

Comparison of Two Fully-Passive Multimodal Damping Techniques: Multi-Branch Shunts and Interconnected Piezoelectric Networks

Pierre Flament, Boris Lossouarn, Éric Ramond, Pascal Audrain, Jean-François Deü

Abstract. Multi-branch resonant piezoelectric shunt damping is an efficient method for mitigating the amplitude of multiple mechanical resonances. In this approach, one or several piezoelectric patches are connected to an electrical circuit designed to control several specific modes, enabling tailored management of broadband structural damping. However, the introduction of capacitors into the multi-branch shunts reduces the overall performance of the circuit. To overcome this limitation, interconnected piezoelectric networks have been proposed in the literature. This solution enhances performance by enabling each piezoelectric patch to contribute to the total coupling factor for each vibration mode. Based on recent studies, a method is introduced to define interconnected piezoelectric networks using a matrix formulation that optimally represents the multi-input multi-output behavior of the network, satisfying the optimal impedance equations. After numerical implementations on a benchmark blade structure, both techniques are compared by evaluating the fully interconnected optimal network with respect to the multi-branch shunts. Experimental results are also proposed to validate the numerical models and the subsequent comparison of performances between the two multimodal techniques.

Key words: Passive control, Multimodal vibration damping, Piezoelectricity, Multi-branch shunts, Interconnected piezoelectric networks, Coupling factors

1 INTRODUCTION

Piezoelectric shunt damping is an efficient vibration control technique that leverages the electromechanical coupling of piezoelectric materials to mitigate structural vibrations of a host structure. When subjected to strain, the material generates an electric field and deformation occurs when an electric field is applied. Thus, a piezoelectric transducer converts part of mechanical energy into electrical energy and can be dissipated through a properly tuned shunt circuit using resistive elements [1]. Introduced by Forward in 1979 [2] and theoretically formalized by Hagood and von Flotow in 1991 [3], these shunts can be resistive (R) or resonant (RL). Resonant shunts use inductive elements to create an electrical resonance

Pierre Flament & Éric Ramond & Pascal Audrain
Naval Group, Place Bruat, 50100, Cherbourg-en-Cotentin, France,
e-mail: pierre.flament@naval-group.com

Pierre Flament & Boris Lossouarn & Jean-François Deü
Laboratoire de Mécanique des Structures et Systèmes Couplés, Conservatoire national des arts et métiers, 292 rue Saint Martin, 75003, Paris, France

with the capacitance of the piezoelectric transducer. Optimized for damping around a specific mode, they may be insufficient for complex structures subjected to broadband excitations.

To damp multiple resonances simultaneously, multi-branch shunts have been proposed, where several electrical branches are connected to a single piezoelectric transducer, each tuned to a specific vibration mode. First introduced by Edberg *et al.* [4] in 1991 and generalized by Hollkamp [5] in 1994, several circuit topologies were proposed thereafter [6, 7], while these works have proven to be rather complex due to the interaction between the different resonant branches. More recently, Raze *et al.* [8] presented a summary of multi-branch shunts proposed in the literature with their respective optimal tuning rules. The potential of a purely passive realization is highlighted, mainly due to its inherent stability and cost-effectiveness [9]. The feasibility of using passive components has also been demonstrated by Lossouarn *et al.* [10], and a complete implementation on a complex structural geometry has been achieved by Flament *et al.* [11]. From his work [12], copper losses are considered through additional series resistors, dielectric losses are modeled using a frequency-dependent dynamic resistance, and iron losses are minimized through selection criteria to build tailor-made inductors. Although strengths and weaknesses are difficult to distinguish numerically [8], the current blocking shunt [13] has been shown to be more efficient and robust for a fully-passive application case [11]. However, a structure endowed with a single transducer may limit damping performance for certain modes, and the introduction of capacitors into the multi-branch shunts reduces the overall performance of the circuit [8]. In order to overcome this limitation, recent advances have explored the distribution of multiple transducers across a structure through an interconnected piezoelectric network, combining the electromechanical coupling factors between all transducers and modes when mechanical and electrical modal properties coincide [14]. Whereas comparisons between multi-branch shunts and networks are limited, pioneering studies show that networks offer superior performances [15]. Experimental validations have confirmed their effectiveness for various simple structures, such as beams [16]. To address complex structures, Raze *et al.* [17] developed a non-iterative, modal-based synthesis method for designing resonant electrical networks optimized for multimodal damping, ensuring realizability with passive components. Later, a matrix formulation was developed [18] to optimally represent the multi-input multi-output behavior of the network, satisfying the optimal impedance equation for the overall vibration reduction of the targeted resonances.

The objective of this work is to compare the vibration reduction performance of multi-branch shunts and interconnected piezoelectric networks applied to a complex geometry. The article is structured as follows. First, the benchmark structure is presented in Section 2. The current blocking shunt is then implemented in Section 3 while accounting for and incorporating parasitic losses associated with inductive components and piezoelectric patches. An optimal tuning strategy is thus proposed, aligning with the theoretical solution in the literature. Meanwhile, in Section 4, interconnected piezoelectric networks are implemented, and their performances are compared with that of the current blocking shunt. Finally, a fully-passive experimental validation is proposed for the current blocking shunt in Section 5.

