

Eigenfrequency Adaptivity of Locally Resonant Metamaterial Resonators using Piezoelectric Elements with Negative Capacitance Shunts

Stefan Janssen, Gabriel Konda Rodrigues, Leopoldo Pisanelli Rodrigues de Oliveira, Carlos De Marqui Jr., Claus Claeyls, Elke Deckers

Abstract. Locally resonant metamaterials (LRMs) have shown exceptional noise and vibration reduction capabilities within targeted frequency ranges. By adding resonant elements onto a flexible host structure on a sub-wavelength scale, free wave propagation can be inhibited in a targeted frequency range, known as a stop band. However, these stop bands are fixed by design, limiting the applicability of LRMs for non-stationary noise and vibration problems. This work explores the addition of piezoelectric elements with negative capacitance shunts onto a beam-shaped resonator to tune the frequency of its first out-of-plane bending mode for application in an LRM context. A coupled electromechanical finite element (FE) model is constructed in COMSOL, with FE matrices exported to MATLAB and connected to negative capacitance shunts in parallel and series configurations. First, the modal electromechanical coupling is numerically calculated for different resonator beam dimensions and three configurations chosen for further study. Eigenfrequency adaptivity using negative capacitance shunts is predicted and experimentally validated. The results highlight the potential of integrating piezoelectric elements to achieve adaptivity in LRM resonators and emphasize the importance of considering the modal electromechanical coupling during the design phase.

Key words: Locally resonant metamaterials, piezoelectric elements, negative capacitance, noise and vibration reduction

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1 INTRODUCTION

The trend towards lightweight design has driven the search for innovative lightweight noise and vibration solutions, especially in the low-frequency range [1]. In this context, locally resonant metamaterials (LRMs) have emerged as a promising potential solution. By integrating or adding resonant elements onto a flexible host structure on a sub-wavelength scale, a stop band can be achieved [2]. In this stop band, elastic waves are prohibited from travelling freely throughout the host structure, therefore typically leading to a targeted frequency range of high noise and vibration reduction [3]. Despite their potential, conventional LRMs come with several drawbacks that limit their versatility. The stop band width is tied to the resonant mass added, which is constrained in lightweight designs, therefore typically resulting in a narrow stop band. Moreover, the stop band frequency is related to an eigenmode of the resonator which exerts a non-zero net force on the host structure. Once fabricated, the resonator's structural properties are fixed, making the LRM solution less effective for systems subject to dynamic changes in time or frequency, as commonly found in automotive, aerospace and industrial applications.

To overcome these limitations, research has increasingly focused on adaptive metamaterials [4]. Piezoelectric elements with synthetic inductance shunts have been explored for their ability to create tunable stop bands [5, 6]. By modifying the electrical impedance of the shunts, the resonance frequencies of piezoelectric elements can be shifted, allowing for adaptivity in the LRM solution. Integrating a negative capacitance element, typically in parallel or series with a shunt circuit, has been shown to enhance the electromechanical coupling of piezoelectric elements by modifying their effective behavior [7, 8]. The piezoelectric element then acts as an equivalent transducer with altered coupling and stiffness: a parallel configuration increases the open-circuit stiffness, while a series setup decreases the short-circuit stiffness [7]. Even without additional shunt components, this stiffness tuning offers a promising route to adaptive stop band behavior in otherwise purely mechanical LRMs through stiffness control [9, 10, 11]. While the principle of tunability via negative capacitance has been studied, previous research has not fully explored the optimal integration of piezoelectric elements onto elastic resonators for maximum adaptivity. Moreover, experimental realization and validation remains limited. In this study, a piezoelectric element is added to an established beam-shaped elastic resonator design [11, 12, 13, 14]. For a fixed piezoelectric element, modal electromechanical coupling for the first out-of-plane bending mode is numerically calculated across various resonator beam dimensions. From these results, three resonator configurations are selected and eigenfrequency adaptivity simulated for both series and parallel negative capacitance configurations. Finally, the resonators are fabricated and their adaptivity measured in an experimental campaign.

