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A 1:1 TUNABLE PIEZOELECTRIC RESONANT SHUNT USING A NON SMOOTH COMPONENT

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Abstract. This paper describe a new tunable circuit used in a piezoelectric dynamic vibration absorber. The circuit consists in a controlled nonlinear voltage in series with the usual inductance. The non-linear voltage act as a nonsmooth bilinear gain multiplying the piezoelectric voltage. One salient feature of this set-up is the ability to tune the electrical resonant frequency. Focusing on a 1:1 resonance between the mechanical and the electrical systems, a tuning based on a numerical study is explained, and the method is experimentally tested.

Key words: Dynamic Vibration Absorber, Smart Structures, Non-smooth Nonlinearities , Tunable Circuits.

1 INTRODUCTION

Dynamic vibration absorber using piezoelectric resonant shunt are an efficient method to attenuate vibration. To improve the integration of this solution in mechanical structures, piezoelectric shunt were proposed in [1]. In the case of resonant RL shunt, the inner capacitance of the piezoelectric resonates with the inductance but parameter drift can hinder the performances. Mechanical non-smooth nonlinearities have been proposed to overcome similar problem e.g. for shock isolation [2] or non linear energy sink [3] to cancel vibrations. In the case of piezoelectric shunts, non linearities consisting in switched circuit connected to piezoelectric can also be used for wideband vibration reduction [4].
In this paper, a nonsmooth nonlinearity is introduced to realize a tunable resonant piezoelectric shunt absorber which can be tuned either on a 1:1 or a 2:1 resonance. Focusing on the 1:1 resonance, analytical and numerical studies presented in the paper shows a behavior similar to a linear resonant shunt. It is shown that the inductance can be downsized without significative

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loss in the vibration attenuation. Experimental results of such a non linear shunt applied to the first twisting mode of an hydrofoil are also presented to validate the theory.

2 Principle and theoretical analysis

2.1 Governing equations

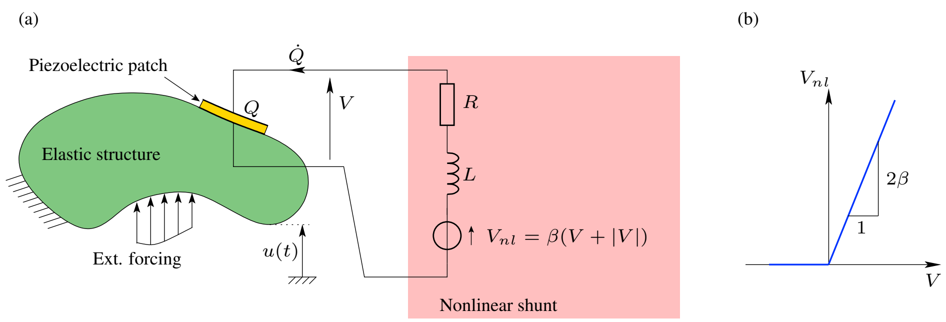

Figure 1: (a) Elastic structure coupled to the nonlinear shunt circuit; (b) graph of the function $V \to V_{nl} = \beta(V + |V|)$.

The system considered is depicted in Fig. 1 consists in an elastic structure excited by an external mechanical loading. The proposed electrical circuit is connected to the piezoelectric patch fixed to the structure. Following the usual assumption [5], the displacement denoted $u(\boldsymbol{x}, t)$ can be approximated by considering the excited mode i.e $u(\boldsymbol{x}, t) = \psi(\boldsymbol{x})q(t)$ where $\psi(\boldsymbol{x})$ is the mode shape and $q(t)$ is the modal displacement. Then, $(Q(t), q(t))$ are given by the dynamic coupled equations:

$$\ddot{q} + 2\xi\hat{\omega}\dot{q} + \hat{\omega}^2 q + \frac{\theta}{mC_\text{p}} Q = \frac{F}{m}\cos\Omega t, \tag{1a}$$

$$\ddot{Q} + 2\xi_e\omega_e\dot{Q} + \omega_e^2 Q + \frac{\theta}{LC_\text{p}} q - \frac{V_\text{nl}}{L} = 0. \tag{1b}$$

In the above equations, m, ξ, $\hat{\omega}$, θ, and F are, respectively, the modal mass, the mechanical damping ratio, the natural frequency in open circuit condition ($Q = 0$), the piezoelectric coupling coefficient, and the forcing, all corresponding to the considered mode. C_p is the effective capacitance of the piezoelectric patch for the mode [6]. The electrical natural frequency is $\omega_e = \frac{1}{\sqrt{LC_\text{p}}}$ and electric damping ratio is $\xi_e = \frac{R}{2}\sqrt{\frac{C_\text{p}}{L}}$, where R and L are the resistance and the inductance in the shunt circuit, respectively.

