Available online at www.eccomasproceedia.org
Eccomas Proceedia SMART (2023) 966-977

ECCOMAS
Proceedia

X ECCOMAS Thematic Conference on Smart Structures and Materials
SMART 2023
D.A. Saravanos, A. Benjeddou, N. Chrysochoidis and T. Theodosiou (Eds)

TOPOLOGICAL DATA ANALYSIS FOR LAMB WAVES BASED SHM METHOD IN OPERATIONAL CONDITIONS — SMART 2023

Arthur Lejeune[*†], **Nicolas Hascoët**[†], **Marc Rébillat**[†], **Eric Monteiro**[†], **Nazih Mechbal**[†]

[*] Safran Composites, a technology platform of Safran Tech
33 Avenue de la Gare, 91760 Itteville, France
Web page: https://www.safran-group.com/

[†] PIMM Laboratory, Arts et Métiers Institute of Technology, CNRS, CNAM, HESAM University
151 Boulevard de l'Hopital, 75013 Paris, France
Web page: https://pimm.artsetmetiers.fr/

Abstract. Structural Health Monitoring (SHM) based on Lamb wave propagation is a promising solution to optimize maintenance, safety and enlarge service life of aeronautical structures. However, it remains a significant challenge to solve requirements for performance and accuracy. In this paper, an original method based on Topological Data Analysis (TDA) is introduced. TDA is a multi-dimensional method which can extract the topological features from time series and point cloud. First, the TDA tool is applied to raw 1D data in order to detect damages. Then, specific pre-processing of the measured time-series based on slicing is developed to improve the persistence homology perception and to leverage topological descriptors to classify different damages. Using a Lamb wave based SHM approach, it is shown that with specific pre-processing of the measured time-series data, the topological analysis (persistent homology) for damage detection and classification can be greatly improved. The temperature of the material has an impact on wave propagation and attenuation properties. It is important to ensure the capacity to detect and classify the damages on material on operational conditions of aerospace structures. The proposed approach enables to consider a priori physical information and provides another way to categorize damages than the traditional approaches. This work aims to characterize the temperature influence on the TDA performance to cluster damages. Finally, a strategy robust to temperature evolution is suggested to classify the plate health state. The dataset used to apply both methods comes from experimental campaigns performed on aeronautical composite plates with embedded piezoelectric transducers where different damage types have been investigated such as delamination and different impacts. In summary, this paper demonstrates that manipulating the topological the features of time-series signals using TDA provides an efficient mean to separate and classify the damage natures. It opens the way for further developments on the use of TDA in SHM.

Key words: Structural Health Monitoring; topological data analysis; time series; damage detection; damage nature clustering; temperature influence; machine learning; model reduction; composite aeronautical structures

Peer-review under responsibility of the organizing committee of SMART 2023.
doi: 10.7712/150123.9848.444519

1 INTRODUCTION

The main purpose of this article is to demonstrate a way to detect and characterize damage inside a composite plate structure in life-cycle conditions. Structural Health Monitoring (SHM) aims to detect damage on structural part. These information about the health of the structure are the first step to computed an estimated Remaining Useful Life (RUL) [1] and to predict maintenance processes needs [2] . Current and previous works show the efficiency of frequency analysis to resolve SHM problems [1, 2, 3, 4] . These methods use physical understanding of the behavior of the structure.

This article focus on the use of a new method and algorithm to extract topological features from a given dataset, these methods are called Topological Data Analysis (TDA). It can be interpreted as an original method to infer damage indexes from high dimensional data spaces by quantifying the "shape" of data. The main algebraic topology tool used in the TDA analysis, is called persistence homology. The persistence homology gives information about the global and local form of the data and consequently extracts topological invariant features from the data. TDA is used in several research field such as Financial networks [5], medicine [6, 7, 8], biomechanics [9]. The article [10] also provide a pionieer work on TDA applyied for SHM purposes.