This paper is structured as follows: **Section 2** introduces the beam-shaped resonator. **Section 3** describes the modeling approach, including finite element analysis and negative capacitance implementation. **Section 4** presents the modal electromechanical coupling results, selection of three resonator designs and simulated eigenfrequency shifts. **Section 5** details the fabrication and experimental validation. Finally, **Section 6** summarizes the conclusions.

2 APPLICATION CASE: BEAM-SHAPED RESONATORS

The resonator under investigation is a beam-shaped resonator (Figure 1), commonly used in the context of locally resonant metamaterials (LRMs) [12, 13, 14, 11]. The first out-of-plane bending mode of this type of resonator is known to interact with and generate a stop band for acoustically-relevant bending waves traveling through host structures. For this study, a piezoelectric element (PZT-5J) with fixed dimensions is selected, as it is typically constrained by the manufacturer. In Section 4, two dimensions of the resonator, length (l_{beam}) and height (h_{beam}), are varied to achieve different modal electromechanical couplings for the first out-of-plane bending mode. Other resonator dimensions are provided in Table 1 and are chosen to ensure that the targeted mode remains within the low-frequency range across a wide range of l_{beam} and h_{beam} values and in view of later manufacturing through laser cutting. Polymethyl methacrylate (PMMA) is selected for the beam-shaped resonator with properties used from [14], while the properties of the piezoelectric element (PZT-5J) are used from the COMSOL Multiphysics® material library.

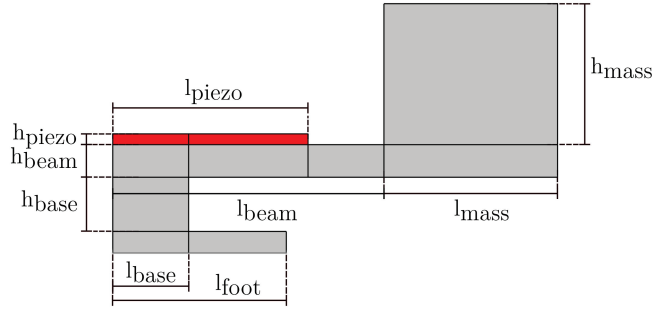


Figure 1: Beam-shaped resonator under study

3 METHODOLOGY

This section briefly outlines the modeling approach used to compute modal electromechanical coupling and predict eigenfrequency adaptivity, following the methodology in [11].

Table 1: Beam-shaped resonator dimensions in [mm].

l_{foot}	l_{base}	l_{beam}	l_{mass}	l_{piezo}	h_{foot}	h_{base}	h_{beam}	h_{mass}	h_{piezo}	w_r
16	7	21.5 – 40	20	21	2	5	1 – 10	20	0.55	6

3.1 Finite element modelling of resonators with piezoelectric patches

The beam-shaped resonator with piezoelectric element of Section 2 is constructed and discretized using COMSOL Multiphysics® 6.1. After discretization, the coupled stiffness $\mathbf{K}$ and mass matrix $\mathbf{M}$ are exported with LiveLink™ for MATLAB®. Following the procedure of [11], the following system of equations is obtained:

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{v}_p \end{Bmatrix} + \begin{bmatrix} \mathbf{H}_{uu} & \mathbf{H}_{uv_p} \\ -\mathbf{H}_{uv_p}^T & \mathbf{H}_{v_p v_p} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ v_p \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ q_p \end{Bmatrix}, \quad (1)$$

where $\mathbf{u}$ and v_p equal the displacement and piezoelectric voltage degrees-of-freedom (DOFs), $\mathbf{f}$ and q_p the nodal external forces and piezoelectric charge. $\mathbf{M}_{uu}$ contains the structural inertia of the structure, $\mathbf{H}_{uu}$ the structural stiffness, $\mathbf{H}_{uv_p}$ the piezoelectric coupling terms that link $\mathbf{u}$ to v_p and $\mathbf{H}_{v_p v_p}$ the capacitance at constant strain. From this set of equations, a lumped negative capacitance element can be connected by imposing a relation between v_p and q_p , further detailed in Section 3.3.