2 BENCHMARK PIEZOELECTRIC STRUCTURE

2.1 Governing equations

Let us consider an elastic structure endowed with a piezoelectric patch characterized by a capacitance C_p and a parallel resistance R_p , connected to a shunt of impedance Z_s , as shown in Figure 1a. The governing equation of the coupled electromechanical system, incorporating mechanical damping, is derived from the strong formulation described by Thomas *et al.* [1]

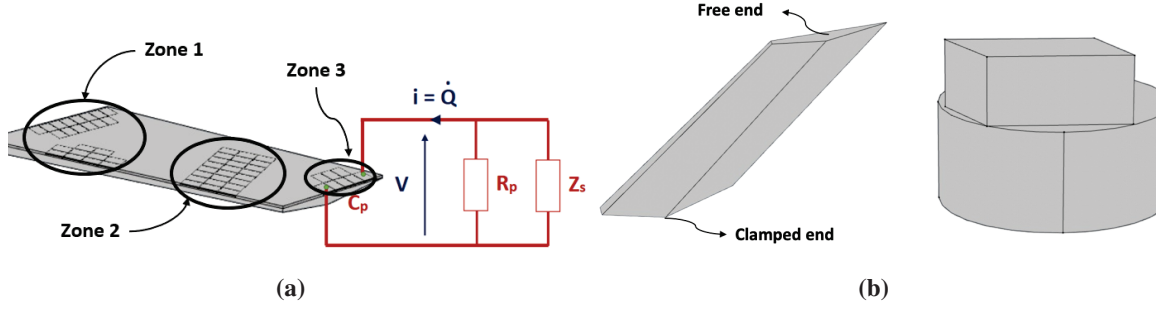


Figure 1: Benchmark piezoelectric structure: (a) electrical scheme of a piezoelectric patch of capacitance C_p and parallel resistance R_p connected to a shunt of impedance Z_s and (b) numerical model of the blade.

$$\begin{pmatrix} \mathbf{M}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{U}} \\ \ddot{\mathbf{V}} \end{pmatrix} + \begin{pmatrix} \mathbf{C}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{U}} \\ \dot{\mathbf{V}} \end{pmatrix} + \begin{pmatrix} \mathbf{K}_{sc} & \mathbf{K}_c \\ -\mathbf{K}_c^T & \mathbf{K}_e \end{pmatrix} \begin{pmatrix} \mathbf{U} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{F} \\ \mathbf{Q} \end{pmatrix}, \quad (1)$$

where $\mathbf{M}_m$, $\mathbf{K}_{sc}$, and $\mathbf{C}_m$ are the mechanical mass, stiffness (short-circuited), and damping matrices, respectively, each of size $M \times M$. The electromechanical coupling matrix is denoted by $\mathbf{K}_c$ of size $M \times P$ and $\mathbf{K}_e$ is the diagonal matrix of the blocked capacitance of size $P \times P$. Here, M and P are the numbers of mechanical degrees of freedom (dofs) and piezoelectric patches, respectively.

The system exhibits two different modal characteristics: its short-circuited configuration ($\mathbf{V} = \mathbf{0}$) and its open-circuited configuration ($\mathbf{Q} = \mathbf{0}$). The corresponding natural frequencies, denoted as ω_{sc} and ω_{oc} , along with their associated mode shapes Φ_{sc} and Φ_{oc} , satisfy the eigenvalue problems formulated for each configuration at $\mathbf{F} = \mathbf{0}$, where $\mathbf{K}_{oc} = \mathbf{K}_{sc} + \mathbf{K}_c \mathbf{K}_e^{-1} \mathbf{K}_c^T$, given by

$$\mathbf{K}_{sc} \Phi_{sc} - \omega_{sc}^2 \mathbf{M}_m \Phi_{sc} = \mathbf{0}, \quad (2)$$

$$\mathbf{K}_{oc} \Phi_{oc} - \omega_{oc}^2 \mathbf{M}_m \Phi_{oc} = \mathbf{0}. \quad (3)$$

2.2 Finite element model and targeted modes

The benchmark piezoelectric structure under consideration represents a simplified model of a generic lifting surface, as described by Lossouarn [19], and clamped into a cylindrical block of 730 kg, as presented in Figure 1b, where fully detailed geometric data are provided in [19]. The blade and the cylindrical block are made of homogeneous and isotropic steel, characterized by a Young's modulus $E_s = 210$ GPa, a Poisson's ratio $\nu_s = 0.3$, and a density $\rho_s = 1780$ kg/m³. The total weight is approximately 920 kg, with boundary conditions defined as follows: the end with the largest cross-section is rigidly fixed to a cylindrical block of 730 kg, while the opposite end is free. the blade is excited with a vertical pressure near the end of the trailing edge. This excitation is distributed to ensure a vertical force of 1 N at each measurement point within the designated area. Rayleigh damping is introduced based on the first and tenth vibration modes, with damping ratios $\xi_1 = 0.2$ % and $\xi_{10} = 0.4$ %, respectively. Three zones of piezoelectric patches are bonded to the upper surface and embedded within a 6.5 mm epoxy layer, following [19]. The epoxy has a Young's modulus $E_e = 3$ GPa, a Poisson's ratio $\nu_e = 0.3$, and a density $\rho_e = 1080$ kg/m³. The piezoelectric material PIC255 is detailed in [9].

Table 1: Short and open-circuit eigenfrequencies and their corresponding electromechanical coupling factors of the first five bending modes and for the 3 zones from the finite element model.