The voltage across the piezoelectric patch [5] and the nonlinear voltage are respectively given by:

$$V = \frac{1}{C_p}(Q + \theta q) \qquad V_{nl} = \beta(V + |V|), \tag{2}$$

and the non-linear voltage is chosen as follows : Combining (1b) and (2) give the new equations for the system:

$$\ddot{q} + 2\xi_i \hat{\omega}\dot{q} + \hat{\omega}^2 q + \frac{\theta}{mC_\mathrm{p}} Q = \frac{F}{m}\cos\Omega t, \tag{3a}$$

$$\ddot{Q} + 2\xi_e\omega_e\dot{Q} + \omega_e^2 Q + \omega_e^2\theta q - \beta\omega_e^2\left(Q + \theta q + |Q + \theta q|\right) = 0. \tag{3b}$$

2.2 Free vibration of the 1DOF system

There are no analytical solutions for Eq. 3, so a 1DOF model is first considered obtained for $\theta = 0$:

$$\ddot{Q} + 2\xi_e\omega_e\dot{Q} + \omega_e^2 Q - \beta\omega_e^2\left(Q + |Q|\right) = P\cos\Omega t, \tag{4}$$

where P is the forcing amplitude. For this part, let $P = 0$ and rewrite Eq. (4) by taking the different cases :

$$\ddot{Q} + 2\xi_e\omega_e\dot{Q} + \omega_e^2 Q = 0 \quad \text{if} \quad Q \leq 0, \tag{5a}$$

$$\ddot{Q} + 2\xi_e\omega_e\dot{Q} + \omega_e'^2 Q = 0 \quad \text{if} \quad Q \geq 0, \tag{5b}$$

where $\omega_e' = \omega_e\sqrt{1-2\beta}$. In the conservative case ($\xi_e = 0$), it is clear from Eqs. (5) that the free response is a succession of one half period of sine of period ω_e and one half period of sine of frequency ω_e', as illustrated Fig. 2(a) . Then, the oscillation period can be expressed as:

$$T = \frac{2\pi}{\bar{\omega}_e} = \frac{\pi}{\omega_e} + \frac{\pi}{\omega_e'}, \tag{6}$$

where $\bar{\omega}_e$ is the modified angular frequency of the free response due to the nonsmooth term, which reads:

$$\bar{\omega}_e = \underbrace{\frac{2\sqrt{1-2\beta}}{1+\sqrt{1-2\beta}}}_{\eta}\omega_e = \frac{2\sqrt{1-2\beta}}{1+\sqrt{1-2\beta}}\frac{1}{\sqrt{LC_p}}, \qquad \eta = \frac{\bar{\omega}_e}{\omega_e}, \tag{7}$$

where η is the ratio between the modified ($\bar{\omega}_e$) and unmodified (ω_e) natural frequencies. It is important to observe from Eq. (7) that *even if the behaviour is nonlinear, the free oscillation frequency of Eq. (3b) does not depend on the amplitude* and that *β controls the free response frequency*. As shown on Fig. 2(b), the ratio η can be lowered to zero (at $\beta = 0.5$) and increased up to a factor of 2 (for $\beta \to -\infty$). Note that β should be less than $\frac{1}{2}$ to avoid unstability.

2.3 Forced response of the 1DOF system

The non-linearity introduced respects the following property $|ax| = a|x|$ for $x \in \mathbb{R}$ and indeed, introducing the dimensionless parameters $\tilde{t} = \omega_e t$, $\tilde{Q} = Q\frac{\omega_e^2}{P}$, $\tilde{\Omega} = \Omega/\omega_e$, Eq. (4) can be rewritten as follows:

$$\ddot{\tilde{Q}} + 2\xi_e\dot{\tilde{Q}} + \tilde{Q} - \beta\left(\tilde{Q} + |\tilde{Q}|\right) = \cos\tilde{\Omega}\tilde{t}. \tag{8}$$