3.2 Modal electromechanical coupling evaluation

To explore how the geometry of the piezoelectric component influences the adaptivity of the chosen resonator type, a parametric analysis is carried out on a FE model to calculate the modal electromechanical (EM) coupling. This parameter captures the ratio of electrical energy stored in the piezoelectric elements to the total mechanical strain energy supplied to the structure when the structure is vibrating in a particular eigenmode [15]. The modal EM coupling for the first out-of-plane bending mode of the beam-shaped resonator is calculated through the difference in the eigenfrequencies with the piezoelectric element in open- and short-circuit conditions [16]:

$$EM = \sqrt{\frac{f_{OC}^2 - f_{SC}^2}{f_{SC}^2}} \times 100\%. \quad (2)$$

To this end, a clamped boundary condition is applied at the bottom surface of the resonator's foot and using the FE formulation from Section 3.1, both open- and short-circuit eigenfrequencies are calculated for different lengths and heights of the resonator beam. To identify the first out-of-plane bending mode, the modal effective mass in the out-of-plane direction is evaluated. The modal effective mass represents the portion of mass that effectively resonates within a specific eigenmode of the structure, which will differ from the total mass. Finally, the modal EM coupling is evaluated according to Eq. 2.

3.3 Negative capacitance implementation

The approach from [11] is used to connect a lumped negative capacitance element C_N , where $C_N < 0$, in either a series or parallel configuration (Figure 2 (a,b)). The effect of both configurations is a piezoelectric transducer with an equivalent capacitance and modified electromechanical coupling and stiffness characteristics (Figure 2 (c)). In the parallel configuration, the following set of equations is derived:

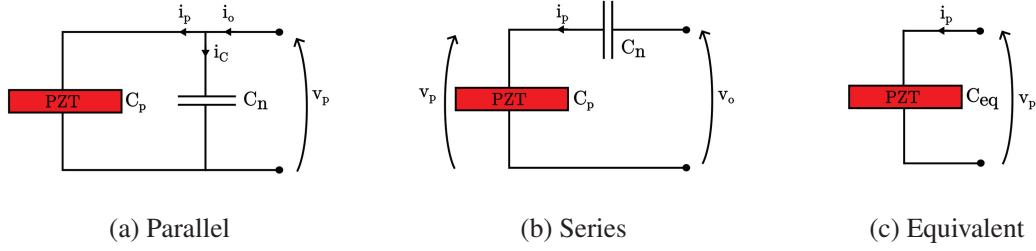


Figure 2: Parallel (a) and Series (b) configurations of negative capacitance shunts. In both cases, the system can be reduced to a system with equivalent capacitance (c).

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{v}_p \end{Bmatrix} + \begin{bmatrix} \mathbf{H}_{uu} & \mathbf{H}_{uv_p} \\ -\mathbf{H}_{uv_p}^T & H_{v_p v_p} + C_N \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ v_p \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ q_o \end{Bmatrix}, \quad (3)$$

while for the series configuration the following set is obtained:

$$\begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{q}_p \end{Bmatrix} + \begin{bmatrix} \mathbf{H}_{uu} + \mathbf{H}_{uv_p} H_{v_p v_p}^{-1} \mathbf{H}_{uv_p}^T & \mathbf{H}_{uv_p} H_{v_p v_p}^{-1} \\ H_{v_p v_p}^{-1} \mathbf{H}_{uv_p}^T & H_{v_p v_p}^{-1} + C_N^{-1} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ q_p \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ v_o \end{Bmatrix}. \quad (4)$$

Using these assembled mass and stiffness matrices, the eigenfrequencies of the coupled electromechanical system are then computed by solving the eigenvalue problem using the built-in *eigs* function in MATLAB. In the parallel configuration, this means $q_0 = i_0 = 0$ and in the series configuration $v_0 = 0$.