In this paper, an extended application of TDA tools to monitor damages on composite structures using guided wave-based techniques is introduced. It's possible to improve distinction between each damage type and then to successfully classify them by slicing the temporal data into several time-windows. Each window is associated to a unique persistence diagram with the Lower Star Filtration (LSF) [11] method. It enables to extract convenient topological features on each data frame. Then the persistence diagrams are compared thanks to the Wasserstein distance [12]. This work shows that TDA tools are able to separate different damage types in several clusters. This original approach is also robust with respect to temperature evolution which could enable the detection on flight components by measuring the temperature as an input parameter.

A description of the experimental setup is done in the paper to fully understand the context of the dataset and the physical phenomena that enable to generate and collect data. Then, some TDA for time series applications and tools will be presented. Finally, the TDA tools will be applied on SHM experimental data to detect and classify the damage on the composite plate.

2 Description of the experimental setup and dataset

The experimental data are provided by piezoelectric transducers (PZT) bonded on composite CFRP plates. The test specimen is a eight-ply CFRP composite laminate with symmetrical stacking [0°, -45+, +45°, 0]s. The dimensions of each laminate are 400 mm ×300 mm×0.28 mm. The mechanical properties of the lamina are listed in Table 1 and the Figure 1 presents the Lamb waves test bench and the composite specimen with its PZT network.

A set of N = 5 piezoelectric (PZT) elements (Noliac NCE51) from NOLIAC© are surface-mounted on the composite plate [17]. Each piezoelectric element is 20 mm in diameter and 0.1 mm in thickness. The pitch/catch principle is considered here, wherein one PZT acts as an actuator while the others act as sensors.

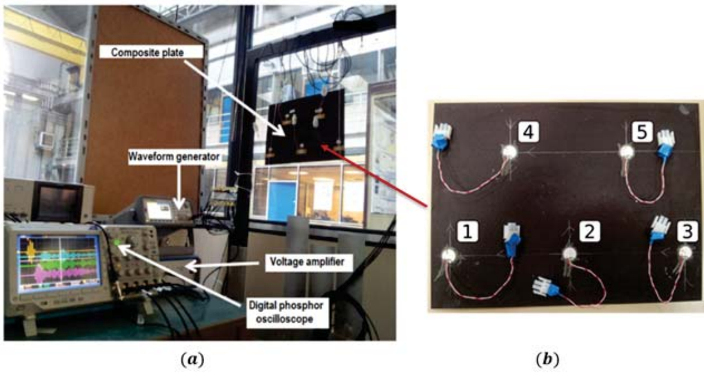

Figure 1: (a)The experimental test bench. (b) The composite plate with the PZT network

In addition to the healthy plate, 3 others similar specimen cut from the same original composite plate have been manufactured and damaged with different types of damage. Each damage is characterized with a position and a radius listed in table 1. As the damages are not produced with the same methods, the physical properties of the damaged area will not be the same. The first type of damage is an impact which is made by dropping masses on the plate figure 2. Impacts induce in fact several damages because it breaks down the carbon fibre in addition of the delamination. An illustration of the impact location and of the induced damage is shown in figure 2. Delamination damage is created, during the manufacturing process, by placing a sheet of PTFE between two plies. This material prevents the two layers to join in order to emulate a delamination. All these damages try to represent different type of mechanical disturbances that can be met in practice during the composite structure life cycle. To ensure the damage detection in life-cycle condition, it is necessary to check the robustness of the algorithm to temperature evolution. The temperature values are : [6°C, 11°C, 16°C, 31°C, 36°C, 56°C, 66°C].

N°	Damage	Size(mm)
1	Healthy	
2	Delamination	10
3	Impact1	14
4	Impact2	24

Table 1: Damages on the composite plate

On the Figure 2 (a) the damage is located at (300, 150) mm. The excitation signal is generated by a 33500B series Waveform generator and amplified to 10 V using a voltage amplifier from FLC Electronics. It is a 5 cycles sinusoidal tone burst with a central frequency of $f_0 = 200$ kHz, modulated by a Hanning window (Morlet wavelet type). The sampling frequency is fixed to 1 MHz. Successively, each PZT becomes the transmitter and the four other the sensors. The

