Available online at www.eccomasproceedia.org
Eccomas Proceedia SMART (2023) 1800-1807

ECCOMAS Proceedia

X ECCOMAS Thematic Conference on Smart Structures and Materials
SMART 2023
D.A. Saravanos, A. Benjeddou, N. Chrysochoidis and T. Theodosiou (Eds)

COMPARISON BETWEEN LINEAR AND NON-LINEAR PERFORMANCE OF VIBRATION-BASED PIEZOELECTRIC ENERGY HARVESTERS

ROBERTO PALMA*, RAFAEL CASTRO†, ESTHER PUERTAS* AND RAFAEL GALLEGO*

*Structural Mechanics and Hydraulic Engineering
University of Granada
Campus Fuentenueva, 18071 Granada, Spain
e-mail: rpalgue@ugr.es

† Mechanics
University of Córdoba
Campus Rabanales, 14071 Córdoba, Spain

Abstract. Nowadays, the consumption of energy is increasing due to the high quality of life, but society demands more and more the use of clean energy. The production of this type of energy at the macro-scale is solved by wind and solar plants but the micro-energy production is still mostly based on electrochemical batteries that pollute the environment after their useful life.

In the context of Structural Health Monitoring (SHM) applied to railways bridges, equipment composed of networks with sensors, actuators and wifi devices that require micro-power supplies is needed. One way to avoid the use of electrochemical batteries in these applications could be the use of vibrating harvesters based on smart materials such as piezoelectric, piezomagnetic, etc. These materials work both in linear and non-linear modes.

On this ground, this work presents a numerical comparison between both operation modes by using a displacement-based Finite Element (FE) formulation with a) small strains and rotations for the linear response and b) a Saint-Venant-Kirchhoff model for the non-linear regime as a first and good approximation. Both formulations are used to model a piezoelectric harvester for which the prescribed time-dependent displacements are numerically simulated. Finally, the voltages generated by both formulations are compared and several conclusions are highlighted.

Key words: Vibrating Harvesters, Piezoelectricity, Non-linear, Finite element method, Experimental measurements

1 INTRODUCTION

Railway bridges have evolved greatly over the last decades due to two main reasons: increasing the public transportation networks and decreasing travel times. The preservation of these

doi: 10.7712/150123.9950.450290

infrastructures has become one of the main research topics for the civil engineering community and, consequently, the developments in Structural Health Monitoring (SHM) are rapidly increasing [1]. In the context of SHM applied to railways bridges, pieces of equipment composed of networks with sensors, actuators and wifi devices that require micro-power supplies are used. Commonly, the power of SHM equipment is supplied by electrochemical batteries presenting two shortcomings:

1. they need to be recharged and/or replaced, which becomes a problem for inaccessible structures,

2. they are not considered as green energy, which is currently demanded by society.

Therefore, despite the fact that vibrating harvesters produce relatively little energy, they could be used to solve the two previous shortcomings.

Vibration harvesters using piezoelectric materials for rotational motion systems are reported in [2], for ambient vibrations in [4] and for multipurpose applications in [5].

With regard to the application of vibrating harvesters in railway bridges, there are two types of harvesters:

- Adhesive patches bounded to the structure could increase the energy production in bridges [6] but they are made out of polymers for which the energy conversion factor is several orders of magnitude lower than the ceramics according to [7].

- Cantilever harvesters that use a substructure and one or several piezoelectric ceramics. There are several works in the literature based on simplified models that assume small strain at the linear regime for both substructure and piezoelectric ceramic, see [10] and [11].

For the best understanding of the authors of the present work, there are no works on the non-linear performance of cantilever harvesters applied to railway bridges.

On this ground, the present work presents a numerical study to compare the performance of cantilever harvesters using piezoelectric and working at linear and non-linear modes. The vibration harvester is modeled by two displacement-based Finite Element (FE) formulations: the first one uses small strain while the second assumes the simplest finite deformation model based on the Saint-Venant-Kirchhoff material. Notice that several of the authors of the present work have experience in multiphysics FE formulations, see [12], [13], [14], [15] .

2 CANTILEVER HARVESTERS

In short, a cantilever harvester is composed of a substructure, which is typically a metal, a piezoelectric ceramics and a proof mass, which is used for tuning the cantilever, at the free end of the substructure. Figure 1 shows a sketch of a cantilever harvester and the boundary conditions: mechanically, it is a fixed-free beam; electrically, the electric potential is set to ground on the bottom electrode and the top one is connected to a piece of electronic equipment.