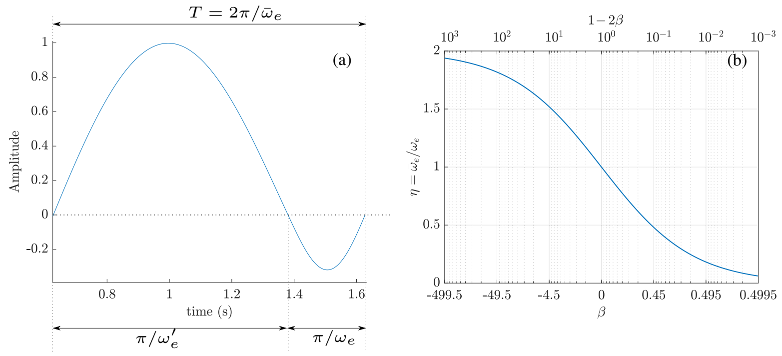

Figure 2: (a) Example of the free response of Eq. (4) over one period. (b) Ratio $\eta = \bar{\omega}_e/\omega_e$ as a function of β.

which depends only on the damping ratio ξ_e and the gain β. As a consequence, *the shape of the resonance curve of Eq. (8) is independent of the excitation level* P.

However, the non-linearity generates harmonics. This was studied by numerical simulations of

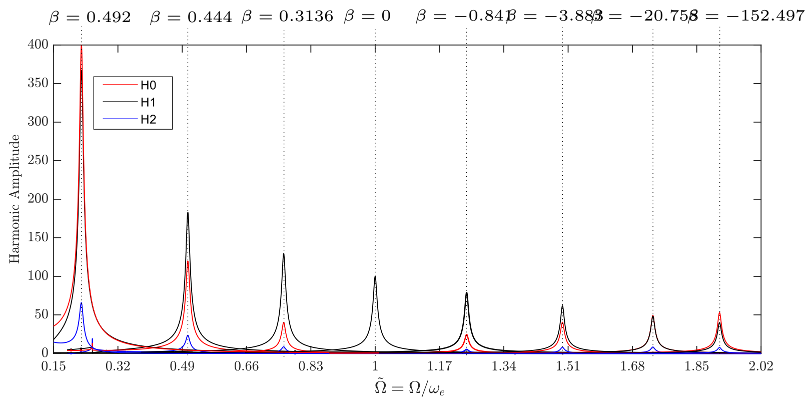

Figure 3: Harmonics H0, H1 and H2 of the periodic solutions of Eq. (4) in the vicinity of the primary resonance, as a function of $\tilde{\Omega} = \Omega/\omega_e$ for $\xi_e = 0.005$. The harmonics' amplitude is normalized with respect to the excitation level. The vertical dotted black lines show the value $\bar{\omega}_e(\beta)$.

Eq. (8) using an Harmonic Balance Method implemented in the MANLAB software [7]. Fig. 3 shows the dimensionless frequency response, normalized with respect to the excitation level, of the zeroth (H0), first (H1), and second (H2) harmonics of the periodic solutions for different values of β (other harmonics are present, but negligible). It was observed that relative amplitude of the harmonics and their damping *remain independent of the excitation level* [8]. In other words,the nonlinearity distributes the power between the different harmonics in proportions independent of the input amplitude and fixed only by the coefficient β.

Thus, it was hypothesized that the dynamic of each harmonic can be modeled as a linear oscillator, so fo $p \in \{0, 1, 2\}$, one has :

$$\ddot{Q}_p + 2\bar{\xi}_e(\beta)\bar{\omega}_e\dot{Q}_p + \bar{\omega}_e^2 Q_p = \phi_p(\beta)P\cos\Omega t, \tag{9}$$

where Q_p is the approximated response of the p-th harmonic of $Q(t)$, $\bar{\xi}_e(\beta) = \frac{\xi_e}{\eta(\beta)}$ (cf. Eq. (7)) is the modified damping ratio due to the nonsmooth term, and $\phi_p(\beta)$ are ad-hoc gains that were identified. Fig. 4 where the identified ϕ_p are depicted as function of β and $1 - 2\beta$ in log scales (shown by markers) and polynomial approximation as $\phi_p(\beta) = \sum_{m=0}^{N} a_m \log_{10}(1 - 2\beta)$.

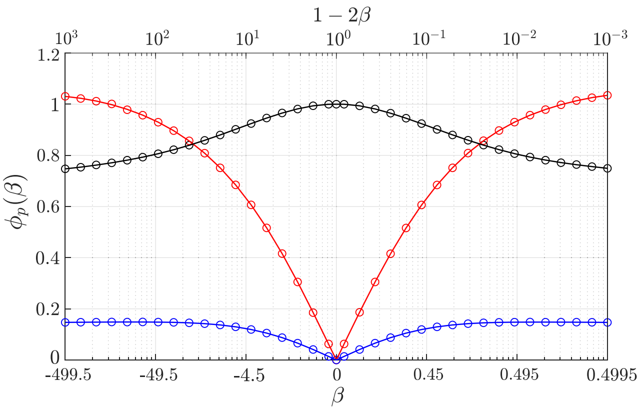

Figure 4: ϕ_p as a function of β corresponding to harmonics H0 (red), H1 (black), and H2 (blue). The circles denote the values estimated by fitting the analytical and the numerical responses for each value of β, and the solid lines denote the closed form approximated polynomial curves (solid lines).