Mode	f_{sc} (Hz)	$f_{oc,k}^{(1)}$ (Hz)	$f_{oc,k}^{(2)}$ (Hz)	$f_{oc,k}^{(3)}$ (Hz)	$k_{c,k}^{(1)}$ (%)	$k_{c,k}^{(2)}$ (%)	$k_{c,k}^{(3)}$ (%)
1	88.62	88.80	88.66	88.62	6.28	2.94	0.24
2	295.32	295.54	295.79	295.34	3.84	5.61	1.25
5	650.98	651.05	651.16	651.27	1.52	2.38	2.99
7	1075.61	1075.61	1076.27	1077.06	0.17	3.49	5.19
10	1524.55	1524.55	1524.59	1528.88	0.21	0.71	7.54

The placement of the piezoelectric patches was initially designed to control bending modes 1, 2, and 10 [19]. The first five bending modes are targeted, where short and open-circuit frequencies are defined in Equations 2 and 3 and presented in Table 1. It is noted that f_{sc} are computed by short-circuiting all the piezoelectric patches and $f_{oc}^{(p)}$ by only open-circuiting the p -th piezoelectric patch, while the others remain short-circuited. The electromechanical coupling factor associated $k_c^{(p)}$ is directly linked to the efficiency of a piezoelectric transducer for vibration damping of the considered vibration mode. It is defined from the open- and short-circuit frequencies f_{oc} and f_{sc} [1], with i the index of the vibration modes and superscript (p) the number of the zone of patch, such as

$$k_{c,k}^{(p)} = \sqrt{\frac{f_{oc,k}^{(p)2} - f_{sc,k}^2}{f_{sc,k}^2}}. \quad (4)$$

3 FULLY-PASSIVE MULTIMODAL DAMPING TECHNIQUES

3.1 Parasitic losses

Since this work focuses on purely passive control, each targeted resonance is managed using passive inductors. The use of these components introduces parasitic losses [10], such as copper losses due to the non-negligible series resistance inherent in wire-wound inductors. Iron losses are also monitored, as they manifest as a frequency-dependent resistance value. Moreover, given the low coupling factors of the benchmark structure, the energy transfer between the mechanical and electrical domains results in increased heat dissipation within the piezoelectric patches [11], which also leads to consider dielectric losses. These losses can degrade the circuit's performance; therefore, they must be quantified and incorporated into the model to ensure optimal vibration reduction of the targeted resonances.

3.2 Multi-branch resonant piezoelectric shunts

Multi-branch resonant shunts extend the single-mode approach by adding multiple electrical branches in parallel to the piezoelectric patch. Each added branch is introduced with inductors and capacitors, allowing the control of several resonances simultaneously [5, 6]. Each branch can target a vibration mode thanks to the introduced capacitors but lead to reduced coupling factors of the circuit [8]. The degraded (effective) electromechanical coupling factors are given by

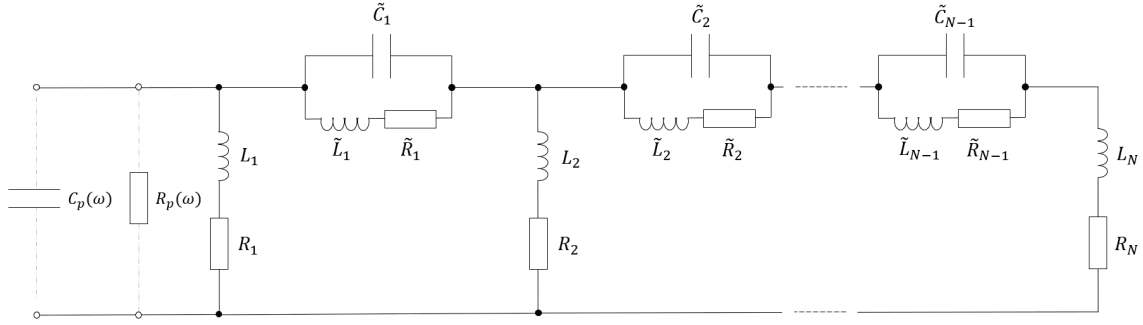


Figure 2: The proposed current blocking shunt circuit.

$$\kappa_{c,k}^{(p)2} = r_k^{(p)} \kappa_{c,k}^{(p)2}, \quad r_k^{(p)} \geq 0, \quad \sum_{k=1}^N r_k^{(p)} \leq 1, \quad (5)$$

where residues r_k serve as weighting factors assigned to each branch k , allowing a customized vibration performance control for each resonance.

The previous section highlighted that copper losses in a branch are included when an inductor is associated in series with a resistor. Since copper losses are often dominant in our applications, only this type of series configuration is considered. Consequently, the two selected solutions are the Hollkamp shunt and the current blocking shunt [8]. Despite comparable vibration reduction performance demonstrated both numerically and experimentally [8], and similar sensitivity to inductance variations [12], the current blocking shunt can offer an interesting advantage through its sequential tuning procedure for a fully-passive application case. It also provides greater resistive margins and typically requires smaller components, making it more efficient and robust for passive implementations [11]. Therefore, the current blocking shunt, illustrated in Figure 2, is adopted as the reference in the present study.

In its optimal configuration [8], inductors in the branches are coupled with series resistors, but not in the filters, accounting for only a portion of the copper losses. To fully consider them, additional series resistors are introduced in series with the filter inductors, resulting in $\tilde{R}\tilde{L}\tilde{C}$ filters. Dielectric losses are also modeled using a dynamic parallel resistor $R_p(\omega)$. Various types of passive inductors are available, and their selection is guided by the target (L, R) values and several feasibility criteria, aiming to minimize associated core losses and avoid nonlinear operating regimes [11].