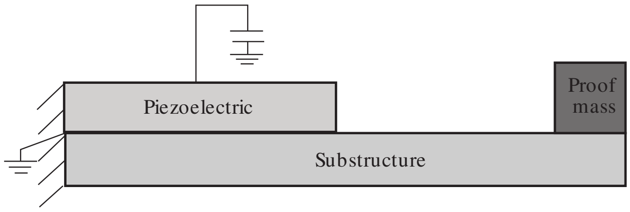

Figure 1: Sketch of a cantilever harvester and boundary conditions.

2.1 Piezoelectric coupling

The piezoelectric coupling is an interaction between the mechanical and electrical energies: stress applied on a piezoelectric generates a voltage (direct effect) or a voltage applied on the piezoelectric generates the deformation of the material (inverse effect).

The present work uses commercial piezoelectric ceramics, which are polarized along the z direction by a polarisation vector P. Then, the piezoelectric coupling is represented by a third-order tensor e_{ijk} that may be expressed in matrix form as:

$$[e_{ijk}] = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{13} & e_{13} & e_{33} & 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

where the Voigt's notation $(11, 22, 33, 23, 13, 12)$ has been used.

2.2 Operation modes

According to the piezoelectric coefficients, there are three coupling types that result in three operation modes of the cantilever harvester, see Figure 2:

- Mode 31 for which the vibration is perpendicular to the material poling direction P.
- Mode 33 for which excitation and P directions are the same.
- Mode 15 for which vibrations produce shear stresses.

According to [16], mode 31 is the most efficient for small vibrations and 33 is more durable and efficient under high excitations.

3 NUMERICAL RESULTS

3.1 Finite element model

The present work uses both small and large strain three-dimensional FE formulations by using eight noded elements with four degrees of freedom per node: displacements u_i and voltage V. The spatial discretization is performed by standard isoparametric shape functions of Lagrange type while the time discretization and non-linearities are solved by the Newmark-β and

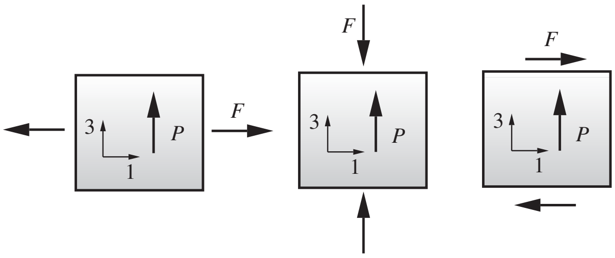

Figure 2: Sketches of operation modes: 31 (left), 33 (middle) and 15 (right).

Newton-Rhapson algorithms, respectively. The FE formulation is implemented in the research code [17], which belongs to the University of California at Berkeley (USA).

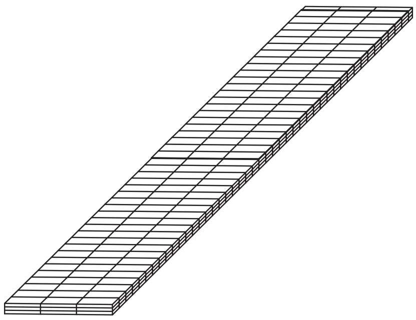

Figure 3: Finite element mesh composed of 603 eight-noded elements.

In order to simulate the cantilever harvester the following materials are used:

- A substructure of aluminum with dimensions 700×25.12×2.4 [mm] and material properties: Young's module $E = 52.82$ [GPa], Poisson's ration 0.304 [-] and mass density 2418 [kg/m^3].
- A commercial piezoelectric PZT-5J ceramic manufactured by *piezo.com*.
- The proof mass, which is assumed to be zero in the present work since the dimensions of the aluminum beam ensure a vibration mode of 3.7 [Hz], see Figure 4.

The FE mesh is composed of 603 elements, as shown in Figure 3. The boundary conditions set to zero the mechanical displacements at the clamped end of the beam and the voltage at the bottom face of the piezoelectric material, as shown in Figure 2. In addition, the body forces are due to the self-weight of the beam and are considered along the polarisation axis z.

3.2 Excitations

For the sake of simplicity, the cantilever harvester is excited with a sinusoidal displacement at the clamped end given by the equation:

$$w = A_0 \sin(2\pi f_0 t), \tag{2}$$

where the vector w denotes the prescribed displacement along the polarisation direction, t the time, $A_0 = 1$ [mm] the amplitude and $f_0 = 3.71$ [Hz] the frequency. Notice that these amplitude and frequency values are obtained from [18] in order to simulate the vibrations produced by the passage of a train through a railway bridge.

3.3 Modal analysis

Figure 4 shows the four first frequencies calculated from the linear model. As observed, the first frequency is 3.718 [Hz] and, consequently, the cantilever is tuned for the railway bridge vibration, see the value of f_0. Furthermore, this frequency agrees with the experimental one obtained by measuring the displacement at the beam's free end versus several excitation frequencies and plotting the dynamic amplification factor.