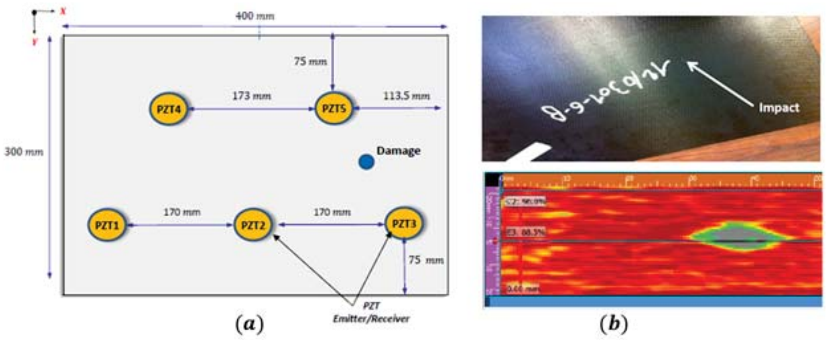

Figure 2: (a) PZT and damage position on the experimental plate. (b) A 14 mm impact (top) C-scan of the impact (bottom)

same experiment is repeated 100 times in order to assess the repeatability of the process. At the end, the dataset is composed of 500 time series for each PZT switch, multiplied by the 4 damage types. In the first step, the measured signals are used without any additional post-processing step. A recording in the case of the healthy and delaminated structure is given in figure 3 as an example.

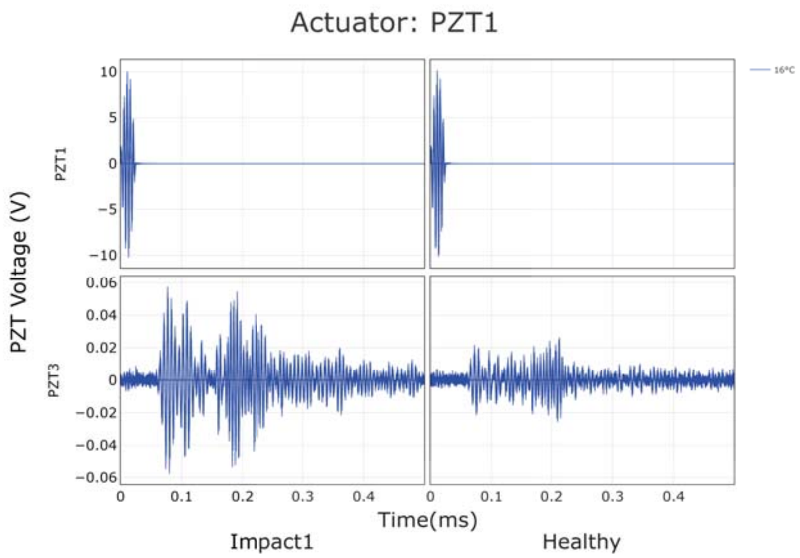

Figure 3: Signal from PZT_1 measured by PZT_1 and PZT_3 on damaged and healthy plate

Each sensor measures different signals associated with the wave because they are not located at the same position (cf. Figure 3). The first detected packet corresponds to the direct path between the actuator and the sensor. After that, the received signal corresponds to a mix between indirect path waves which are due to the boundary and to the damage. The dataset consists of 10 repetitions for each damage case and each temperature.

3 Topological Data Analysis for time series

3.1 Lower Star Filtration

The Lower star filtration (LSF) [13] is also called Sublevel filtration is a method to extract topological features from 1-dimensionnal data. These topological features are called Homology classes. The LSF approach first need to set up a distance between neighbors. Due to the lack of simple distance such as euclidian distance in 1-dimensionnal space, it is necessary to settle a way to compute these. To do so, each point in the time serie has two neighbors except the first and last ones. For each pair of points, the assigned distance is the difference between the values of points. The Lower-star filtration (LSF) , describes the evolution of the time series on several scale . It has an intuitive and graphic representation:

