

## **ENHANCED ENERGY HARVESTING USING LOCALIZED VIBRATION MODES IN PLATES WITH AN APERIODIC ARRAY OF SCATTERERS**

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**Abstract.** Vibration energy harvesting has proven to be an effective method to power milliwatt sensors, devices that are quickly gaining attention for online monitoring, within the framework of the Internet of Things. The most common and least invasive electromechanical conversion method uses piezoelectric transducers, thanks to their high power density and ease of integration. The main difficulty lies in optimizing the combined mechanical-electrical system. The piezo elements are typically attached to resonating structures, and the electrical harvesting network is often optimized for the particular narrow frequency band of interest. Broadband harvesting can be achieved by exploiting system non-linearity, or using a rainbow-trapping device consisting of multiple resonators.

Higher vibration levels evidently lead to an increase of harvested energy, but can threaten the structural integrity of the mechanical system. The use of metamaterials can be of advantage, slowing waves sufficiently, to increase the energy conversion efficiency, even for low vibration levels. We propose a different approach, based on energy localization in plates decorated with a quasi-periodic pattern of scatterers. Our previous research has shown that phononic plates with non-resonant scatterers on points defined by a Penrose pattern exhibit rich dynamics: effective band gaps in the bulk of the plate, and narrow-band energy localization in well-defined regions. The plate thus acts as a mechanical filter for broadband excitation, limiting the overall vibration amplitude and guiding certain spectral components to well-defined locations. There, the accumulated power can be harvested using a piezoelectric disk. Finite element simulations show the increase in harvested power compared to a bare plate with the same mass and stiffness.

**Key words:** Energy harvesting, metamaterials, phononic crystals, localized modes

## 1 INTRODUCTION

The ever-increasing demand for energy, coupled with the limited availability of traditional sources of energy, has made the search for alternative and renewable energy sources crucial [1] Energy harvesting, the process of converting energy from ambient sources into usable electrical energy, has emerged as a promising solution for meeting certain energy needs.

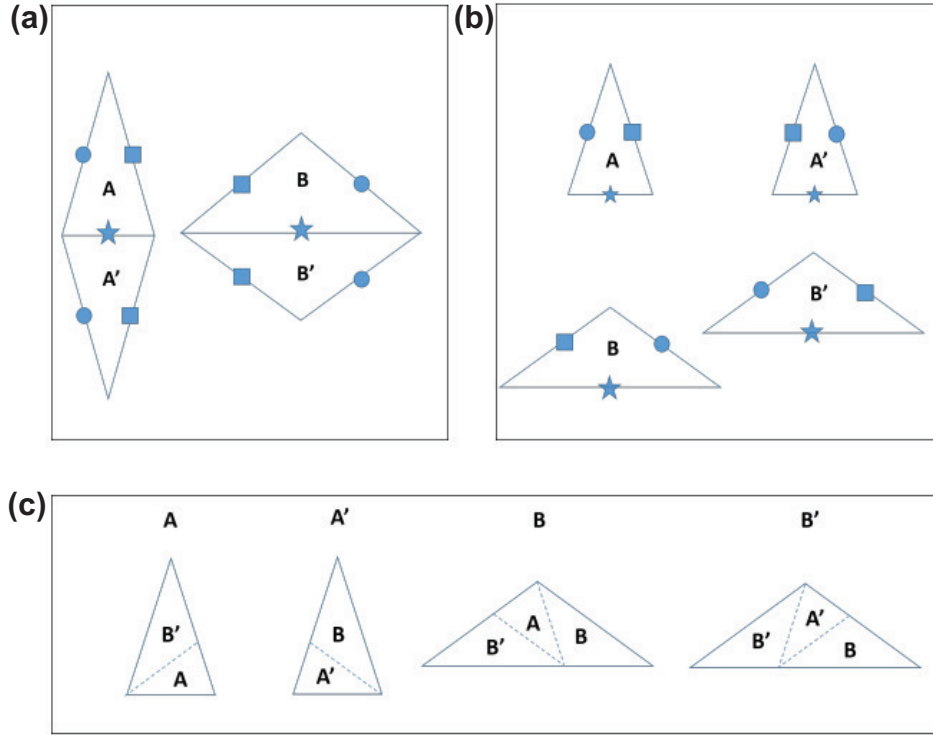
Energy harvesting can be broadly defined as the process of capturing and converting ambient energy sources into electrical energy that can be stored or used to power devices. The sources of ambient energy can be diverse, ranging from solar, thermal, and mechanical energy to electromagnetic radiation, vibrations, and chemical reactions. Vibrational energy harvesting, in particular, involves the conversion of mechanical energy from ambient vibrations into electrical energy [2, 3]