The optimal admittance of the shunt $Y_s(s)$ is expressed as

$$Y_s(s) = \frac{1}{sL_1 + R_1} + \frac{1}{\frac{s\tilde{L}_1}{s^2\tilde{L}_1\tilde{C}_1 + 1} + \frac{1}{\frac{1}{sL_2 + R_2} + \frac{1}{\frac{s\tilde{L}_2}{s^2\tilde{L}_2\tilde{C}_2 + 1} + \dots + sL_N + R_N}}}. \quad (6)$$

The aim is to compensate for the two parasitic losses thanks to an update of the inductors and resistors of the main branches, ensuring that the true admittance maintains the same level of performance as the optimal one. The optimal admittance of the shunt $Y_s^*(s)$ considering parasitic losses is expressed as

$$Y_s^*(s) = \frac{1}{R_p(\omega)} + \frac{1}{sL_1^* + R_1^*} + \frac{1}{\frac{s\tilde{L}_1 + \tilde{R}_1}{s^2\tilde{L}_1\tilde{C}_1 + s\tilde{R}_1\tilde{C}_1 + 1} + \frac{1}{sL_2^* + R_2^*} + \frac{1}{\frac{s\tilde{L}_2 + \tilde{R}_2}{s^2\tilde{L}_2\tilde{C}_2 + s\tilde{R}_2\tilde{C}_2 + 1} + \dots + sL_N^* + R_N^*}}, \quad (7)$$

where the compensation tuning procedure of L_k^* and R_k^* is fully described in [11, 12].

3.3 Interconnected piezoelectric networks

Interconnected piezoelectric networks offer another multimodal approach to mitigate vibration, overcoming certain limitations of the multi-branch strategy [15, 17]. Networks enable the combination of coupling factors generated by each piezoelectric patch for each vibration mode [18]. That allows to maximize the coupling factors of each structural mode and, in some cases, eliminate the need of additional capacitors in the network, which are often in charge of vibration reduction performance.

The governing equations of the interconnected piezoelectric network formulation are derived by applying the Ritz-Galerkin projection method to the electromechanical system presented in Equation (1). This is achieved by expressing $\mathbf{V}$ from Equation (1) in terms of $\mathbf{U}$ and $\mathbf{Q}$ and then pre-multiplying the first row of the reformulated electromechanical system by Φ_{oc} with $\mathbf{U} = \Phi_{oc}\mathbf{u}$, yielding

$$\begin{pmatrix} \Phi_{oc}^T \mathbf{M}_m \Phi_{oc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{Q}} \end{pmatrix} + \begin{pmatrix} \Phi_{oc}^T \mathbf{K}_{oc} \Phi_{oc} & \Phi_{oc}^T \mathbf{K}_c \mathbf{K}_e^{-1} \\ \Phi_{oc} \mathbf{K}_c^T \mathbf{K}_e^{-1} & \mathbf{K}_e^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \Phi_{oc}^T \mathbf{F} \\ \mathbf{V} \end{pmatrix}. \quad (8)$$

Let us consider a passive application [18], then the dynamic host system becomes

$$\begin{cases} (s^2 \mathbf{I}_{N \times N} + \Omega_{oc}^2) \eta_{oc} - \Gamma \Phi_p \eta_e = \Phi_{oc}^T \mathbf{f} \\ (s^2 \Omega_e^{-2} + 2s \mathbf{Z}_e \Omega_e^{-1} + \mathbf{I}) \eta_e - \Phi_p^T \Gamma^T \eta_{oc} = 0 \end{cases}, \quad (9)$$

where s is the Laplace variable, $\Omega_{oc} = \text{diag}(\omega_{oc,1}, \dots, \omega_{oc,N})$, $\Omega_e = \text{diag}(\omega_{e,1}, \dots, \omega_{e,N})$, and $\mathbf{Z}_e = \text{diag}(\xi_{e,1}, \dots, \xi_{e,N})$ are the diagonal $N \times N$ matrices containing the mechanical natural frequencies in open-circuit configuration, the electrical natural frequencies, and the electrical damping ratios of the passive network, respectively. Φ_{oc} are the eigenvectors of size $M \times N$ associated to the N considered eigenvalues, and Φ_p are the electrical mode shapes of the passive network of size $N \times P$. Finally, $\Gamma = -\Phi_{oc}^T \mathbf{K}_c \mathbf{K}_e^{-1}$, η_{oc} and η_e are the mechanical and electrical vectors of modal coordinates. The objective is to calculate the state-space matrix model described in [18], given by

$$\mathbf{A}_e = \begin{pmatrix} \mathbf{0}_{N \times N} & \mathbf{I}_{N \times N} \\ -\Omega_e^2 (\mathbf{I} - \Phi_p^T \mathbf{E}_p \Phi_p) & -2\Omega_e \mathbf{Z}_e \end{pmatrix}, \quad \mathbf{B}_e = \begin{pmatrix} \mathbf{0}_{N \times P} \\ \Omega_e^2 \Phi_p^T \end{pmatrix}, \quad \mathbf{C}_e = (\mathbf{0}_{P \times N} \quad \Phi_p), \quad (10)$$

where $\mathbf{A}_e$, $\mathbf{B}_e$, and $\mathbf{C}_e$ form the state-space model of the network.

The optimal admittance of the network [20] can be derived from Equation (10), as

Table 2: Comparison of the computed coupling factors with their corresponding residues and attenuation levels between three resonant shunts and the interconnected piezoelectric network for the modes 1, 2, and 10.