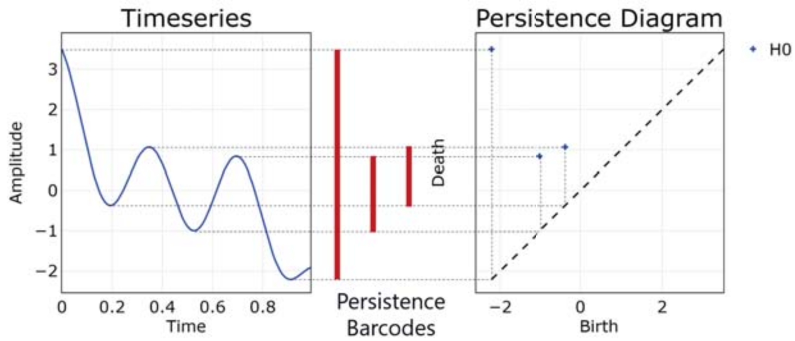

Figure 4: Persistence diagram computed for the signal: $f(t) = -20 \times (x - 0.5)^3 + cos(5.7\pi x)$

Homology classes can be computed by following a rising level of water on the Y-axis starting at the minimum amplitude of the time series. When the water's threshold exceeds a local minimum, a new pool forms, it corresponds to a homology class birth. When the water's level reaches a local maximum and two different pools are merging. In fact, the biggest will gulp the smaller one, this makes the newest homology class die. The persistence of the homology class is the ability of the feature to continue to grow with the threshold rising. Persistence of a homology is equal to: $Persistence(h_i) = Death(h_i) - Birth(h_i)$. The t=0 axis and $\mathrm{t} = \mathrm{t}_{\max}$ are considered as walls with infinite height.
The LSF algorithms and other TDA tools used in the paper come from several Python packages: Giotto-tda [14], Scikit-tda Ripser [11].

3.2 Wasserstein and bottleneck distance

Each time series is now linked to a corresponding persistence diagram, in order to compare time series with topological descriptor the diagrams needs to be compared. The Wasserstein distance and the Bottleneck distance are two methods to do so. These two algorithms calculate the pairwise distance between points in the two diagrams.

For two persistence diagrams D_1 and D_2, $p > 0$, the p-Wasserstein distance is:

$$W_p\left(D_1, D_2\right) = \left(\inf_{\varphi} \sum_{x \in D_1} d_\infty\left(x, \varphi\left(x\right)\right)^p\right)^{\frac{1}{p}} \tag{1}$$

$\varphi : D_1 \rightarrow D_2$ is the function which gives the pairwise point of $x \in D_1$ in the diagram D_2 and $d_\infty\left(x, y\right) = \max\left\{\left|x_1, y_1\right|, \left|x_2, y_2\right|\right\}$. The function φ is linked with the transportation theory because it links points in pairs by minimizing a cost function which represents the distance between the two points. Because, the Wasserstein and Bottleneck distance is computing pairwise distance, it is necessary to give two diagrams with the same number of points. In real use case, persistence diagrams cannot be the same even with similar input data. Some technics enable to compute the Wasserstein matrix with different shape diagrams by bonding lower persistence points with the diagonal line (y=x). This special version of the Wasserstein distance compute is implemented in the module Gudhi [15].

4 Application and results

The Lower Star Filtration could be used directly on the whole time series to generate the persistence diagram. This simple application of the TDA is not adequate to detect damages on the plate. In fact, analysing the topology of data without preprocessing is not the best way to classify defects because it does nott consider the temporal behavior of the wave like the Time of Fly (ToF). That is why, an enhanced method of the TDA is presented in the article. In order to give physical understanding, this approach slices the time series into several same size smaller time series. By comparing the corresponding persistence diagrams on each window of time series, it is possible to split the differents damages. In this paper, the size of the slicing window is set up equal to the time duration of the input signal ($\sim 3.10^{-5}$s). An example of diagrams given by slicing a time series is represented in figure 5.

In order to improve the results it could be possible to optimize the size and the stride of windows along the time series. This optimization will not be discussed here. Each generated diagram is strongly related to his corresponding window, to obtain good results it is necessary to compair and classify diagrams coming from the same window.

After all these processes, it is possible to construct a dataset of diagrams to work with. Each time series is divided into 20 series and each one contains 30 points, each window gives one persistence diagram. This means that each measurement time series can be represented by 20 persistence diagrams similarly to the figure 5.