Negative capacitance stability

Since the equivalent capacitance (C_{eq}) of the piezoelectric element connected to a negative capacitance cannot be negative for practical stability reasons, several limits upon C_N are imposed [7]. In the parallel case, the equivalent capacitance is:

$$C_{eq} = C_p + C_N, \quad (5)$$

where C_p is the piezoelectric capacitance. Since $C_N < 0$, the magnitude of the negative capacitance element $|C_N|$ can therefore not be larger than the capacitance of the piezoelectric element C_p . For C_N connected in series, C_{eq} is:

$$C_{eq} = \frac{C_p C_N}{C_p + C_N}. \quad (6)$$

Again to ensure system stability, $|C_N|$ must in this case always be greater than C_p . The capacitance of the piezoelectric element is obtained in COMSOL Multiphysics® by applying a voltage (V) to the piezoelectric element and capturing the electric displacement on the top electrode surface (A) in the normal direction (D_Z):

$$C_P = \frac{1}{V} \int_A D_Z dA \quad (7)$$

4 RESONATOR EIGENFREQUENCY ADAPTIVITY SIMULATIONS

This section presents numerical results based on the methodology from Section 3. In Section 4.1 the modal EM coupling for various resonator beam dimensions are shown and three configurations selected. Section 4.2 then evaluates eigenfrequency adaptivity using negative capacitance shunts.

4.1 Modal EM coupling evaluation

The methodology from Section 3.2 is followed, varying the resonator beam length l_{beam} (Figure 1) from 21.5 mm , maintaining a minimum gap of 0.5 mm with the piezoelectric element, up to 40 mm . The beam height h_{beam} is varied from 1 mm , the minimum allowed for laser cutting, up to 10 mm .

The resulting modal EM coupling for the first out-of-plane bending mode of the beam-shaped resonator is depicted in Figure 3a. A strong variation can be observed within the chosen parameter range. For a h_{beam} of 3 mm and l_{beam} of 21.5 mm , the piezoelectric element is able to exert the most influence to the resonator under beam bending, as obvious from the highest modal EM-coupling. With these dimensions, the highest combination of elastic strain energy within the piezoelectric element and its ability to influence the dynamics of the host structure within the composite piezoelectric/PMMA beam section is observed. When h_{beam} increases or decreases from this optimal value, the piezoelectric element is less able to change the dynamics of this resonator and therefore also a lower eigenfrequency adaptivity with negative capacitance is expected [11]. Likewise, it is clear that a thicker piezoelectric element is not necessarily better to achieve a stronger influence on the dynamics of the resonator. Also, the non-linear and complex nature of the results presented in Figure 3a highlight the importance of performing this analysis before the design of the electromechanical resonator.

