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OPTIMIZATION OF PASSIVE SHUNTED DAMPING CONFIGURATIONS FOR VIBRATION ATTENUATION

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Abstract. Vibration control is essential to reduce excessive oscillation amplitudes, suppress undesirable resonances, and avoid early fatigue failure of structures and structural elements. The use of piezoelectric materials with shunted circuits for energy dissipation is a technique that can be used to address this problem. A recently proposed tuning method developed for resistive-inductive (RL) shunts is implemented in a commercial finite element software, followed by an optimization approach to improve the circuit configuration. The shunted circuits configurations that minimize the displacement amplitude are obtained for two benchmark problems using the GLODS algorithm (Global and Local Optimization using Direct Search). Results are compared with the ones produced by the tuning method.

Key words: Piezoelectricity, Resonant Shunt Tuning, Vibration attenuation, Optimization

1 INTRODUCTION

Different approaches to vibration control have been developed over the last few decades. These methods can be classified as passive, active or hybrid. Active damping, as the name suggests, involves energy inputs from an external source and uses control actuators and sensors whereas passive damping does not. Piezoelectric materials can be used in either active or passive vibration control techniques, as sensors or actuators in a control loop in active control or

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bonded to the vibrating structure to convert the mechanical energy into electrical energy, which is later dissipated in a shunted electrical network. Hybrid techniques still use a resistance to dissipate energy, however the current flow should be managed by an external controller to increase the energy dissipation [1]. Piezoelectric shunt damping has long been studied as a means of attenuating noise and vibration in mechanical constructions.

Allik and Hughes [2] developed a general finite element formulation for piezoelectric materials in 1970, which is when the literature on piezoelectric numerical implementation first appeared. In 1979, Forward [3] proposed and demonstrated experimentally the first RL shunt circuit. Calibration methods were first developed for series circuits, by Hagood and von Flotow [4] and later for parallel circuits by Wu [5]. More recently Høgsberg and Krenk [6, 7, 8] have developed several calibration procedures. In [6] a procedure that maximizes the damping factor was obtain, while in [7] the quasi-static background flexibility was considered. The more recent work has used residual mode correction to calibrate the RL shunts [8]. Calibration methods using mode correction have been further developed by Toftekær et al in [9] and [10]. The shunt tuning method in [10] uses the EMCC (effective electromechanical coupling coefficient) at resonance and the modal charge in short circuit (SC) boundary conditions. The influence of non-resonant vibration modes is studied further and the modal voltage is included as shunt tuning parameter, allowing a direct evaluation of the effective modal capacitance in [11], where a tuning method is derived for an arbitrary number of piezoelectric shunts and it is extended to multi-mode damping. The squared EMCC is used in all calibration methods, governing the attainable damping and shunt tuning.

The main goal of this work is to obtain the piezoelectric patch resonant shunt tuning parameters that can maximize the damping of the first mode of vibration and compare the resulting circuit configurations values with those calculated using the calibration method from [10].

2 FINITE ELEMENT FORMULATION

Piezoelectric materials have the ability to convert mechanical energy into electrical energy and vice-versa, due to fact that these materials strain when an electrical field is applied and produce voltage when under strain [4]. The constitutive equations for piezoelectric materials describe the relation between strain, stress and electric field. These relations, in strain form, can be written as [12]:

$$\begin{aligned} \boldsymbol{\varepsilon} &= \mathbf{S}^E \boldsymbol{\sigma} + \mathbf{d}^T \mathbf{E}_f \\ \mathbf{D} &= \mathbf{d} \boldsymbol{\sigma} + \boldsymbol{\epsilon}^{\sigma} \mathbf{E}_f \end{aligned} \tag{1}$$

where $\boldsymbol{\sigma}$ is the mechanical stress vector, $\boldsymbol{\varepsilon}$ is the mechanical strain vector, $\mathbf{D}$ is the electric displacement vector, $\mathbf{E}_f$ is the electric field vector, $\mathbf{S}^E$ is the compliance matrix, $\mathbf{d}$ is the piezoelectric coupling matrix in strain form and $\boldsymbol{\varepsilon}^{\sigma}$ is the dielectric matrix measured at constant stress.