The potential of energy harvesting technologies lies in their ability to generate electricity from these energy sources without the need for an external power source or battery, thus enabling the development of self-powered and energy-efficient devices. Over the past few decades, energy harvesting technologies have attracted significant research interest and investment due to their potential applications in a wide range of fields, including wireless sensors, wearable electronics, biomedical devices, and internet of things (IoT) devices [4, 5]

Vibration energy harvesting has proven to be an effective method to power milliwatt sensors, devices that are quickly gaining attention for online monitoring. The most common and least invasive electromechanical conversion method uses piezoelectric transducers, thanks to their high power density and ease of integration. The main difficulty lies in optimizing the combined mechanical-electrical system. The piezo elements are typically attached to resonating structures, and the electrical harvesting network is often optimized for the particular narrow frequency band of interest. Broadband harvesting can be achieved by exploiting system non-linearity [6], or using a rainbow-trapping device consisting of multiple resonators [7, 8].

Higher vibration levels evidently lead to an increase of harvested energy, but can threaten the structural integrity of the mechanical system. The use of metamaterials can be of advantage, slowing waves sufficiently, to increase the energy conversion efficiency, even for low vibration levels. We propose a different approach, based on energy localization in plates adorned with a quasiperiodic pattern of scatterers. Our previous research has shown that phononic plates with non-resonant scatterers on points defined by a Penrose pattern exhibit rich dynamics: effective band gaps in the bulk of the plate, and narrow-band energy localization in well-defined regions [9]. The plate thus acts as a mechanical filter for broadband excitation, limiting the overall vibration amplitude and guiding certain spectral components to well-defined locations. There, the accumulated power can be harvested using a tuned resonator coupled to a piezoelectric patch.

We first present the layout of the scatterers, leading to an overall bandgap with localized vibration modes. After this, we present finite element simulations of the energy harvesting capabilities when piezoelectric patches are placed in appropriate positions on the quasiperiodic plate. The results are compared to a flat plate with the same overall density and bending stiff-



**Figure 1:** Decomposition rule for a P3 Penrose plate (From []). Two triangles can only be combined if their edges have the same symbol. We start from the  $BB'$  rhombus, decomposing it 5 times according to the scheme shown in (c). After the decomposition, point masses are placed in the center of mass of each triangle [9].

ness. Finally, we present our conclusions.

## 2 LOCALIZED MODES IN PLATES WITH A QUASI-PERIODIC PATTERN OF SCATTERERS

### 2.1 The Penrose phononic quasi-crystal

In our previous work [9], we have introduced a plate with mass scatterers placed according to a P3 Penrose pattern on a thin steel plate. These patterns consist of two different rhombic tiles filling the plane in a non-periodic way. Each rhombus can be divided into two triangles, leading to 4 different shapes that act as building blocks. The tessellation can be constructed by decomposing one of the rhombi into triangles, after which each triangle is further split according to the decomposition rule shown in Fig. 1.

In this paper, we start with the big rhombus  $BB'$  and go through five steps of decomposition. This leads to a set of 288 triangles, whose centers are decorated with a single mass. In order to agree better with reality, we choose masses with a finite size, and thus non-zero moments of inertia.

The resulting pattern has no translational periodicity, although locally a certain level of symmetry can be noticed. In certain areas the scatterers are closely spaced, whereas many open places are present. The latter will act as vibrating membranes, and they have a wide range of sizes. They can be delimited by as many as 10 masses, or as few as 4.

We chose an initial rhombus with edge length 0.776 m and a thickness of 1 mm. The resulting open membranes have sizes ranging from 5 to 12 cm between the two furthestmost scatterers.