Mode	Patch	k_c (%)	r (%)	A_{multi} (dB)	K_c (%)	w	A_{net} (dB)
1	1	6.28	100	-22.2	6.89	1	-23.3
2	2	5.61	100	-23.1	6.91	1	-24.6
10	3	7.54	100	-17.9	7.55	1	-17.9

$$\mathbf{Y}_{net}(s) = \mathbf{C}_e(s\mathbf{I}_{2N \times 2N} - \mathbf{A}_e)^{-1}\mathbf{B}_e. \quad (11)$$

In order to tune the network, $\mathbf{C}_p$ is introduced into the tuning equations. By defining $\mathbf{E}_p = \mathbf{C}_p^{-1}$, using two singular value decompositions (SVD) [18], the electrical mode shapes of the network are equal to

$$\begin{cases} \mathbf{U}_D \Sigma_D \mathbf{V}_D^T = \text{SVD}(\mathbf{E}_p) \\ \mathbf{U}_M \Sigma_M \mathbf{V}_M^T = \text{SVD}(\mathbf{W} \Omega_{oc}^{-1} \Gamma \mathbf{V}_D \Sigma_D^{-1/2}) \end{cases} \Rightarrow \Phi_p = \mathbf{V}_D \Sigma_D^{-1/2} \Psi, \quad (12)$$

where $\mathbf{W} = \text{diag}(w_1, \dots, w_N)$ with w_k representing the arbitrarily assigned control authority over the targeted modes and $\Psi = \mathbf{V}_M \mathbf{U}_M^T$ the fully-passive realization criterion defined such that $\mathbf{I} - \Psi^T \Psi \succeq 0$. Then, the total electromechanical coupling factor of the network is given by [18]

$$K_{c,rk}^2 = \frac{\alpha_{rk}^2}{1 - \alpha_{rk}^2}, \quad \text{with } \alpha_{rk} = \frac{1}{\omega_{oc,k}} \|\gamma_r \phi_r\|, \quad (13)$$

with k the index of the structural resonance, γ_r and ϕ_r are the r -th row and column of Γ and Φ_p , respectively. Then, the components of the electrical matrices Ω_e and $\mathbf{Z}_e$ of the network are calculated as

$$\omega_{e,k} = \sqrt{\frac{3K_{c,rk}^2 - 4r_{e,k} + 8}{4K_{c,rk}^2 + 4}} \omega_{oc,k}, \quad \xi_{e,k} = \sqrt{\frac{27K_{c,rk}^4 + 80K_{c,rk}^2 + 64 - 16r_{e,k}(4 + 3K_{c,rk}^2)}{\sqrt{2}(5K_{c,rk}^2 + 8)}}, \quad (14)$$

where $r_{e,k}$ is an intermediate parameter given in [17, 18]. To sum up, Ω_e and $\mathbf{Z}_e$ can be build with Equation (14), which allows to compute Equation (10), enabling the calculation of the admittance matrix of the passive network given in Equation (11).

4 MULTI-BRANCH SHUNTS VS INTERCONNECTED PIEZOELECTRIC NETWORKS

4.1 Case study $N = P$

The case $N = P$, where $N = \text{rank}(\Phi_p)$, represents the simplest and most optimal scenario. Indeed, the problem is well-posed [18]. No additional capacitors are introduced into the network, thus preventing any degradation of the total coupling factors defined in Equation (13). Three classical resonant shunts are connected to each zone of patches as well as the associated interconnected network. FRF are presented in Figure 3. The different coupling factors k_c , K_c , and the attenuation levels A are presented in Table 2. The interconnected network solution enhances the coupling factors of modes 1 and 2. However, since mode 10 is exclusively coupled with zone 3, it does not exhibit any significant change.

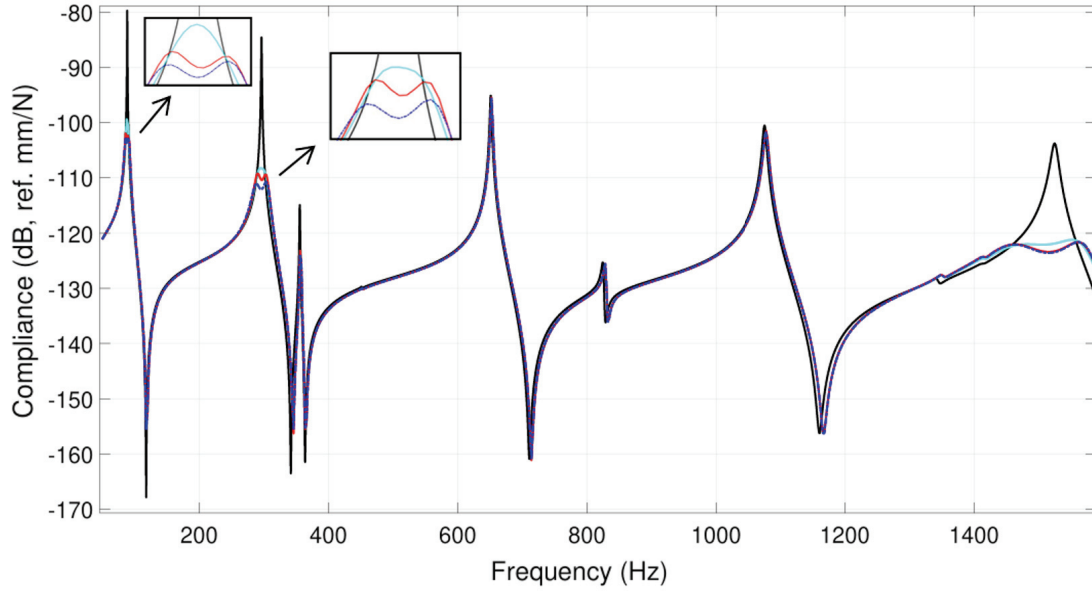


Figure 3: Numerical compliance in short-circuit (—), with three classical resonant shunts with the compensation of parasitic losses (—), without the compensation (—) and with the interconnected piezoelectric network (· · ·) for the modes 1, 2, and 10.

