil structures, including their design and analysis principles. An overview of recent research on adaptive structures can be found in Neuhäuser et al. [6].
The roots of the redundancy concept trace back to the works of Bahndorf [7] and Ströbel [8]. They are the first to mention the derivation of redundancies for truss structures and apply it to cable-strut structures. Bahndorf also shows the redundancies for frame structures but does not provide a method for their calculation yet. The concept was later revisited by von Scheven et al. [9], where a derivation of redundancies for discrete beam structures is given. A further extension of the theory to continuous models can be found in Gade et al. [10].
Most recent research applied the redundancy concept to the design and analysis of adaptive structures. In this way, information on the redundancy distribution has been used for example for optimization in the design of adaptive structures (Geiger et al. [11]) and actuator placement in (Wagner et al. [12]). The first connection between redundancies and the safety of structures has been made by Kou et al. [13]. By investigating structural failure in connection with redundancies they pointed out the possible application to load path analysis.

This paper builds upon the perspective of using information about the redundancy distribution to gain insight into the structure's load paths and investigates the case of structural failure in truss structures. As for redundancies providing information about actuation modes in adaptive structures, the knowledge of failure behavior is then applied to the methodologies of analyzing adaptive structures and consequently the diagnosis of failure in these structures. The methods derived from the investigation are then applied to the analysis of a sample truss structure and the result is discussed. A special focus lies on the sensitivity of the structural response to the failure of elements, which yields information on the failure detectability. Furthermore, the ability to localize structural failure through means of the redundancy distribution is discussed.

The presented methods are novel to the application of adaptive structures, provide a representation of the theory of redundancies for truss structures and the analysis in case of structural failure. A relation can be formed to the method of using influence vectors for self-diagnosis, as in Adam and Smith [14]. Therein, the diagnosis is carried out on a tensegrity structure.

2 METHODS

2.1 Fault Diagnosis

Fault detection and diagnosis in civil structures is a part of structural health monitoring (SHM). SHM aims for early detection of failure to ensure safety and the possibility of reacting early with repairs to keep the resulting consecutive damage low. Since adaptive structures are complex and highly correlated systems, a model-based approach for fault detection and following isolation is necessary. For the detection of faults, measurements are usually compared to an expected value derived from a model of the intact system. For this paper, the compared values are the elongations of the elements in the truss structure. The comparison is formed by residuals that show the deviation of the measurements $\Delta\hat{\mathbf{l}}$ from the expected elongations $\Delta\mathbf{l}$

$$\mathbf{e} = \Delta\hat{\mathbf{l}} - \Delta\mathbf{l}. \tag{1}$$

If these residuals are evaluated to zero, the system is fault-free and behaves like the intact model. If the residuals are non-zero, a fault can be assumed. Usually, for detection, a certain threshold is defined, which depends on uncertainties that stem from the quality of measurements and the model. After a fault has been detected, further diagnosis can lead to the isolation of the fault. For this, the residuals are compared to a previously generated table of fault modes. If the residual pattern can be found in the table, the failure can be isolated. For the isolation to result in a single fault, the modes have to be linearly independent. The higher the difference between the fault modes, the better the fault can be isolated. For this work, only the occurrence of structural failure is considered to ensure the focus on their detection and isolation.

2.2 Redundancy and Actuation Model

The redundancy matrix provides the spatial distribution of redundancies in a structure. Based on the concept of static indeterminacy it gives insight into the load-bearing behavior of the structure. If a structure or part of a structure is statically determinate, failure of an element results in collapse. If a structure is statically indeterminate, there are multiple load paths by which the load can be carried. Redundant members can fail without resulting in total failure of the system. If they fail, alternative load paths are activated for carrying the load. The redundancy matrix for the case of linear elastic truss structures with discrete topological information can be calculated as

$$\mathbf{R} = \mathbf{1} - \mathbf{A}\left(\mathbf{A}^{\mathrm{T}}\mathbf{C}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{C}. \tag{2}$$

The matrix $\mathbf{A}$ is called the equilibrium matrix and contains the topological information of the structure, describing the kinematic modes. The material matrix $\mathbf{C}$ contains information about the cross-section of the elements and the Young's modulus.
The resulting redundancy distribution maps the initial elongations to the elastic elongations.