3 Tuning strategy

3.1 Simplified coupled system

For the 1:1 resonant shunt tuning, the harmonic H_1 is considered, and the hypothesis discussed above is used to establish the approximation:

$$\ddot{q} + 2\xi\hat{\omega}\dot{q} + \hat{\omega}^2 q + \frac{\theta}{mC_\text{p}}Q_1 = \frac{F}{m}\cos\Omega t, \tag{10a}$$

$$\ddot{Q}_1 + 2\bar{\xi}_e(\beta)\bar{\omega}_e\dot{Q}_1 + \bar{\omega}_e^2 Q_1 + \bar{\omega}_e^2\phi_1(\beta)\theta q = 0. \tag{10b}$$

Inspecting the model, one observes that the usual coupling factor k characterizing the energy conversion in piezoelectric [9] must be modified to account for the energy dispatching between the various harmonics. Thus, we define:

$$\bar{k}(\beta) = k\sqrt{\phi_1(\beta)} \text{ with } k^2 = \frac{\theta^2}{\hat{\omega}mC_\text{p}} \tag{11}$$

Since, $\phi_1 < 0$ for all β within the feasible range (cf. Fig. 4), it follows that the coupling is degraded compared to the linear case.

3.2 Optimal tuning

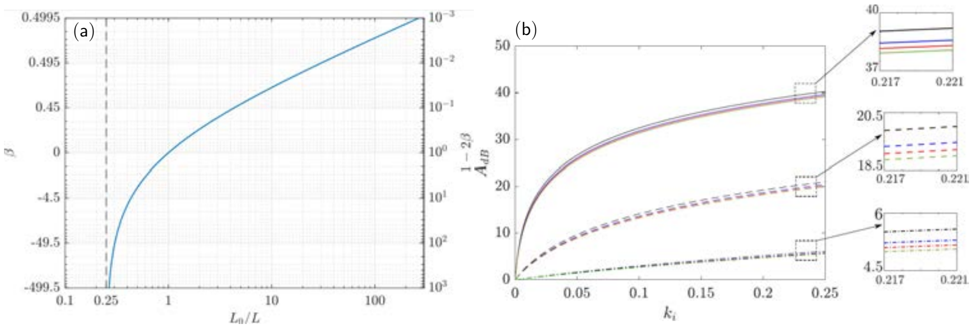

Figure 5: Optimal value $\beta^{\rm op}$ of β to tune the RL-shunt as a function of (a) $r_i = \omega_e/\hat{\omega}_i$ and (b) as a function of the reduction ratio of the shunt inductance with respect to its optimal value for a linear RL-shunt L_0/L.

The tuning is basically the same optimization procedure as a linear RL-shunt considering the modified set of parameters appearing in Eq. (10). Therefore, one must have $\bar{\omega}_e^{\rm op} = \hat{\omega}$.Thus, following [5], the optimal settings can be derived as given in Tab. 1. Note that the linear case is

Table 1: Optimal parameters and optimal attenuation for the 1:1 and 2:1 tuning cases.

Optimal β	Optimal R	Optimal attenuation [dB]
$\beta^{\rm op} = \frac{1}{2}\left[1-\left(\frac{1}{2r-1}\right)^2\right]$	$R^{\rm op} = \frac{\sqrt{6}}{2C_{\rm p}r^2\hat{\omega}}k\sqrt{\phi_1(\beta^{\rm op})}$	$A_{\rm dB} = 20\log_{10}\left(1+\frac{k_i}{\xi_i\sqrt{6}}\sqrt{\phi_1(\beta^{\rm op})}\right)$

obtained for $r = 1$ which gives $\beta = 0$ where only H1 appears (cf. Fig 3).
Furthermore, the inductance is

$$L = \frac{L_0}{r^2}. \tag{12}$$

Therefore, in practice the inductance can be reduced by a factor r^2 as shown in Fig. 5(a) compared to L_0 the inductance required in the case of a standard RL-shunt. For instance, choosing $r = 10$ results in $\beta \simeq 0.47$ that is $\frac{L_0}{L} = 100$.
Fig. 5(b) compares the attenuation $A_{\rm dB}$ for various mechanical damping ξ and ratio r. Clearly, there is only a slight dependance on the ratio r, which means that the inductance can be reduced without signicantly degrading the attenuation.