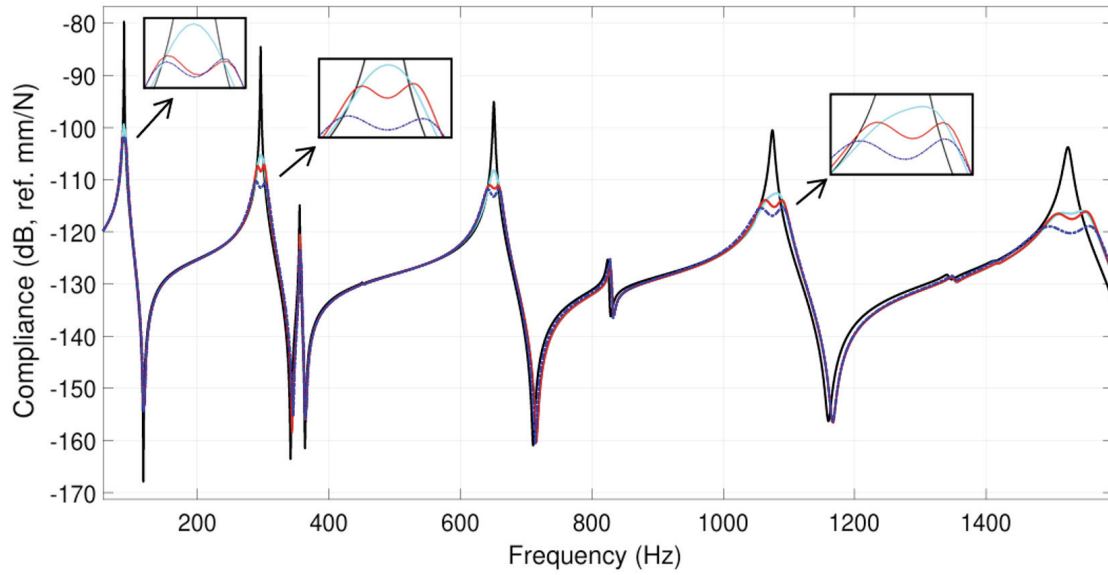


Figure 4: Numerical compliance in short-circuit (—), with three current blocking shunts with the compensation of parasitic losses (—), without the compensation (—) and with the interconnected piezoelectric network (· · ·) for the first five bending modes.

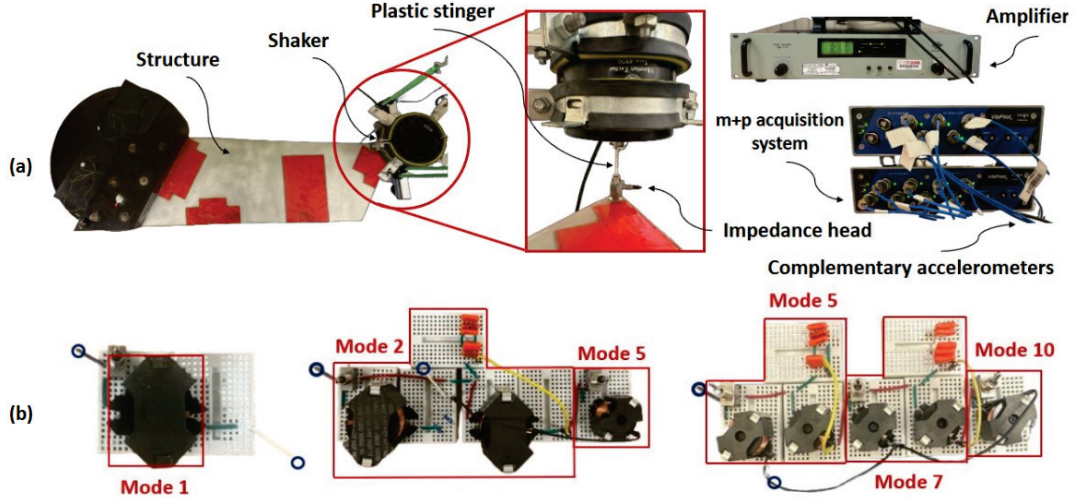


Figure 5: Experimental validation: (a) setup for the forced vibration analysis of the benchmark structure and (b) picture of the current blocking shunts - the targeted modes are highlighted with red boxes and connections to the piezoelectric patches are marked with blue circles.