$$\mathbf{R}\,\Delta\mathbf{l}_0 \;=\; -\Delta\mathbf{l}_{\mathrm{el}}. \tag{3}$$

Initial elongations correspond to constraints like temperature load cases or prescribed displacements of supports. The elastic elongations are related to the inner forces in the structure. The sum of both forms the total elongations, which can be derived from measurements

$$\Delta\mathbf{l} = \Delta\mathbf{l}_0 + \Delta\mathbf{l}_{\text{el}}. \tag{4}$$

Actuation in adaptive structures can be modeled as a constraining load case with the actuator stroke $\hat{u}$ as the initial elongation of the actuated element. In this case, the actuation is in series with the passive element. With this model, the columns of the redundancy matrix can be interpreted as actuation modes. Each mode represents the elastic elongations resulting from a unit actuation of an element. The total elongations for actuation can then also be expressed by the use of the redundancy matrix as

$$\Delta\mathbf{l} = (\mathbf{1} - \mathbf{R})\Delta\mathbf{l}_0. \tag{5}$$

2.3 Structural Failure and the Redundancy Matrix

Methods to analyze structural failure in truss members usually include solving a linear system of equations. In each step, the stiffness matrix is inverted for every possible case of member failure. This tends to be intensive use of processing power for systems with higher numbers of structural members and is not feasible to be used for online calculation. For the update of the redundancy matrix, Kou et al. [13] provide a formula to recalculate the redundancy matrix $\mathbf{R}'_k$ for a system reduced by one member, only by using only the entries of the initial redundancy matrix:

$$\mathbf{R}'_k \text{ with } \mathrm{r}'_{ij} = \mathrm{r}_{ij} - \frac{\mathrm{r}_{ik}\mathrm{r}_{kj}}{\mathrm{r}_{kk}}. \tag{6}$$

The update of the redundancy matrix represents a reduction by the constraint that member k is acting out on the other members of the structure.

3 RESIDUAL GENERATION WITH THE REDUNDANCY MATRIX

From the given formula to update the redundancy matrix in case of member failure, the actuation modes for structural failure of member k are derived as

$$\Delta\mathbf{l}'_k = (\mathbf{1} - \mathbf{R}'_k)\Delta\mathbf{l}_0. \tag{7}$$

By taking the difference to the actuation modes in the initial state of the structure, the residuals in elongations for the fault states can be derived. Selection of the columns corresponding the actuation of element α forms a matrix of residual patterns

$$\mathbf{e}^{\alpha} \text{ with } \mathrm{e}^{\alpha}_{ik} = -\frac{\mathrm{r}_i\mathrm{r}_{k\alpha}}{\mathrm{r}_{kk}}. \tag{8}$$

The matrix contains the deviations from expected elongations that occur for the actuation in an element α. Each column corresponds to a fault mode for the failure of one element k in the structure. These residual patterns can be used for fault diagnosis in adaptive structures by carrying out test actuations. For fault diagnosis, uncertainties play a major role. They emanate from the quality of the measurement and uncertainties in the reference model. Therefore, thresholds are to be defined for the residuals. If the difference between the fault vectors and the expected elongations in the reference model is higher than these thresholds, a fault is detected. If the fault vectors are pairwise linearly independent and the distance between them is high enough to overcome the threshold, the structural failure can be localized.