4.1 Detection at 16°C

First, for each the time series in the dataset sliced is several corresponding persistence diagram. The Wasserstein distance is used in order to assure the classification step to predict a damage. In fact, the Wasserstein distance is able to compare digrams corresponding to different damage cases. The 5 PZTs are bonded around the plate at precise location. Because of that, the wave path will be different for each actuator, sensor pair. The path of the wave is strongly related

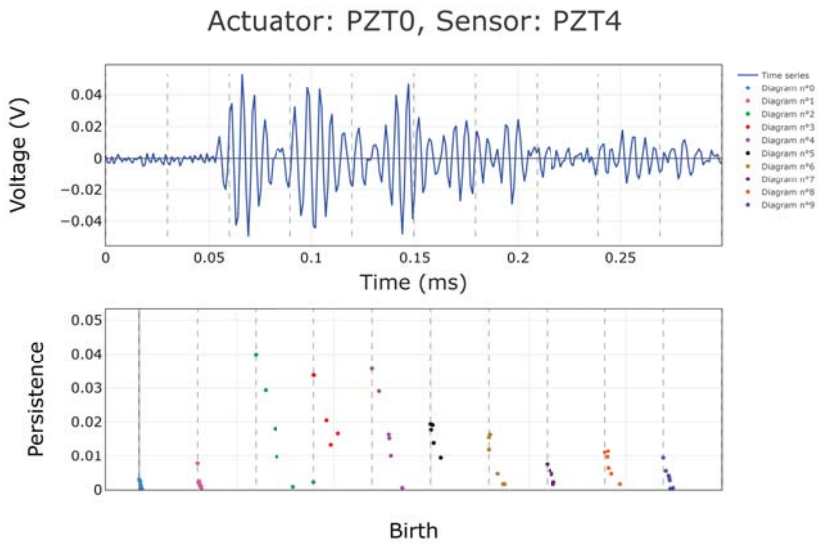

Figure 5: Persistence diagram computed for the signal

to the received signal. For example, the amplitude attenuation will be more important for longer way, in addition, the time of flight will also increase and modify the measured time series. For all these reasons, several results will be displayed by representing only one or two path. The Wasserstein matrices contain the average Wasserstein distance between persistence diagram as a classical distance matrix. The figure 6 shows that different damages can easily be seperate in several classes thanks to their persistence diagrams, in fact, two diagrams corresponding to the same damage will be more similar which results in a reduced Wasserstein distance. In order to optimize the classification of damage, the values of the distance matrix diagonal should be minimized when other are maximized. For the sake of visiblity, every Wasserstein distance is multiplied by 1000.

A confidence interval at 95% is computed for each values in the matrix and included in the figure 6 and figure 7. Considering the confidence intervals still make the clustering efficient because the maximum values of the diagonal is still inferior to minimum extra-diagonal values.

4.2 Temperature effects on persistence diagrams

SHM technologies aim to detect and predict damages in flight, in order to make the algorithm efficient during the entire flight time, it is mandatory to study the effect of temperature on the waves and the results. The Wasserstein distance is computed between diagrams from same damage case at different temperature. The results are represented in a Distance matrix in figure 7.

It shows the temperature impact on the diagrams distances. The temperature effects are significant because the distances of the figure 7 are comparable to figure 6 values. In fact, the distance error made without considering temperature is the same order of magnitude than the