The discrete equilibrium equations for free vibrations in a displacement based finite element formulation, considering the piezoelectric effect, can be written as:

$$\left[\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi}^T & \mathbf{K}_{\phi\phi} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\right] \left\{ \begin{array}{c} \mathbf{u} \\ \boldsymbol{\phi} \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{0} \\ \mathbf{Q} \end{array} \right\} \tag{2}$$

where $\mathbf{K}_{uu}$, $\mathbf{K}_{u\phi}$ and $\mathbf{K}_{\phi\phi}$ are the stiffness matrices associated with the mechanical displacements, electromechanical coupling and electric potentials, respectively, $\mathbf{M}$ contains the physical mass associated with the inertia of the plate and piezoelectric patches, $\mathbf{Q}$ is the applied electric charges vector, ω is the circular frequency, $\mathbf{u}$ is the vector of mechanical degrees of freedom and $\boldsymbol{\phi}$ is the vector of electric potentials.

The electromechanical coupling coefficient (EMCC) is a key parameter for the tuning of an electrical shunt, which typically involves an inductor (L) - calibrated to assure that the piezoelectric patch works in resonance with the host structure - and an resistor (R) - to dissipate the energy into heat [8]. The effective EMCC k_e^2 evaluates the piezoelectric material capacity to inter-convert mechanical and electric energy and logically provides also damping for a specific target mode. This coefficient is defined as:

$$\kappa_e^2 = \frac{\omega_{OC}^2 - \omega_{SC}^2}{\omega_{SC}^2} \tag{3}$$

where ω_{SC} is the natural frequency of the system with the electrodes of the patches in short circuit, obtained from equation (2), and ω_{OC} is the natural frequency in open circuit (zero electric charge on the electrodes). It should be noted that equipotential conditions should be imposed on the top and bottom surface of each patch, in order to simulate the electrodes.

An inductance L and a resistance R connected either in parallel or in series constitute the resonant shunt circuit. The tuning procedure and expressions for the parallel and series RL resonant shunt circuit, according to [10], are presented bellow.

For the parallel shunt circuit, the inductance calibration is

$$L_p = \frac{k_e^2}{Q_r^2} \tag{4}$$

and the optimum resistance tuning

$$R_p = \frac{k_e^2 \omega_r}{Q_r^2} \sqrt{\frac{1}{2k_e^2}} \tag{5}$$

For the series shunt circuit, the inductance calibration is

$$L_s = \frac{k_e^2}{Q_r^2 (1 + k_e^2)^2} \tag{6}$$

and the optimum resistance tuning

$$R_s = \frac{k_e^2 \omega_r}{Q_r^2} \sqrt{\frac{2\kappa_e^2}{(1 + \kappa_e^2)^3}} \tag{7}$$

In the calibration formulas presented above, Q_r and ω_r are, respectively, the modal charge and resonant frequency, both in short circuit, obtained from equation (2).

2.1 Implementation in ANSYS

The electromechanical structures previously analysed in [10] were implemented in ANSYS. Two cases were studied: a simply supported plate with a single pair of patches and a simply supported plate with four pairs of patches. The plate dimensions are 414mm $\times$ 314mm $\times$ 1mm. The single pair of patches with dimensions 82.8mm $\times$ 62.8mm $\times$ 0.5mm is slightly off-centered from the center of the plate at $\frac{13}{28}$ of the plate lenght in x and $\frac{15}{28}$ of the plate width in y. For the second case four pairs of patches were placed symmetrically in the quarter points of the plate and each patch is a quarter of the size of the single patch of the first case, so that the total mass of the patches is the same in both cases.

The material of the plate is aluminum, with elastic properties $E = 70$ GPa, $\nu = 0.33$ and specific mass $\rho = 2700$ kg/m ^{3}. Lead-zirconate-titanium (PZT-5H) is the material used for the piezoelectric patches, its properties were taken from [13] and are presented in Table 1.