4.2 Case study $N > P$

The case $N > P$, where $N > \text{rank}(\Phi_P)$, means that there are fewer transducers than targeted structural modes, and Φ_P has more columns than rows. Therefore, internal dofs I have to be introduced to make the problem well-posed, as $I = N - \text{rank}(\Phi_P)$ [18]. Capacitors are then introduced into the network, and the total coupling factors are degraded. FRF are presented in Figure 4, where three current blocking shunts are connected to each patch as well as the associated interconnected network. The coupling factors κ_c and K_c and the attenuation levels A are presented in Table 3. The use of an interconnected piezoelectric network to target the first five bending modes provides greater vibration reduction compared to current blocking shunts. By selecting an appropriate weight vector w , the influence of each targeted mode can be selectively emphasized. In this case, to achieve performance at least comparable — and potentially superior — to that of the multi-branch shunts, the set of weights has been set to $w = [2, 2, 3, 1, 1]$.

5 EXPERIMENTAL VALIDATION

A fully-passive experimental validation is conducted for the first time on a well-known circuit architecture, which is the current blocking shunt. The objective was to validate the proposed compensation method to account for parasitic losses and the feasibility criteria developed in [11] for the fabrication of tailor-made inductors. To analyze the dynamics of the piezoelectric structure, an impedance head is glued at the free end, near the trailing edge. A plastic stinger is positioned between the impedance head and the shaker, ensuring alignment and enabling fine adjustments to achieve optimal orientation. The shaker, connected through an impedance head, is equipped with two sensors to measure the excitation force and the acceleration. Both signals are acquired through an m+p acquisition system and amplified accordingly. Eigenfrequency extraction is performed with the PolyFREQ algorithm. The experimental setup and the electrical circuits are presented in Figure 5. The FRF illustrated in Figure 6 is computed

Table 3: Comparison of the computed coupling factors with their corresponding residues and attenuation levels between three current blocking shunts and the interconnected piezoelectric network for the first five bending modes.

Mode	Patch	κ_c (%)	r (%)	A_{CB} (dB)	K_c (%)	w	A_{net} (dB)
1	1	6.28	100	-22.2	6.29	2	-22.2
2	2	4.45	60	-22.4	6.59	2	-25.8
5	2	1.51	40	-16	3.12	3	-17.2
5	3	2.14	50	-16	3.12	3	-17.2
7	3	2.89	30	-13.7	3.57	1	-15.4
10	3	3.49	20	-12.5	4.97	1	-15

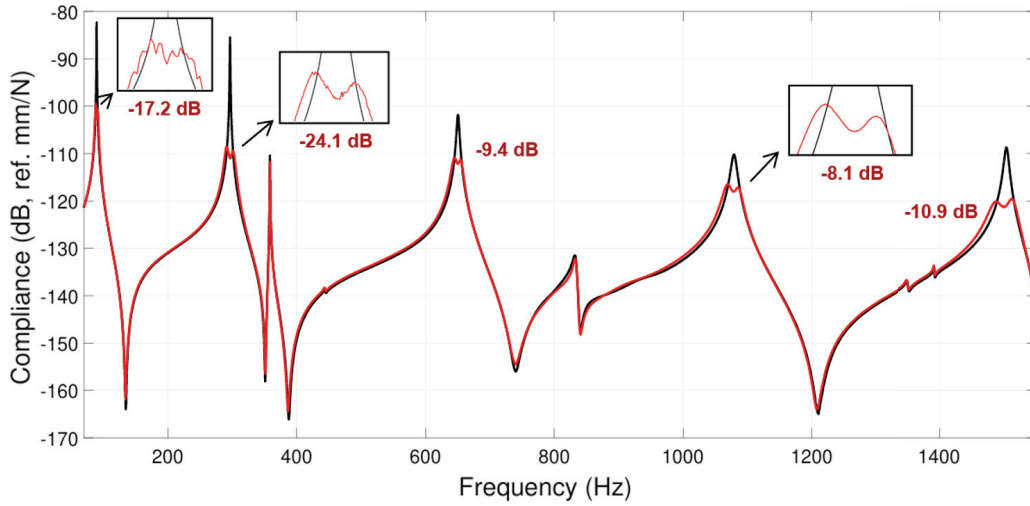


Figure 6: Experimental compliance in short-circuit (—) and with three current blocking shunts (—) with their respective attenuation levels for the first five bending modes.

using a processing block, with a complete acquisition comprising 10 blocks for a total duration of 40 seconds. The experimental FRF is consistent with the numerical model, with slightly lower attenuation levels due to weaker experimental coupling compared to the numerical predictions. Using purely inductive components, the vibration reduction profiles observed align with the optimal equal peaks behavior reported in the literature.

6 CONCLUSION

This study aims to compare the performance of multi-branch shunts and interconnected piezoelectric networks in the context of a fully-passive implementation. The different losses associated with passive inductors and piezoelectric patches are accurately quantified and implemented into the current blocking shunt — and multi-branch shunts in general — through a novel compensation method. The actual performance of the shunt is therefore in line with the expected theoretical performance, both numerically

and experimentally. Once this intermediate step is completed, interconnected piezoelectric networks once again demonstrate their superiority in terms of vibration reduction performance when compared to multi-branch shunts, yielding a gain between 1 and 4 dB on our benchmark structure depending on the mode to be controlled.

Future work will focus on describing the electrical network in a configuration of purely passive components, allowing fully-passive application of interconnected piezoelectric networks to any type of structure. Although this remains a challenge since explicit network topologies have not yet been derived, the electrical matrices of the network can be leveraged. Finally, the objective will be to implement a tuning correction that accounts for the same parasitic losses already considered into multi-branch shunts but applied to interconnected piezoelectric networks. Such a solution could be regarded as the most effective fully-passive approach for multimodal piezoelectric vibration damping.