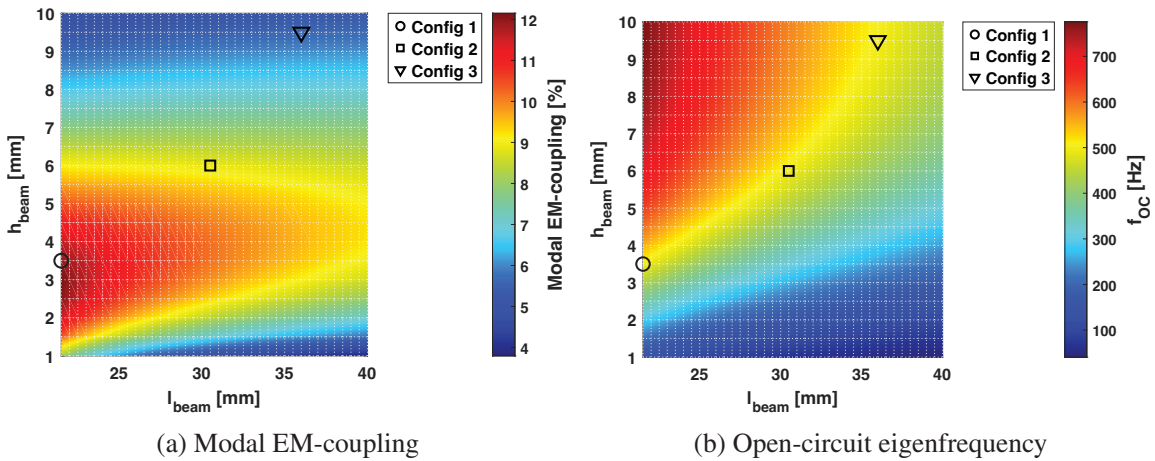


Figure 3: Results of parametric study with configurations 1, 2 and 3 marked

Table 2: Parameters of the three selected configurations

Config	l_{beam} [mm]	h_{beam} [mm]	EM [%]	f_{OC} [Hz]
1	21.5	3.5	12.0	499.6
2	30.5	6.0	8.90	496.6
3	36.0	9.5	5.65	500.4

Additionally, the open-circuit eigenfrequency of the first out-of-plane bending mode f_{OC} of the resonator is captured and displayed in Figure 3b. To properly compare the effects of the modal EM-coupling to the frequency adaptivity with negative capacitance shunts, three configurations with different modal EM-coupling, however approximately the same f_{OC} are chosen, marked as Config 1, 2 and 3. The dimensions and properties of these configurations are summarized in Table 2.

4.2 Eigenfrequency adaptivity simulations

Using the methodology outlined in Sections 3.1 and 3.3, lumped negative capacitance elements are connected to the piezoelectric element of the three selected configurations in both parallel and series. For the parallel case, as described in Eq. 5, the negative capacitance C_N can be varied from $C_N = 0$ to $C_N = -C_p$, which represents the theoretical stability limit. In the series configuration (Eq. 6), C_N can theoretically range from $C_N = -\infty$ to $C_N = -C_p$, again reaching the same stability limit. In the series case, C_N is varied from $-2C_P$ to $-1C_p$. A step size of $0.01C_N/C_P$ is chosen for both series and parallel simulations.

The results are shown in Figure 4. In both the series and parallel configurations, configuration 1, with the highest modal EM-coupling, exhibits the strongest eigenfrequency adaptivity. Adaptivity increases exponentially as the negative capacitance approaches the theoretical stability limits for all three configurations. However, the greater the modal EM-coupling, the stronger the adaptivity effect, also at points further from the stability limits. This suggests that for practical applications, a high modal EM-coupling is essential.

5 EXPERIMENTAL VALIDATION OF EIGENFREQUENCY ADAPTIVITY

This section first describes the manufacturing of the beam-shaped resonators with integrated piezoelectric elements and the practical implementation of the negative capacitance circuits. Section 5.2 outlines the experimental test setup and Section 5.3 presents the measurements.

5.1 Resonators and negative capacitance circuit realization

The beam-shaped resonators (Config 1, 2 & 3) were fabricated by laser cutting PMMA panels using a Trotec Speedy 100R laser cutter. Piezoelectric plates (PZT-5J, *SMPL21W21T05410*) with dimensions $21 \times 21 \times 0.55$ mm were cut to size using waterjetcutting. Thin copper tape was soldered to the bottom electrodes of the piezoelectric elements using a reflow oven and the