## 2.2 Bending waves in a Penrose phononic plate

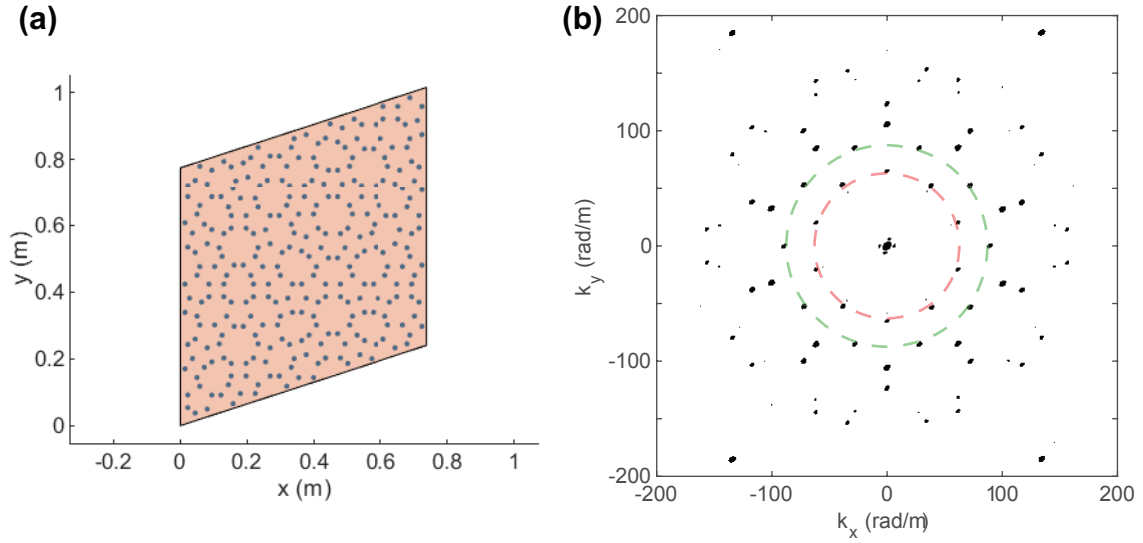
Bending waves in the resulting plate are scattered by the masses, resulting in an intricate interference pattern. We have shown before how the resulting vibrational amplitude, averaged over a chosen area, varies with frequency. At wave numbers equal to high-symmetry points in reciprocal space, destructive interference of waves results in broad frequency ranges with high wave attenuation, so-called band gaps. These band gaps occur at lower frequencies than in a similar plate with an equal amount of periodically placed scatterers. This can be seen in reciprocal space, since several high-symmetry points of the Penrose pattern are located closer to the origin than those of the honeycomb pattern. In real space, these low wave numbers refer to the areas with a low density of masses, effectively blocking long wavelengths and thus low frequencies. The higher wave number points refer to areas with high scatterer densities, blocking higher frequency waves. The overlap of this wide range of high-symmetry points leads to a wide band gap, which is however less deep than in the case of a periodic phononic crystal.

The wide band gap shows the interesting phenomenon of localized wave modes. Since the overall band gap is composed of many different wave numbers in reciprocal space, the mass scatterers are only efficient in certain areas of the plate for a given frequency. In areas where the masses do not have the right spacing to lead to destructive interference, energy can accumulate. If the frequency of the waves coincides with a natural frequency of an open membrane, this leads to a well defined localized vibration mode. Given the more or less circular shape of the membranes, fairly symmetric mode shapes with an increasing number of nodes can be distinguished.

## 2.3 Finite element model

Since the tessellation of masses shows no translational periodicity, the plate has to be modeled explicitly. No unit cell can be defined to reduce the calculation effort, as is typically done for the study of phononic crystals. We have shown that the band gap starting frequency can be estimated from analytical principles. The effect of an array of scatterers placed on an infinite plate can be estimated from multiple scattering theory. However, to take the edges of the plate into account, only a finite element model can achieve the desired level of accuracy.

