

Experimental Calibration of Digital Piezoelectric Shunts: Practical Considerations, Parameter Estimation and Dielectric Losses

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Abstract. Piezoelectric shunts have been studied extensively over the past decades for vibration mitigation of flexible structures. The admittance of the electrical circuit in a piezoelectric shunt can be synthesized by a digital control unit, commonly referred to as a digital piezoelectric shunt. While the digital control unit requires power (and is therefore semi-active), it allows for a straightforward implementation and adaptation of, in theory, any electrical circuit, making it effective for prototyping. Furthermore, the hardware of the digital shunt allows for a direct measurement of the dynamic capacitance of the piezoelectric transducer by applying a known current and measuring the resulting voltage. All required parameters for calibration of a shunt targeting a single vibration mode can be obtained directly from such capacitance measurements, eliminating the need for additional sensors in the shunt calibration. This study considers practical issues for experimental implementation of digital shunts from capacitance measurements, such as the influence of delay introduced by the digital control unit on the performance of the shunt. Furthermore, it is demonstrated how the performance of a resonant digital shunt is affected by the parallel resistance in the used digital shunt hardware and dielectric losses in the piezoelectric transducer. Corrections for these effects are introduced by modification of the implemented transfer function and their effectiveness is evaluated experimentally for a composite blade.

Key words: Piezoelectric Shunt Damping, Dielectric Loss, Shunt Calibration, Digital Shunt

1 Introduction

Piezoelectric shunt damping has been considered extensively for vibration reduction in flexible structures by connecting an electrical circuit to a piezoelectric transducer embedded in the structure. Resonant shunts are often used for single-mode vibration reduction where an inductor and resistor are tuned to the targeted vibration mode. While allowing effective vibration

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reduction, resonant shunts are sensitive to mistuning [1] and may require large custom inductors with a small series resistance [2]. Experiments are therefore often performed with synthetic inductors [3, 4] or by introducing a synthetic impedance [5, 6].

Synthesizing the electric impedance of the shunt circuit with a synthetic impedance was first considered by Fleming et al. in 2000 [5] by the introduction of a device that prescribes the relationship between the current and voltage between two terminals. While the device can contain either a current or a voltage source, a voltage-controlled current source is typically used to minimize hysteretic nonlinearities of the piezoelectric transducer and because implementation of improper transfer functions, where the order of the numerator is greater than the denominator, is impractical [7]. The desired electrical impedance is then implemented by synthesizing the corresponding admittance by measuring the voltage across the terminals of the piezoelectric transducer and prescribing the current.

The device used to implement a synthetic impedance consists of a digital control unit (DCU) where the desired transfer function is implemented, as well as additional hardware to convert between the input/output of the DCU and the voltage/current at the two terminals of the device. As the impedance relation is governed by the DCU, the device is, in the present paper, referred to as a digital shunt. Following the digital shunt design proposed by Fleming et al. [5], different digital shunt hardware have been applied in the literature, for instance, with the current being generated using the voltage compensation method [6, 8, 9] or based on a Howland current pump [10, 11, 12]. A comparison between a digital shunt based on the voltage compensation method and on the Howland current pump can be found in [10]. One advantage of the Howland current pump is that it allows one of the electrodes of the piezoelectric transducer to be grounded. Other modifications of the digital shunt hardware have been used, for instance, to scale and transform the voltage between the two terminals to the input of the DCU. As demonstrated in [13] and [14], impedance measurements with the digital shunt hardware can be used for the estimation of the modal parameters required for shunt tuning.

While the digital shunt, in theory, allows for the implementation of almost any shunt impedance, care must be taken to obtain an accurate representation of the shunt impedance. In [15], Raze et al. demonstrated the importance of accounting for the delay introduced by the DCU for the stability of the digital shunt. Similarly, the present paper describes practical considerations for the implementation of an accurate digital shunt, observed during experiments conducted in [14].