4 CASE STUDY

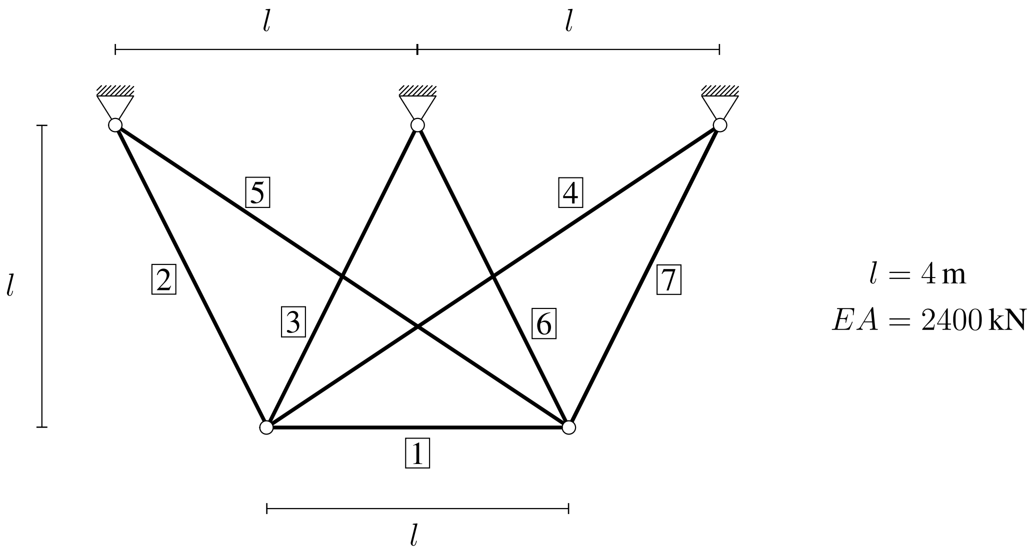

Figure 1: Example truss system

To investigate the detection and isolation, an example truss structure with seven members and five nodes is analyzed. The system, its dimensions and material parameters are given in Figure 1.

The system's redundancy matrix is

$$\mathbf{R} = \begin{bmatrix} 0.259 & 0.195 & -0.100 & -0.152 & -0.152 & -0.100 & 0.195 \\ 0.218 & 0.273 & -0.302 & 0.046 & -0.128 & -0.084 & 0.164 \\ -0.112 & -0.302 & 0.478 & -0.284 & 0.066 & 0.043 & -0.084 \\ -0.275 & 0.075 & -0.458 & 0.618 & 0.161 & 0.106 & -0.207 \\ -0.275 & -0.207 & 0.106 & 0.161 & 0.618 & -0.458 & 0.075 \\ -0.112 & -0.084 & 0.043 & 0.066 & -0.284 & 0.478 & -0.302 \\ 0.218 & 0.164 & -0.084 & -0.128 & 0.046 & -0.302 & 0.273 \end{bmatrix}. \tag{9}$$

The degree of statical indeterminacy of the structure is equal to 3, which complies with both the rank and the trace of $\mathbf{R}$. Its highly connected topology distributes the indeterminacy over the whole structure. This can be seen by the fully populated redundancy matrix. The columns of the matrix represent actuation modes for unit actuation in the elements. Since there are no zero entries in the redundancy matrix, an actuation in any element of the structure leads to elastic deformations in all elements. The fact that there are no zero entries in the redundancy distribution means also, that any element can fail without resulting in a total collapse.

For the following investigations, we choose member number six to be actuated. The resulting elastic elongations in the initial state are therefore in the sixth column of the redundancy matrix. In addition to the initial unit elongations in member six the total elongations are

$$\Delta \mathbf{l}^6 = \begin{bmatrix} -0.100 & -0.084 & 0.043 & 0.106 & -0.458 & 1+0.478 & -0.302 \end{bmatrix}^{\mathrm{T}}. \qquad (10)$$

This is the reference model state that the residuals as deviations will be calculated with. The resulting residual modes for structural failure according to (8) are displayed in Figure 2.