ACKNOWLEDGMENTS

This work is part of a three-year Cifre doctoral project funded by Naval Group and the Agence de l'innovation de défense with supervision from the Conservatoire national des arts et métiers.

REFERENCES

- [1] O. Thomas, J. Ducarne, and J.-F. Deü. Vibrations of an elastic structure with shunted piezoelectric patches: efficient finite element formulation and electromechanical coupling coefficients. *International Journal for Numerical Methods in Engineering*, 2009. <https://doi.org/10.1002/nme.2632>.
- [2] R. L. Forward. Electronic damping of vibrations in optical structures. *Applied optics*, 18:690–7, 1979. <https://doi.org/10.1364/AO.18.000690>.
- [3] N. W. Hagood and A. H. von Flotow. Damping of structural vibrations with piezoelectric materials and passive electrical networks. *Journal of Sound and Vibration*, 146:243–268, 1991. [https://doi.org/10.1016/0022-460X\(91\)90762-9](https://doi.org/10.1016/0022-460X(91)90762-9).
- [4] D. L. Edberg, A. S. Bicos, and J. S. Fechter. On piezoelectric energy conversion for electronic passive damping enhancement. *Proceedings of Damping*, pages 717–724, 1991. <https://contrails.library.iit.edu/item/163767>.
- [5] J. J. Hollkamp. Multimodal Passive Vibration Suppression with Piezoelectric Materials and Resonant Shunts. *Journal of Intelligent Material Systems and Structures*, 5:49–57, 1994. <https://doi.org/10.1177/1045389X9400500106>.
- [6] S.-W. Wu. Method for Multiple Mode Piezoelectric Shunting with Single PZT Transducer for Vibration Control. *Journal of Intelligent Material Systems and Structures*, 9:991–998, 1998. <https://doi.org/10.1177/1045389X9800901204>.
- [7] A. J. Fleming, S. Behrens, and S. O. R. Moheimani. Reducing the inductance requirements of piezoelectric shunt damping systems. *Smart Materials and Structures*, 12:57–64, 2003. <https://doi.org/10.1088/0964-1726/12/1/307>.

- [8] G. Raze, J. Dietrich, and G. Kerschen. Tuning and performance comparison of multiresonant piezoelectric shunts. *Journal of Intelligent Material Systems and Structures*, 33(19):2470–2491, 2022. <https://doi.org/10.1177/1045389X221088031>.
- [9] J. Ducarne. Modélisation et optimisation de dispositifs non-linéaires d’amortissement de structures par systèmes piézoélectriques commutés. *Conservatoire national des arts et metiers - CNAM*, 2009. <https://theses.hal.science/tel-00464513v2>.
- [10] B. Lossouarn, M. Aucejo, J.-F. Deü, and B. Multon. Design of inductors with high inductance values for resonant piezoelectric damping. *Sensors and Actuators A: Physical*, 259:68–76, 2017. <https://doi.org/10.1016/j.sna.2017.03.030>.
- [11] P. Flament, B. Lossouarn, É. Ramond, P. Audrain, and J.-F. Deü. Fully-passive implementation of multi-branch piezoelectric shunts for multimodal vibration damping. Manuscript in preparation, 2025.
- [12] P. Flament, B. Lossouarn, É. Ramond, P. Audrain, and J.-F. Deü. Passive damping of several resonances using multi-branch resonant piezoelectric shunts. *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, 270(9):2207–2218, 2024. https://doi.org/10.3397/IN_2024_3151.
- [13] A. Agneni, D. S. Matteoa, M. Francoa, and P. Gian Marioa. Multi-modal damping by shunted piezopatches: Possible aeroelastic applications. *International Journal of Applied Electromagnetics and Mechanics*, 24:1–24, 2006. <https://doi.org/10.3233/JAE-2006-709>.
- [14] S. Vidoli and F. dell’Isola. Modal coupling in one-dimensional electromechanical structured continua. *Acta Mechanica*, 141:37–50, 2000. <https://doi.org/10.1007/BF01176806>.
- [15] R. Darleux. Development of analogous piezoelectric networks for the vibration damping of complex structures. *HESAM Université*, 2020. <https://theses.hal.science/tel-02902755>.
- [16] B. Lossouarn, M. Aucejo, and J.-F. Deü. Multimodal vibration damping of a beam with a periodic array of piezoelectric patches connected to a passive electrical network. *Smart Materials and Structures*, 24:11, 2015. <https://doi.org/10.1088/0964-1726/24/11/115037>.
- [17] G. Raze, J. Dietrich, B. Lossouarn, and G. Kerschen. Modal-based synthesis of passive electrical networks for multimodal piezoelectric damping. *Mechanical Systems and Signal Processing*, 176:109120, 2022. <https://doi.org/10.1016/j.ymssp.2022.109120>.
- [18] G. Raze, J. Dietrich, B. Lossouarn, and G. Kerschen. Shunts vs networks: tuning and comparison of centralized and decentralized piezoelectric vibration absorbers. *Smart Materials and Structures*, 31:115006, 2022. <https://doi.org/10.1088/1361-665X/ac92b0>.
- [19] B. Lossouarn. Optimal placement of thin piezoelectric patches for maximum modal coupling. *Journal of Intelligent Material Systems and Structures*, 35(20):1591–1605, 2024. <https://doi.org/10.1177/1045389X241282054>.
- [20] C. Alexander and M. Sadiku. *Fundamentals of Electric Circuits Fourth Edition*. 2008.