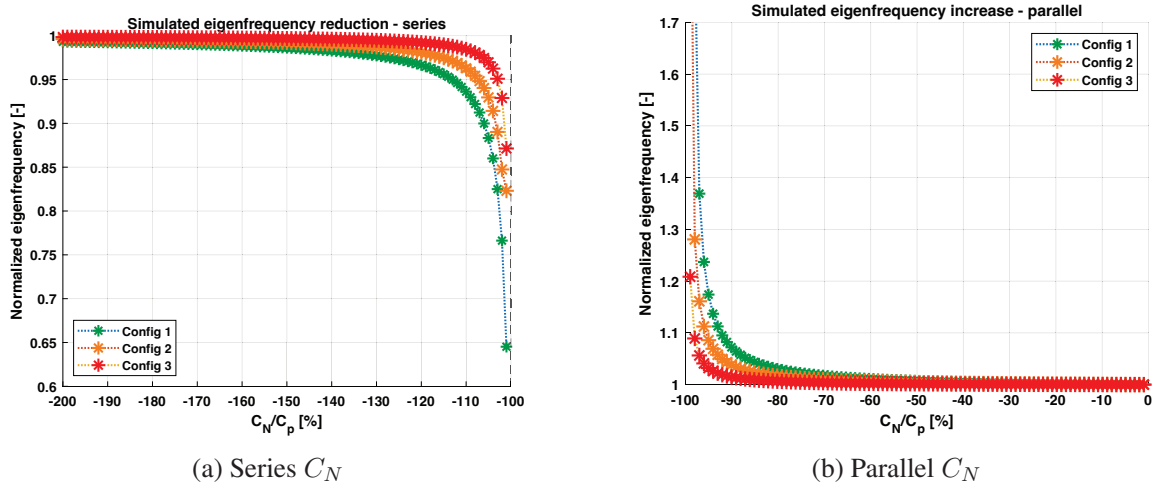


Figure 4: Eigenfrequency decrease (4a) and increase (4b) adaptivity simulations of configurations 1, 2 & 3

piezoelectric elements were bonded to the PMMA resonator beams with 3M DP460 glue. To ensure good adhesion, custom supports were 3D-printed and a clamp was used to apply pressure for a minimum of six hours. Finally, copper wires were soldered to the top electrodes and to the copper tape to establish electrical connections with the bottom electrodes. The realized resonators are shown in Figure 5.

To implement negative capacitance shunts in practice, active circuits are required. In this work, the circuits from [10] are realized and shown in Figures 6a & 6b for parallel and series configurations, respectively. By using a potentiometer for resistor R_1 , the value of the negative capacitance can be tuned according to $C_N = -CR_1/R_2$. Component values for R_1 ($5k\Omega$), R_2 ($10k\Omega$) and C ($100nF$) were chosen to provide sufficient resolution in C_N when adjusting R_1 . Also a test was performed with $C = 50nF$, however the results yielded equivalent performance. Resistor R_s was experimentally chosen with a potentiometer to provide a small amount of damping and stabilize the circuit in the parallel configuration, while R_p in the series case was chosen high to achieve a high pass filter and stop low frequency signals that can cause bias-current and offset-voltage errors [8]. In both circuits, an operational amplifier of type OP-551 was used.

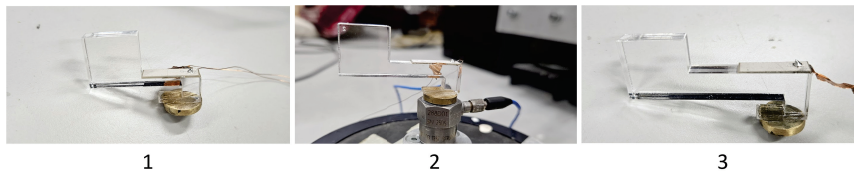


Figure 5: The manufactured beam-shaped resonators (Config 1, 2 and 3)