Table 1: Material properties for PZT-5H [13]

$S^E\ [\times 10^{12}\ \text{Pa}^{-1}]$	$d\,[\times 10^{12} C/N]$	$\epsilon_R = \frac{\epsilon^\sigma}{\epsilon_0}$	$\rho\,[\text{kg/m}^3]$
$S_{11}^E = 16.5$	$d_{31} = -274$	$(\epsilon_R)_{11} = 3130$	7500
$S_{12}^E = -4.78$	$d_{33} = 593$	$(\epsilon_R)_{33} = 3400$	
$S_{13}^E = -8.45$	$d_{15} = 741$		
$S_{33}^E = 20.7$			
$S_{44}^E = 43.5$			
$S_{66}^E = 42.6$			

Note that $\epsilon_R = \frac{\epsilon^\sigma}{\epsilon_0}$ is the relative permittivity and the reference value is the absolute dielectric permittivity of vacuum, $\epsilon_0 = 8.854 \times 10^{-12}$ F/m.

The first step was to create and set up the model, which includes defining the geometry, material properties, element types, mesh, applied loads and equipotential (EP) conditions. The element type for the aluminum base plate was SOLID186 and for the piezoelectric patches SOLID226 elements with piezoelectric capabilities were used. In the second step (SC solution), zero electric potential is applied to the master nodes to obtain SC piezoelectric patches. The SC resonant frequency, $\omega_r = \omega_{SC}$, and modal charge, Q_r, are obtained by conducting a modal analysis. The third step (OC solution) includes deleting the zero electric potential constraints on the master nodes to obtain OC piezoelectric patches and conducting a modal analysis to obtain the OC resonant frequency, $\hat{\omega}_r = \omega_{OC}$. Using equation (3) the effective EMCC k_e^2 is calculated and the resulting value is used to compute the optimal inductance and resistance values, given by equations (4), (5), (6) and (7). The final step is to implement the optimum inductance and resistance between the grounded and ungrounded master nodes using CIRC94 elements. The patch connections to the electrical shunt circuit are such that the electroded surfaces of the patches that are connected to the base plate are grounded and the shunt circuit connects the outer surfaces of each pair of patches. Harmonic analysis was used to validate the shunt tuning with a concentrated force of 100 N applied at the center of the plate.

3 OPTIMIZATION

The single-objective optimization algorithm GLODS is appropriate for bound constrained, derivative-free, and global optimization [14]. The algorithm was developed to determine the largest number of local minima, using a search procedure based on direct search of a directional type, from which the global minimum will be identified. Similar to a classical direct-search method of directional type, the method alternates between a search and a poll step. During the search step the whole feasible region is explored to locate potentially good regions, which are locally explored by the poll step. A point is added to the list of feasible points as active when it belongs to a part of the feasible region not yet explored or if the new point decreases the corresponding objective function value. When a new point presents a better objective function value than an active point but another point already stored equals or decreases this value, inactive points may be added to the list. There are three types of iterations: *successful*, *unsuccessful* and *merging*. When at least a new active point is added to the list, *successful iteration* occurred and the step sizes corresponding to the new active point could be increased or kept constant. If no points are added to the list, it is an *unsuccessful iteration* and the step size parameter corresponding to the poll center is decreased. *Merging iteration* happens if only inactive points were added.

Two Single Objective Optimization (SOO) problems were formulated:

- Problem 1, referring to simply supported plate with one pair of patches;
- Problem 2, referring to simply supported plate with four pairs of patches.

Problem 1 was defined as

$$\begin{aligned} &\min \quad F(\mathbf{x}) \\ &x \in \Omega \\ &\qquad \mathbf{x} = (R, L, C) \\ &\qquad R \in [10000 : 200 : 100000] \\ &\qquad L \in [0.5 : 0.1 : 1] \cup [2 : 0.2 : 100] \\ &\qquad C \in \{1; 2\} \end{aligned} \tag{8}$$

where R is the resistance value, L is the inductance value, C is a design value that indicates the type of circuit implemented - parallel when $C = 1$ or series when $C = 2$ - and $F(\mathbf{x})$ the maximum displacement amplitude of the center of the plate around the first resonant frequency. Note that the values that R, L and C can take are written in accordance with Matlab notation.

Problem 2 was defined as

$$
\begin{aligned}
&\min \quad F(\mathbf{x}) \\
&x \in \Omega \\
&\qquad \mathbf{x} = (R_1, L_1, C_1, R_2, L_2, C_2, R_3, L_3, C_3, R_4, L_4, C_4) \\
&\qquad R_i \in [100000 : 200 : 1000000]_{i=1,2,3,4} \\
&\qquad L_i \in [0.5 : 0.1 : 1] \cup [2 : 0.2 : 400]_{i=1,2,3,4} \\
&\qquad C_i \in \{1; 2\}_{i=1,2,3,4}
\end{aligned}
\tag{9}
$$

where R_i is the resistance value, L_i is the inductance value, C_i is a design value that indicates the type of circuit implemented - parallel when $C_i = 1$ or series when $C_i = 2$ - and $F(\mathbf{x})$ the maximum displacement amplitude of the center of the plate around the first resonant frequency.