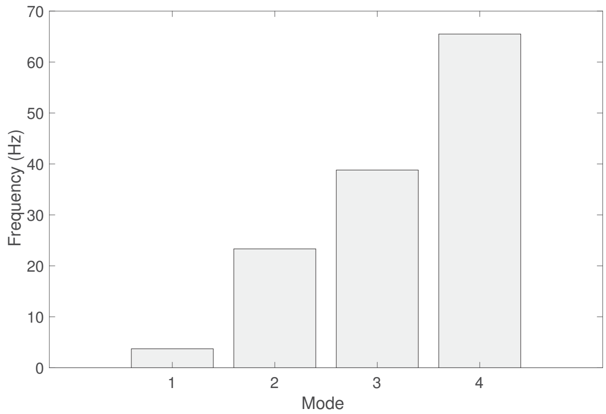

Figure 4: First four frequencies for the modal analysis of the LIN model.

3.4 Generated voltage

In order to obtain the voltage generated, a time-step FE simulation with $\Delta t = 0.01$ [s] and $n_t = 200$ is performed in a 2.3 [GHz] Intel Core i5 processor. For the sake of simplicity, the linear model is denoted by LIN and the Saint-Venant-Kirchhoff by SVK.

Figure 5 shows the difference $\Delta V = (V^{SVK} - V^{LIN})$ between the generated voltage obtained by both LIN V^{LIN} and SVK V^{SVK} models. This difference is approximately $RMS(\Delta V) = 0.5$ [V] while the CPU times for both calculations are 178.67 [s] and 289.77 [s], respectively. The relative error produced by the assumption of the LIN model may be calculated as:

$$error = \frac{|RMS(V^{SVK}) - RMS(V^{LIN})|}{RMS(V^{LIN}} \times 100 = 4.3\%. \tag{3}$$

In conclusion, the optimization of the cantilever harvester for this application and with this kind of materials and excitations should be simulated by linear elastic models to minimize the CPU time.

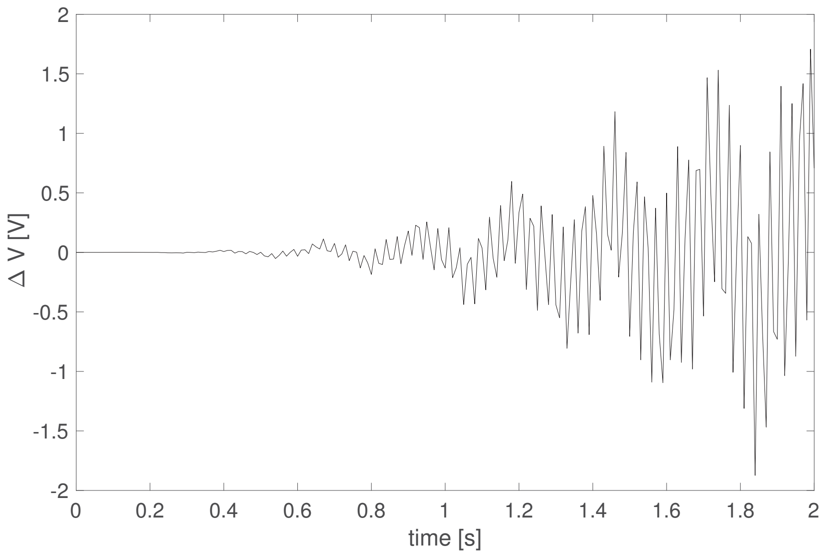

Figure 5: Difference $\Delta V = (V^{SVK} - V^{LIN})$ between generated voltage calculated using linear V^{LIN} and Saint-Venant-Kirchhoff models V^{SVK}.

4 ACKNOWLEDGEMENTS

This research was funded by Ministerio de Ciencia e Innovación of Spain: grant numbers PID202-116644RB-I00 and PLEC2021-007798.

5 CONCLUSIONS

This work presents a numerical simulation of cantilever harvesters based on piezoelectric ceramics for applications in railway bridges. In particular, two finite element models are considered: i) linear elastic assuming small strain, displacements and rotations, denoted by LIN and ii) a Saint-Venant-Kirchhoff (SVK) model that considers large strains. For the numerical comparison, a cantilever beam composed of an aluminum substructure and a PZT-5J piezoelectric ceramic is tuned to the vibrations generated by the passage of a train through a railway bridge. It is concluded that:

1. The difference in generated voltage between both models is approximately 0.5 [V], with a relative difference of 4.3 %.

2. The difference in CPU time for each simulation between both models is 112 [s].

In conclusion, a LIN model could be enough for future simulations and optimizations in this type of harvester application.