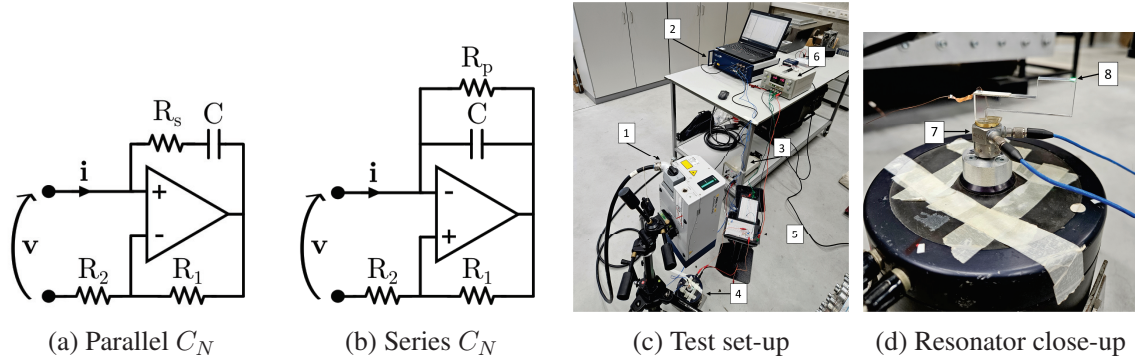


Figure 6: Schematics of the negative capacitance shunts (a–b) and the experimental test set-up (c–d) used for validating eigenfrequency adaptivity.

5.2 Test set-up

An overview of the experimental set-up is provided in Figures 6c & 6d. The resonators were excited by an electrodynamic shaker (4), driven by a white noise signal up to 1 kHz. A 0.1 V input signal was amplified using a power amplifier (3). The resulting input force of about 200 mN RMS and acceleration were measured with an impedance sensor (7). The tip velocity of the resonator (8) was measured using a Polytec PSV Qtec laser vibrometer (1) and captured with a data acquisition system (2), averaging over 10 measurements. The negative capacitance circuit and the power supply are depicted in (5) and (6), respectively. Frequency response functions (FRFs) of the tip velocity over the input force were measured and recorded for various levels of negative capacitance.

5.3 Experimental validation results

Prior to each test with series and parallel negative capacitance shunts, the capacitance of the piezoelectric elements was measured using a B&K 878B handheld LCR meter. These values fluctuated slightly across configurations and tests, ranging from 4.03 to 4.28 nF. During the experiments, R_1 was gradually adapted to vary the negative capacitance C_N until the system reached instability. At each step, the eigenfrequency of the first out-of-plane bending mode and the corresponding value of C_N were recorded. As the system approaches the theoretical stability limits, the FRFs become increasingly noisy. This behavior is attributed to the voltage and current limitations of the operational amplifiers in the negative capacitance circuits. The onset of instability was defined as the point where the eigenfrequency no longer increased (parallel case) or decreased (series case), indicating circuit saturation and dynamic instability.

The measured open-circuit eigenfrequencies of the manufactured resonators were close to the targeted 500 Hz, however not exact. These discrepancies arise due to several factors, i.e. damping is not included in the simulations, small variations in the lasercut geometry, waterjet-cutting of the piezoelectric elements and thickness of the adhesive. To reduce the impact of