We model the plate in Ansys 2022R2. The 1 mm thick steel plate is discretized using SHELL281 elements, quadratic elements with 6 degrees of freedom at each node, suitable for thin to moderately thick plates. The element size is 1 cm. The masses have a value of 10 g



**Figure 2:** (a) Geometry of the plate with an array of mass scatterers according to a P3 Penrose pattern. (b) Representation of the scatterers in reciprocal space, showing a first circle of high-symmetry points (red) at  $k = 65$  rad/m and a second one (green) at 90 rad/m.

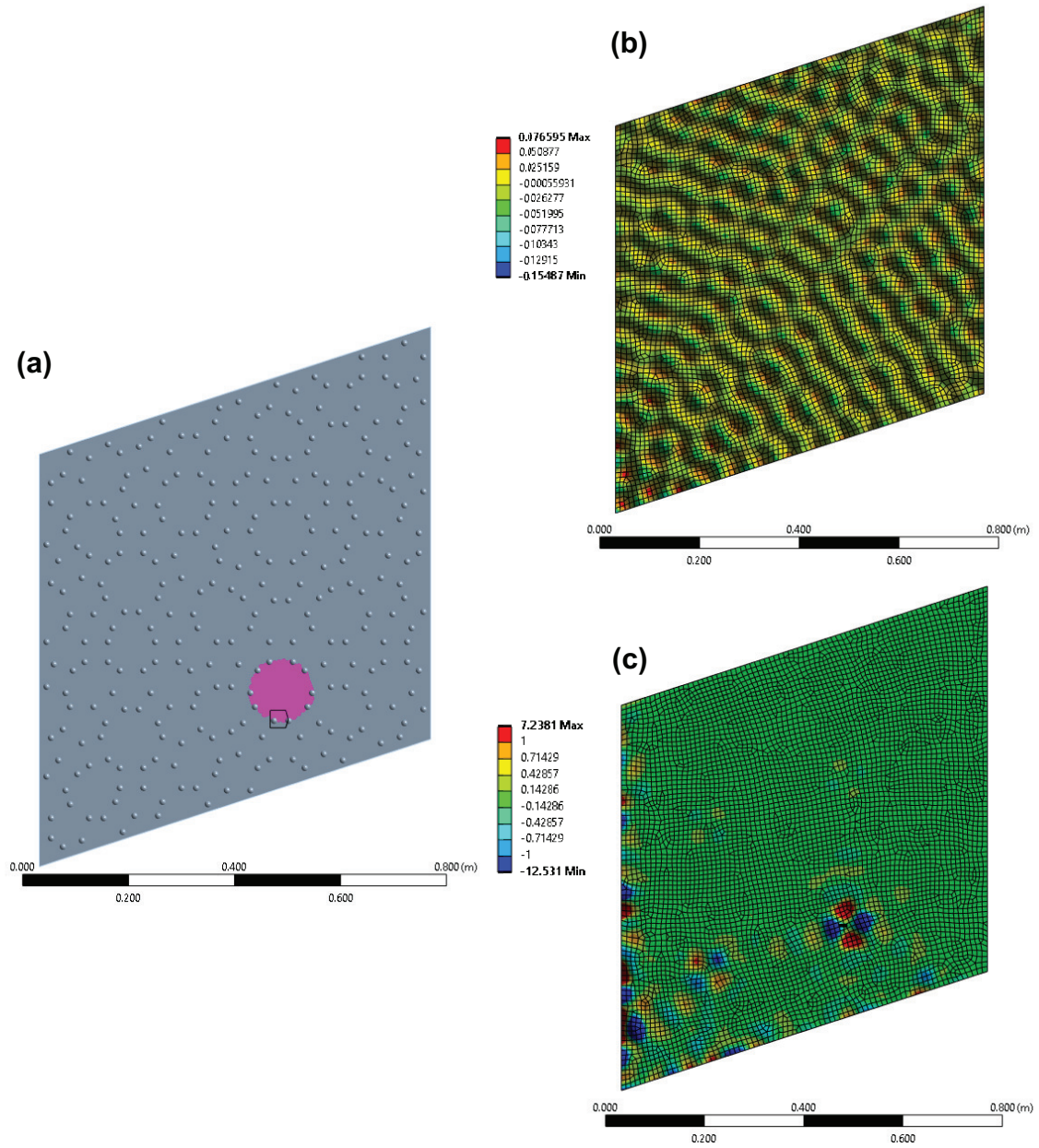
and a rotational inertia around  $x$  and  $y$  equal to  $0.46 \text{ mg.m}^2$ . Since the bending waves lead to a rotation of the rods, the moment of inertia lowers the location of the band gap. The plate is excited in the left bottom corner (Fig. 3 (a)) by a force of 1 N to perform a harmonic analysis between 2200 and 2600 Hz in 400 frequency steps. The result in each node yields the frequency response function, and this allows us to identify the location of the band gap and the localized modes.