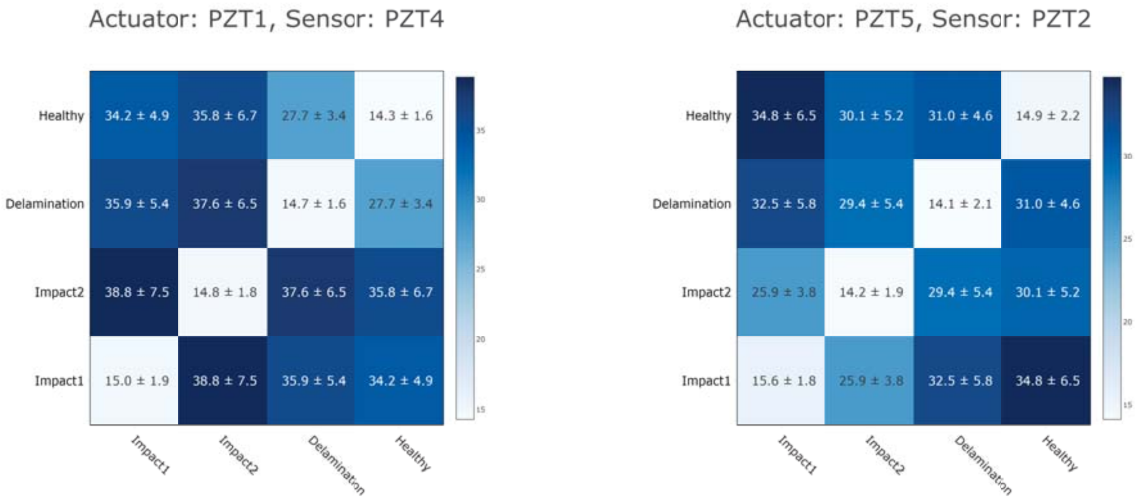

Figure 6: Wasserstein distance Matrices

seperation distance. The figure 7 is well illustrating the issue not to consider the temperature as an input. Most of the time, the distance between diagram coming from different damage and superior to those linked to same damage. This result is hopeful to classify efficiently the damage with the method. However, the Wasserstein distance for two equaly damaged data is increasing linearly with the temperature difference. In several case, the distance computed is is not enough to seperate correctly two damages. Considering temperature as inpu data could avoid this issue. In fact, slicing the temperature range into several interval should enhance the results in each local temperature intevals. As presented in the figure 7 the Wasserstein distance in increasing with the temperature gap. For example, a classification model could be train on data measure between 6°C and 16°C to predict the damages in the same interval.

4.3 Damage clustering at different temperatures

The Wasserstein distance is a good tool to separate data coming from different damages but the results are still depending on the temperature. In this section, another approach is performed to improve the result to be able to classify damage in the whole temperature range. Currently, every distance matrix are computed with the average values on each slicing window. This method considers the involvement of each window equaly. In reality, some windows have a bigger impact on persistence diagram similarity than other. A step of model reduction is enforced by Tuncated Singular Value Decomposition (truncated SVD) on the sliced time series in order to extract the most significant mode. By using Lower Star Filtration, the obtained results is a single persistence diagram for each input time series. From these diagrams, the Wasserstein distance is performed to build the distance matrix between each measured time series. After these processes, the Multidimensional Scaling (MDS) enables to project the distance ma-