In the current study, the four pairs of piezoelectric patches have identical shunted circuits, this is $R_1 = R_2 = R_3 = R_4$, $L_1 = L_2 = L_3 = L_4$ and $C_1 = C_2 = C_3 = C_4$. In this way, instead of twelve optimization variables, the optimization problem has only three design variables.

3.1 Simply supported plate with a single pair of patches

The GLODS algorithm was first initialized using random sampling and later the cache was used as starting point with a smaller comparison radius, r. A total of 140 local minima were found and the 5 best are presented in Table 2.

Table 2: Local minima computed by GLODS for a single pair of patches

Solution	R [Ω]	L [H]	C	F [m]
1	51600	36.8	1	0.14678
2	50600	36.8	1	0.14679
3	49600	36.8	1	0.14680
4	67000	36.8	1	0.16154
5	42600	37.4	1	0.16280

Note that the maximum displacement amplitude measured at the center point of the plate when the RL values were calculated based on the effective EMCC is $F = 0.15964$ m for $R = 42070\ \Omega$ and $L = 36.12$ H, also using a parallel circuit ($C = 1$). From the observation of the results in Table 2 we conclude that three of the optimal solutions are slightly better in terms of the amplitude reduction than the values predicted by the circuit tuning method [10].

3.2 Simply supported plate with four pairs of patches

Initially, the algorithm was initialized with the RL-shunt tuning based on the effective EMCC, followed by two consecutive initializations starting from a random point. Lastly, the algorithm used the points in the cache as staring points and a smaller comparison radius, r, to have a considerable number of evaluations. A total of 5910 local minima were found and the best 20 points are displayed in Table 3.

Table 3: Local minima computed by GLODS for four pairs of patches

Solution	R [Ω]	L [H]	C	F [m]
1	358400	123.6	1	0.37062
2	402200	123.2	1	0.37169
3	453400	122.8	1	0.37241
4	353600	123.6	1	0.37244
5	346600	123.6	1	0.37724
6	370200	123.2	1	0.37844
7	463600	123.0	1	0.38006
8	481600	122.8	1	0.38122
9	363400	124.0	1	0.38231
10	493200	122.8	1	0.38594
11	457600	123.2	1	0.38634
12	434400	123.4	1	0.38721
13	476600	123.2	1	0.39146
14	428000	123.6	1	0.39220
15	398000	124.0	1	0.39469
16	388000	122.6	1	0.39536
17	519400	122.8	1	0.39672
18	325400	123.0	1	0.39773
19	505400	122.2	1	0.39901
20	341400	124.4	1	0.40292

Note that the maximum displacement amplitude measured at the center point of the plate when the RL values were calculated based on the effective EMCC is $F = 0.40178$ m for $R = 332800\ \Omega$ and $L = 122.8$ H, also using a parallel circuit ($C = 1$). From the observation of the results in Table 2 we conclude that nineteen of the optimal solutions are slightly better in terms of the amplitude reduction than the values predicted by the circuit tuning method [10].

4 CONCLUSIONS

The main goal of this work was to obtain the optimal piezoelectric patch resonant shunt tuning parameters that can maximize the damping of the first mode of vibration and compare the resulting circuit configurations values with those calculated using the calibration method from [10].

The models of a simply supported aluminium plate with a single pair of patches slightly off-centered and a simply supported aluminium plate with four pairs of patches placed symmetrically were developed and implemented in ANSYS.

For both of the case studies the optimal shunt tuning parameters are in very close agreement with the ones obtained by the calibration method from [10]. For the case of the single pair of patches three solutions were found to be slightly better that the one obtained by the calibration method and for the case with four pairs of patches nineteen solutions were slightly better than the corresponding solution obtained with the calibration procedure. The best solutions always correspond to parallel shunt circuits, however many local minima were also found using series shunt circuits, but with higher values of maximum displacement.

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