The model of the Penrose plates is compared to the properties of a plate with the same size, bending stiffness, and overall mass. In order to achieve this, we use the same Young's modulus and plate thickness but adapt the mass density of the plate to account for the addition of the point masses.

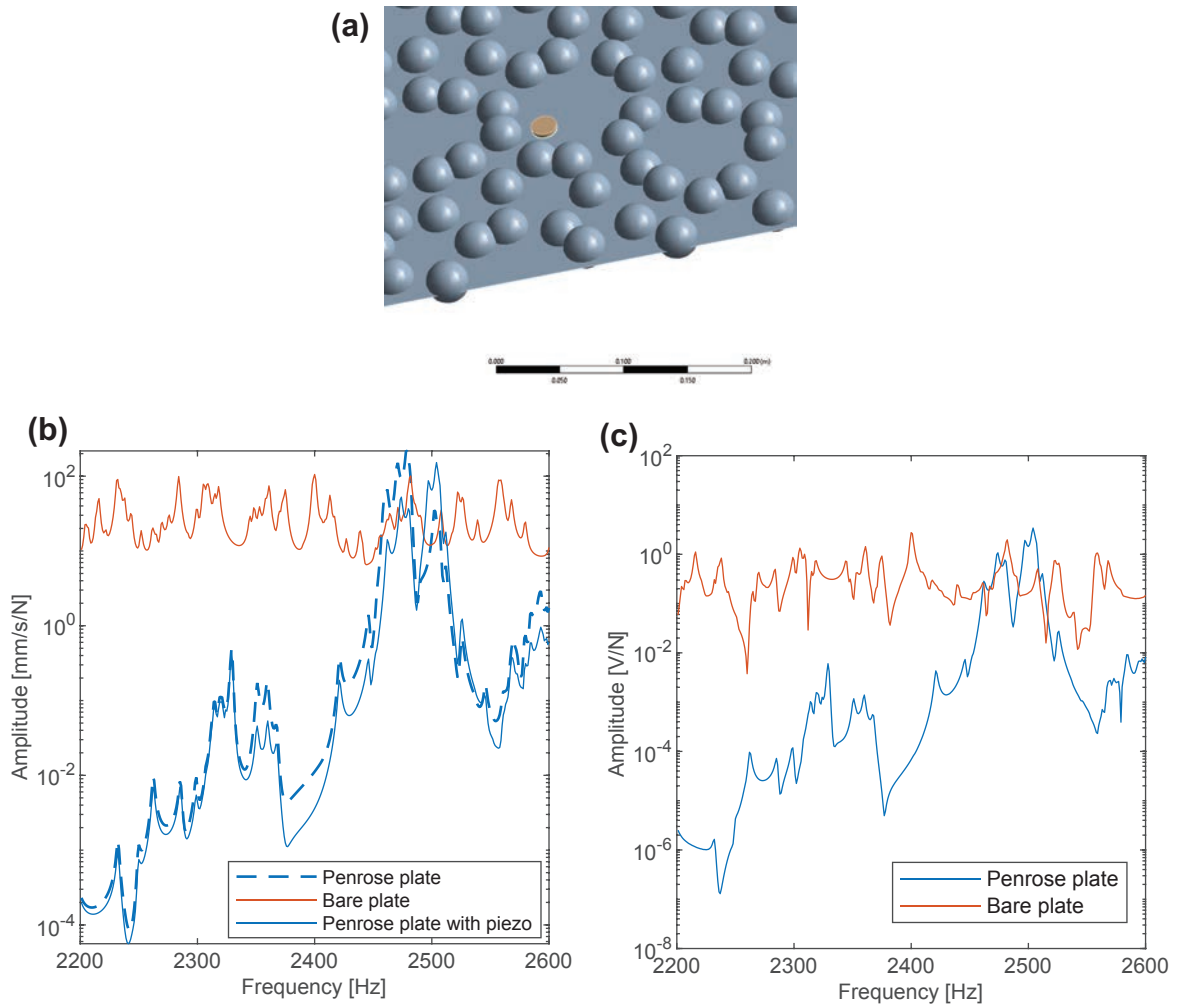
## 2.4 Wave localization effect

The dynamics of the two plates are compared in an area of interest defined by the largest open area in the Penrose plate. It is highlighted in pink in Fig. 3 (a). In this area we could identify a localized membrane mode with 2 node lines at 2480 Hz. For comparison, the bare plate shows an equal energy distribution at that frequency.

The average response in the area of interest was calculated using Matlab R2020b and the results are shown in Fig 4 (b). We clearly show the presence of a band gap with much lower amplitudes of the Penrose plate as compared to the bare plate. At the frequency of the localized mode, the velocity amplitude of the area of interest is 10 times higher than in the bare plate, showing an energy focusing due to the quasi-crystal structure.



**Figure 3:** (a) Plate with point masses, showing the area of interest for the plate velocity analysis. (b) Vibration pattern of the equivalent bare plate at 2480 Hz. (c) Vibration pattern of the Penrose plate at 2480 Hz, showing the localized mode at the area of interest. The unit of the amplitude scale is m/s.



**Figure 4:** (a) Placement of the piezo element for energy harvesting, on a large free membrane. (b) Average velocity of the area of interest on a Penrose plate (blue curves) and bare plate (orange curve). The piezo disk shifts the peak of maximum amplitude to a higher frequency. (c) Generated voltage on the top electrode of the piezo disk, which follows the displacement velocity.

### 3 ENERGY HARVESTING IN LOCALIZED MODES

#### 3.1 Piezoelectric harvester

For the simulations, we propose a disk of piezo material bonded to an antinode of the localized mode. The disk has PZT properties (density  $7500 \text{ kg/m}^3$ , Young's modulus  $120 \text{ GPa}$ ), polarized through the thickness. The piezoelectric coupling coefficient is  $23.3 \text{ As/m}^2$ . On top of the piezo disk, a 3 mm thick steel back load is attached to increase the strain. The total mass of the harvesting structure is 8 g, as compared to the total weight of 7.37 kg for the plate and scattering masses. The setup is shown in Fig. 4 (a).