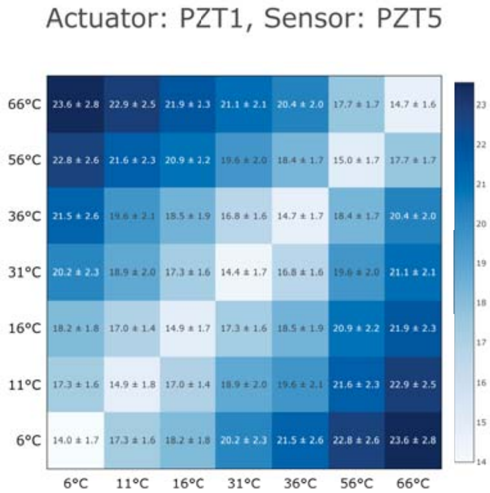

Figure 7: Wasserstein matrix depending on the temperature

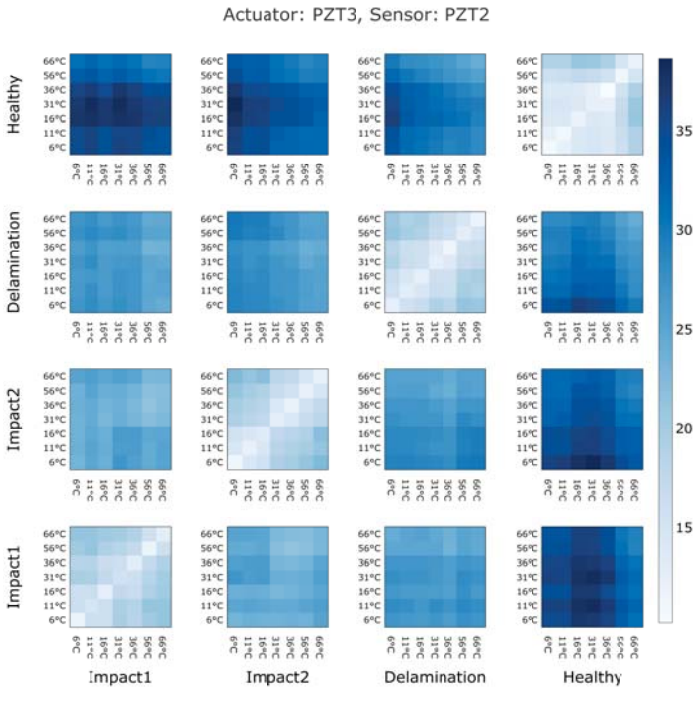

Figure 8: Wasserstein matrix depending on the temperature and on damages

trix computed with the Wasserstein distance in a 2-dimensionnal point cloud, this improve the visualization of the damage groups dissimilarity. The MDS algorithm preserves the distance between each point in the 2-dimensionnal space.

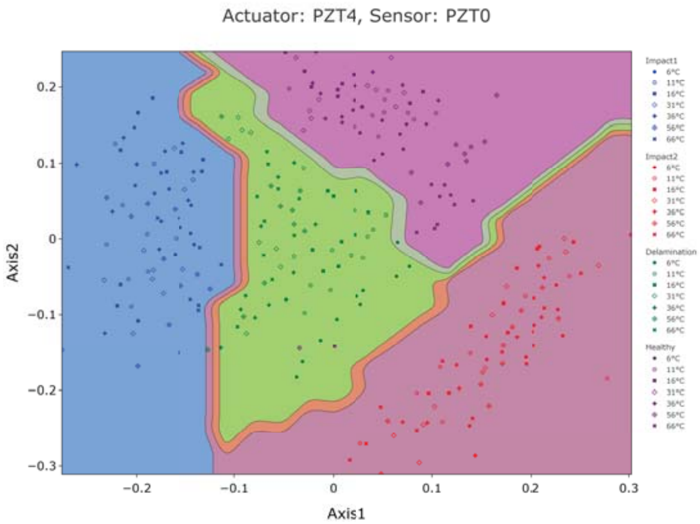

Figure 9: Dissimilarities of damage computed with MDS and K-Neighbors border

As shown in the figure 9, the 4 damages are well seperated thanks to the homology persistence. The K-Neighbors classification algorithm with uniform weights is reliably able to find the borders to classify each damage on the plate. All the dimensionnal reduction and projections algorithms used in this section come from the Python package Scikit-learn [16].

5 Conclusion

The enhanced topological analysis of time series for damage detection on structures presented in the paper is a convincing alternative to classical SHM methods. Indeed, using a guided wave SHM approach, the TDA tools are able to discriminate between different damage type (impacts and delamination) by analyzing the piezoelectric transducer measurements. We have shown that at homogeneous and constant temperature (16°C), the TDA tools are fully able to solve clustering and classification problems for damage detection on a composite plate embedded with PZTs. However, in the case of temperature evolution (representing the case of a, in service use) the wave behavior changes and compromises (without any compensation approach) the efficiency of classical guided wave SHM approaches. Using our former TDA approach, we show that the Lower Star Filtration is not completely robust to temperature changes when it considers all sliced windows equally. Therefore, we propose to bypass this issue by considering the temperature as an input data in order to compute an operable clustering and classification. These temperature effects can be limited by restraining the range of temperature to predict the health of the structure. It is also possible to improve the results by performing MOR (MOdel

Reduction) algorithms such as SVD in order to have a better use of the time series slicing. The results show that enhancing the method with MOR methods is efficiently improving the separation between each damage to enable the prediction on the full-temperature interval. This work focused on the potentialities of the TDA algorithms to solve SHM problemsUsing a Lamb wave based SHM approach, it is shown that with specific pre-processing of the measured time-series data, the topological analysis (persistent homology) for damage detection and classification can be greatly improved even in the presence of environmental perturbations as temperature variation. This approach could be used for future works to estimate the location or the size of the damage inside the structure.