$\mathbf{e}^6 =$

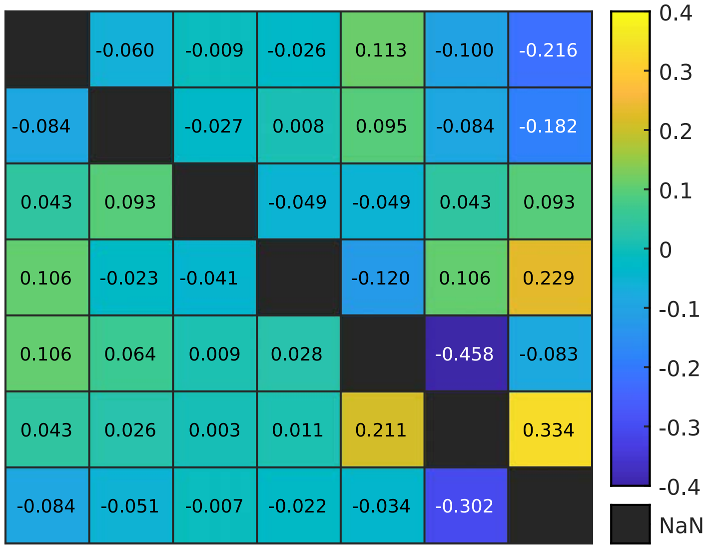

Figure 2: Failure modes for residual evaluation

Each column in the matrix represents the residuals corresponding to the failure in an element. The rows of the table represent the residuals in the measurements of one element for that failure state. The black entries in the table are therefore the residuals in the elements that failed and cannot be determined. The higher the entries, the more reliable the failure can be detected and higher uncertainties can be tolerated. Pairwise linear independence between these the residuals means that the failure can be located to the exact member and the higher the difference between them, the lower the threshold for locating the failure can be chosen.

4.1 Fault detection

As a measure for fault detectability, the euclidean norm of the fault vector is used. The higher the norm of the vector, the better the fault can be detected. The evaluation of the euclidean norm for each fault vector is

$$||\mathbf{e}_k^6||_2 = \begin{bmatrix}0.202 & 0.143 & 0.052 & 0.067 & 0.291 & 0.576 & 0.510\end{bmatrix}. \tag{11}$$

It is reasonable that the fault vector for the failure of element six has the highest norm, since actuation in this element would not have any influence on the surrounding structure anymore. Therefore, failure of element six results in the biggest difference to the intact state of the structure. It is also reasonable that the failure of members one, five, and seven are the failure modes best detectable after that, because they share a node with the actuated element. The least detectable failure mode is the one for element three. If the fault detection aims to detect a failure in any element of the structure with an actuation in element six, the threshold for detection has to be set according to this mode.

4.2 Fault isolation

To isolate the failure, the difference between the fault vectors has to be sufficiently high. Again, this is measured by the euclidean norm. As an example, the distances from the residuals in failure mode two to all modes in the residual table are given as

$$||\mathbf{e}_2^6 - \mathbf{e}_k^6||_2 = \begin{bmatrix}0.156 & 0.0 & 0.118 & 0.222 & 0.633 & 0.750 & 0.526\end{bmatrix}. \tag{12}$$

If the evaluated distance to the failure mode is zero, the failure can be located in the corresponding element. In this case, the distance is zero for failure case two. The highest difference can be evaluated as a failure in element six. Again, by considering the topology of the structure, this seems reasonable since they do not share a direct connection. The minimum distance to the residual of other failure modes can be linked to element three. It is found to be the minimum distance for the evaluation of all failure modes. To isolate the fault and detect the location of failure, the threshold that the residuals have to overcome must be higher than this value. Interestingly, the threshold for locating the failure, in this case, is decisive for the setup of the overall failure diagnosis. If the system is accurate enough to detect structural failure, it is automatically accurate enough to locate it.

4.3 Additional Load Cases

Since the method assumes linear elastic model behavior, the principle of superposition is applicable for additional load cases. If unknown disturbances can be assumed to be constant during the time of the actuation, failure can be detected in the same way as described before. Therefore, measurements are taken before and after actuation. The residuals are then generated based on the change between these two measurements and, due to of the principle of superposition, represent solely the elongations coming from the actuation.

5 CONCLUSIONS AND FURTHER WORK

This paper provides a method for using the redundancy matrix for the detection and isolation of structural failure in adaptive structures. The method is based on the actuation of elements and fault detection using a model-based residuals. The results of a case study show that structural failure can be detected and isolated for the example structure. Prerequisites like thresholds for the detection can be defined after evaluation of the residuals. In conclusion, the redundancy matrix is a promising tool for detecting and identifying structural failure. This allows for timely and effective repair and safer structures. Further research includes investigations on suitable thresholds with realistic uncertainties to validate the proposed methodology.

FUNDING

The work described in this paper was conducted in the framework of the Collaborative Research Center 1244 Adaptive Skins and Structures for the Built Environment of Tomorrow, funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under project number 279064222. The authors are grateful for the generous support.

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