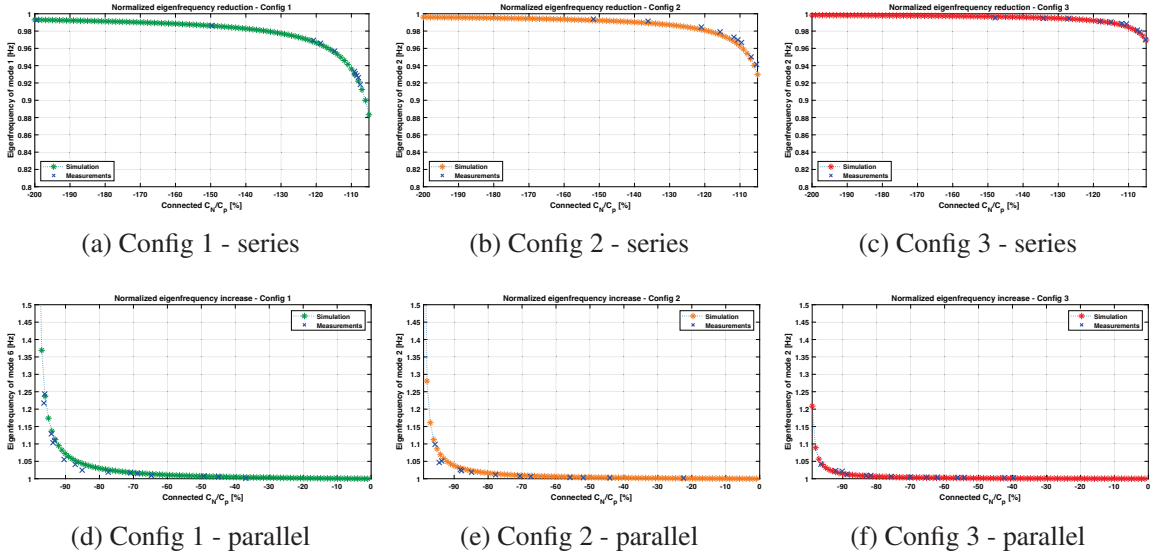


Figure 7: Normalized eigenfrequency shift for each configuration comparing simulations and experiments: series (top row) and parallel (bottom row) negative capacitance

this variability and enable a clearer comparison, both experimental and simulation results were normalized with respect to f_{OC} and f_{SC} for the parallel and series configurations, respectively. Figure 7 presents the normalized change in eigenfrequency as a function of C_N , demonstrating excellent agreement between the two. This close match validates both the simulation approach and the practical implementation of the negative capacitance shunting strategy. The maximum experimentally achieved eigenfrequency shifts are summarized in Table 3. As the resonator was removed and remounted between the series and parallel measurements, slight differences occurred in the open- and short-circuit frequencies. Nevertheless, considering the shift relative to f_{SC} in the series case and to f_{OC} in the parallel case, Config 1 showed the best performance, with a maximum reduction of -8.8% and an increase of 24% , clearly outperforming Configs 2 and 3.

Table 3: Maximum experimentally achieved frequency shifts (Δ) for all configurations.

Config	Series					Parallel				
	f_{SC} [Hz]	f_{OC} [Hz]	$\frac{C_N}{C_p \max}$ [%]	f_{min} [Hz]	$\Delta_{\downarrow}$ [%]	f_{SC} [Hz]	f_{OC} [Hz]	$\frac{C_N}{C_p \max}$ [%]	f_{max} [Hz]	$\Delta_{\uparrow}$ [%]
1	477.8	481.3	-107.5	439.1	-8.8	477.8	481.3	-96.1	596.9	+24.0
2	507.8	509.8	-105.3	478.1	-6.2	502.2	503.8	-95.6	553.1	+9.8
3	529.7	530.3	-105.2	514.1	-3.0	535.6	535.9	-96.4	558.1	+4.1

6 CONCLUSIONS

This study presented a comprehensive investigation into the eigenfrequency adaptivity of beam-shaped elastic resonators enhanced with surface-bonded piezoelectric elements connected to negative capacitance shunts. A multi-physical finite element framework was used to systematically explore how variations in beam length and height affect the modal electromechanical (EM) coupling of the first out-of-plane bending mode, which is relevant for stop band creation in metamaterials. Results showed a nonlinear and complex dependence of the modal EM coupling on these parameters, underscoring its importance in resonator design. Three resonator configurations with similar open-circuit eigenfrequencies but varying EM coupling were selected. Simulations demonstrated that stronger EM coupling leads to larger eigenfrequency shifts under both series and parallel negative capacitance shunts. In an experimental campaign, these findings were validated. While absolute eigenfrequencies differed slightly due to fabrication tolerances, normalized results showed excellent agreement with simulations, confirming the accuracy and robustness of the modeling approach. The results highlight the critical role of EM coupling in achieving effective adaptivity. These findings provide a solid foundation for achieving adaptivity in LRM resonators, enabling noise and vibration control across a broader frequency range. Future research will extend the approach to full metamaterial structures, targeting practical implementation of tunable stop bands for real-world applications.

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