REFERENCES

[1] M. Fouladirad, A. Grall, and L. Dieulle, “On the use of on-line detection for maintenance of gradually deteriorating systems,” *Reliability Engineering & System Safety*, vol. 93, no. 12, pp. 1814–1820, Dec. 2008, number: 12. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0951832008001038

[2] W. Briand, “Lamb waves based active sparse tomography for damage size quantification in composite structures : data-driven and parameter inversion methods,” PhD Thesis, HESAM Université, Paris, France, 2022.

[3] S. Guo, M. Rébillat, and N. Mechbal, “Prediction of frequency and spatially dependent attenuation of guided waves propagating in mounted and unmounted A380 parts made up of anisotropic viscoelastic composite laminates,” *Structural Health Monitoring*, vol. 0, pp. 1–27, May 2022, publisher: SAGE Publications. [Online]. Available: https://journals.sagepub.com/doi/10.1177/14759217221099967

[4] S. da Silva, L. G. G. Villani, M. Rébillat, and N. Mechbal, “Gaussian Process NARX Model for Damage Detection in Composite Aircraft Structures,” *Journal of Nondestructive Evaluation, Diagnostics and Prognostics of Engineering Systems*, vol. 5, no. 1, p. 011007, 2021, number: 1 Publisher: The American Society of Mechanical Engineers. [Online]. Available: https://hal.archives-ouvertes.fr/hal-03675467

[5] M. Gidea, “Topological Data Analysis of Critical Transitions in Financial Networks | SpringerLink,” in *Springer Proceedings in Complexity*, Tel-Aviv, Israel, 2017, pp. 47–59.

[6] R. Iniesta, E. Carr, M. Carrière, N. Yerolemou, B. Michel, and F. Chazal, “Topological Data Analysis and its usefulness for precision medicine studies,” *SORT-Statistics and Operations Research Transactions*, pp. 115–136, Jun. 2022. [Online]. Available: https://raco.cat/index.php/SORT/article/view/401140

[7] R. Rabadán and A. J. Blumberg, *Topological Data Analysis for Genomics and Evolution: Topology in Biology*. Cambridge University Press, Oct. 2019, google-Books-ID: BADDDwAAQBAJ.

[8] Y. Skaf and R. Laubenbacher, "Topological data analysis in biomedicine: A review," *Journal of Biomedical Informatics*, vol. 130, p. 104082, Jun. 2022. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1532046422000983

[9] L. M. Seversky, S. Davis, and M. Berger, "On Time-Series Topological Data Analysis: New Data and Opportunities," in *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 2016, pp. 59–67.

[10] T. Gowdridge, "(PDF) On Topological Data Analysis for SHM: An Introduction to Persistent Homology," in *Data Science in Engineering Proceedings of the 39th IMAC, A Conference and Exposition on Structural Dynamics*, vol. 9, 2021, pp. 169–184.

[11] U. Bauer, M. Kerber, J. Reininghaus, and H. Wagner, "Phat – Persistent Homology Algorithms Toolbox," *Journal of Symbolic Computation*, vol. 78, pp. 76–90, Jan. 2017. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0747717116300098

[12] R. Flamary, M. Cuturi, N. Courty, and A. Rakotomamonjy, "Wasserstein discriminant analysis," *Machine Learning*, vol. 107, no. 12, pp. 1923–1945, Dec. 2018, number: 12. [Online]. Available: https://doi.org/10.1007/s10994-018-5717-1

[13] "Lower Star Time Series — Ripser.py 0.6.4 documentation." [Online]. Available: https://ripser.scikit-tda.org/en/latest/notebooks/Lower%20Star%20Time%20Series.html

[14] G. Tauzin, U. Lupo, L. Tunstall, J. B. Perez, M. Caorsi, A. M. Medina-Mardones, A. Dassatti, and K. Hess, "giotto-tda: A Topological Data Analysis Toolkit for Machine Learning and Data Exploration."

[15] T. Lacombe and M. Glisse, "Wasserstein distance," in *GUDHI User and Reference Manual*, 3rd ed. GUDHI Editorial Board, 2023. [Online]. Available: https://gudhi.inria.fr/python/3.7.1/wasserstein_distance_user.html

[16] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay, "Scikit-learn: Machine Learning in Python," *Journal of Machine Learning Research*, vol. 12, pp. 2825–2830, 2011.