4 Experimental validation

The tuning is tested on a foil equipped with a macrofiber composite M8557-F1 to actuate the twisting mode (cf. [8] for further details). The electronic circuit connected to the piezoelectric patch is shown in Fig. 6. The identified parameters and the components values are summed up in Table 2. In Fig 7, the FRF of harmonic H1 is shown for different values of β with

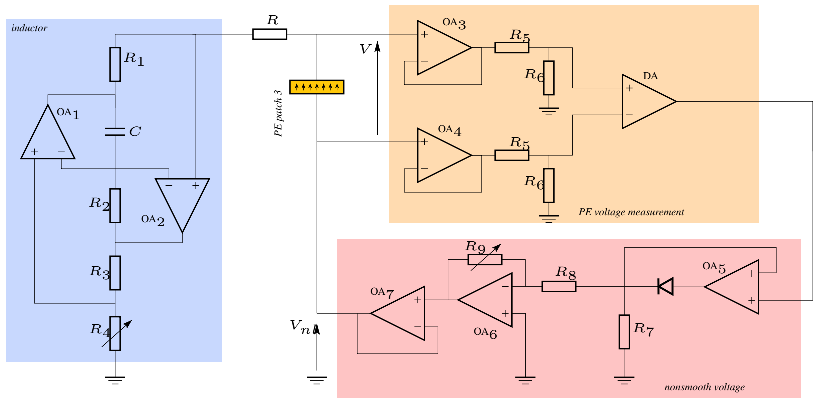

Figure 6: Shunt circuit schematic.

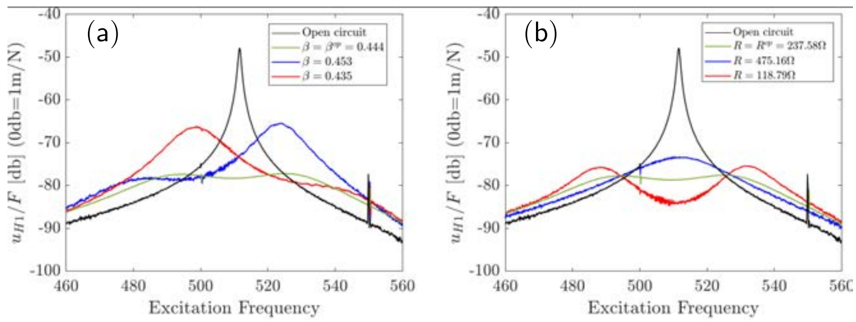

Figure 7: Experimental frequency response of the first harmonic of the velocity u_{H1} normalized with respect to the excitation level, for the 1:1 tuning case.

$R = R^{\text{op}} = 237.58\Omega$ (first column) and different values of R with $\beta = \beta^{\text{op}} = 0.444$ (second column). The inductance value used in the simulations is $L = 0.737\text{H}$ i.e $r = \omega_e/\hat{\omega} = 2$. The displacement and voltage responses are plotted in dB and linear scales, respectively.
It can be observed that the devised tuning for β^{op} and R^{op} is correct (green curves on each graphs). In Fig7(a), the detuning of β does shift the resonance as predicted, while in Fig7(b) the detuning of R will degrade the performance of the shunt.

5 Conclusion

A new nonlinear DVA has been proposed by introducing a nonsmooth component in a RL piezoelectric shunt which features are 1) multiple resonance tuning 2) downsizing of the inductance w.r.t the linear RL shunt. Rules for the optimal settings have been derived from numerical simulation on the adimensioned model in the 1:1 cas. Experiments show that 1) the circuit is tunable and 2) the tuning rules are correct.

Table 2: Electro-mechanical modal parameters.

Parameter	$\tilde{\omega}/(2\pi)$	$\hat{\omega}/(2\pi)$	ξ	m	C_{p}	θ	k	$\psi(\boldsymbol{x}_m)$
Value	509.4 Hz	511.36 Hz	0.0012	10.42 g	32.8 nF	5.2 mN/V	0.0875	1
Component	R_1 [kΩ]	R_2 [kΩ]	R_3 [kΩ]	C [μF]	R_5 [kΩ]	R_6 [kΩ]	R_7 [kΩ]	R_8 [kΩ]
Value	2	1	1	10	82	22	10	10

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