#### 3.2 Energy harvesting

The simulations are performed using a coupled harmonic analysis. The piezo disk is defined as a body with electric properties (in the charge regime), whereas the plate and backing mass are structural bodies. The bottom of the piezo is grounded, and its top is defined as an electrode.

The results for the voltage are shown in Fig. 4 (c). First of all, it can be seen in panel (b) that the location of the localized mode slightly shifts to 2505 Hz. This can be explained by the added stiffness of the piezo disk. The amplitude at this frequency is higher than the one of the bare plate. This consequently leads to an increased piezo voltage at that frequency.

### 4 CONCLUSIONS

A thin plate with a quasi-periodic array of mass scatterers, placed according to a P3 Penrose pattern, shows a wide overall bandgap. However, due to the irregular placement of the masses, energy can accumulate in certain areas, leading to localized modes. These localized modes can have an amplitude ten times higher than the vibrational amplitude of a bare plate with identical properties, leaving the rest of the plate effectively motionless.

The energy accumulation can be used to efficiently harvest vibrational energy by a piezoelectric disk. The disk is placed in an antinode of the localized mode for optimal coupling. Although the mode's amplitude is reduced because of the added mass, the harvested energy at the frequency of localization is significantly larger than when placing the piezo element on a bare plate.

This approach shows that quasi-periodic metamaterials can be used to simultaneously protect the bulk of a structure from high vibration levels, and take advantage of the energy localization for harvesting purposes. However, optimal harvesters should be investigated. It can be foreseen that piezoelectric beams, tuned to the specific mode resonance, might further increase the efficiency of this approach.

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