REFERENCES

[1] J. Ko and Y. Ni, "Technology developments in Structural Health Monitoring of large-scale bridges," *Eng Struct*, vol. 27, no. 12, pp. 1715–1725, 2005.

[2] M. Gua and W.-H. Liao, "Design and analysis of a piezoelectric energy harvesters for rotational motion system," *Energy Conversion and Management*, vol. 111, pp. 239–244, 2016.

[3] H.-X. Zou, W.-M. Zhang, W.-B. Li, K.-X. Wei, Q.-H. Gao, and Z.-K. Peng, "Design and experimental investigation of a magnetically coupled vibration energy harvester using two inverted piezoelectric cantilever beams for rotational motion," *Energy Conversion and Management*, vol. 148, pp. 139–1398, 2017.

[4] K.-Q. Fan, F.-B. Chao, J.-G. Zhang, W.-D. Wang, and X.-H. Che, "Design and experimental verification of a bi-directional nonlinear piezoelectric energy harvester," *Energy Conversion and Management*, vol. 86, pp. 561–567, 2014.

[5] K.-Q. Fan, J. Chang, F. Chao, and W. Pedrycz, "Design and development of a multipurpose piezoelectric energy harvester," *Energy Conversion and Management*, vol. 96, pp. 430–439, 2015.

[6] R. Cahill, N. Nuallain, N. Jackson, A. Mathewson, R. Karoumi, and V. Pakrashi, "Energy harvesting from train-induced response in bridges," *Journal of Bridge Engineering*, vol. 19, no. 9, pp. 1–1, 2014.

[7] Z. Zhang, H. Xiang, and Z. Shi, "Mechanism exploration of piezoelectric energy harvesting from vibration in beams subjected to moving harmonic loads," *Composite Structures*, vol. 179, pp. 368–376, 2017.

[8] M. Karimi, A. Karimi, R. Tikani, and S. Ziaei-Rad, "Experimental and theoretical investigations on piezoelectric-based energy harvesting from bridge vibrations under travelling vehicles," *International Journal of Mechanical Sciences*, vol. 119, pp. 1–11, 2016.

[9] P. Cahill and V. Jaksic, "Effect of road surface, vehicle, and device characteristics on energy harvesting from bridge-vehicle interactions," *Computer-Aided Civil and Infrastructure Engineering*, vol. 31, pp. 921–935, 2016.

[10] M. Macías, R. Castro-Triguero, R. R. Sola-Guirado, D. Bullejos, and M. Friswell, "A feasibility study on piezoelectric energy harvesting from the operational vibration of a highway bridge," *Advances in Structural Engineering*, vol. 26, no. 2, p. 136943322211201, 2022.

[11] J. Cámara-Molina, E. Moliner, M. Martínez-Rodrigo, D. Connolly, D. Yurchenko, P. Galvín, and A. Romero, "3D printed energy harvesters for railway bridges-design optimisation," *Mechanical Systems and Signal Processing*, vol. 190, pp. 1–22, 2023.

[12] J. Perez-Aparicio, H. Sosa, and R. Palma, "Numerical investigations of field-defect interactions in piezoelectric ceramics," *International Journal of Solids and Structures*, vol. 44, no. 14-15, pp. 4892–4908, 2007.

[13] G. Rus, R. Palma, and J. Perez-Aparicio, "Optimal measurement setup for damage detection in piezoelectric plates," *International Journal of Engineering Science*, vol. 47, no. 4, pp. 554–572, 2009.

[14] J. Perez-Aparicio, R. Palma, and R. Taylor, "Multiphysics and thermodynamic formulations for equilibrium and non-equilibrium interactions: non-linear finite elements applied to multi-coupled active materials," *Archives of Computational Methods in Engineering*, pp. 535–583, 2016.

[15] R. Palma, J. Pérez-Aparicio, and R. Taylor, "Dissipative finite-element formulation applied to piezoelectric materials with the debye memory," *IEEE/ASME Transactions on Mechatronics*, vol. 23, no. 2, pp. 856–863, 2018.

[16] S. Saadon and O. Sidek, "A review of vibration-based MEMS piezoelectric energy harvesters," *Energy Conversion and Management*, vol. 52, pp. 500–504, 2011.

[17] R. Taylor and S. Govindjee, *FEAP A Finite Element Analysis Program, Programmer Manual*, 2020.

[18] R. Palma, J. L. Prez-Aparicio, and P. Museros, *Finite element modeling of energy harvesters: application to vibrational devices*. Berlin, Boston: De Gruyter Oldenbourg, 2019, pp. 3–36. [Online]. Available: https://doi.org/10.1515/9783110445053-001