In Sec. 2, the experimental setup is described for context, and the hardware used for the digital shunt setup is introduced. Next, in Sec. 3, the discretization of the transfer function in the digital control unit is described together with the additional delay introduced by the digital shunt unit. A correction minimizing the influence of delay is then presented using simplified terms to ease the implementation of the correction. The influence of a bias resistance in the digital shunt hardware and dielectric losses in the piezoelectric transducer is briefly discussed in Sec. 4, while conclusions are given in Sec. 5.

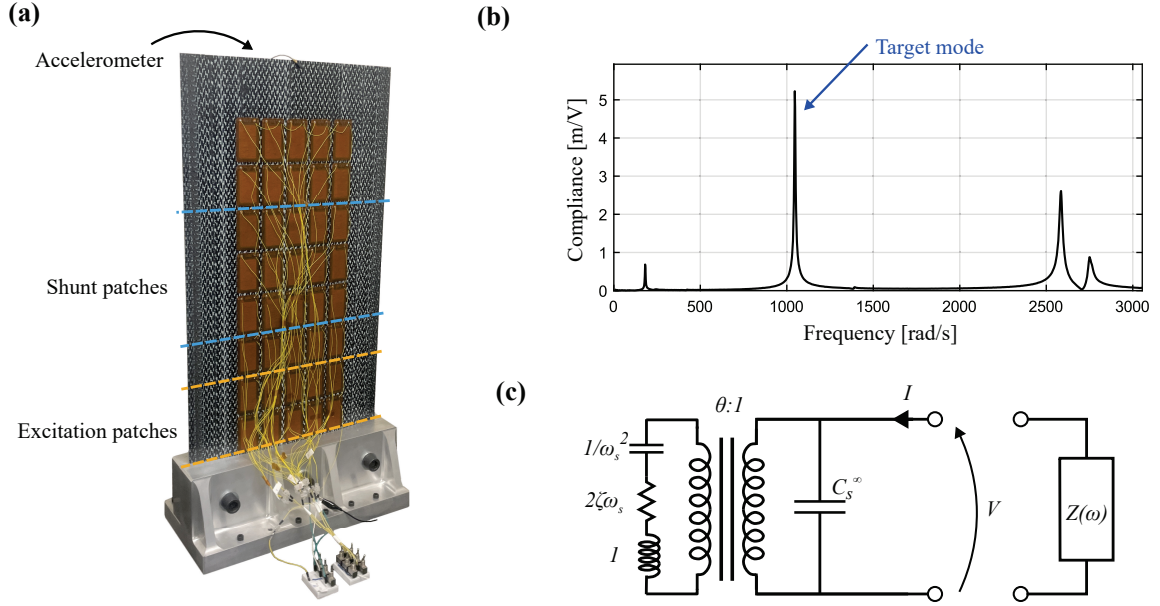


Figure 1: Experimental setup with a composite blade (a), the frequency response function at the top of the blade with the shunt in the OC condition (b), and an equivalent electrical circuit (c) of the targeted vibration mode with $\theta = \omega_s \kappa_s \sqrt{C_s^\infty}$.

2 Experimental setup

This section describes the experimental setup, including the governing equations of the considered electromechanical system and the hardware of the used digital shunt.

2.1 Composite blade

The experimental setup considered in the present paper is the composite blade shown in Fig. 1(a) with 40 piezoelectric patches (DuraAct -P-876.A15, PI Ceramic). One of the electrodes of each patch is grounded, whereby any number of patches can easily be connected in parallel and used as part of a single shunt. The initial analysis is limited to the calibration of a resonant shunt targeting the second bending mode with the short circuit (SC) natural frequency $\omega_s = 1047$ rad/s, corresponding to 167 Hz when only the patches highlighted in Fig. 1(a) are used for shunt damping. The frequency response for the displacement at the top of the blade is shown in Fig 1(b) with the shunt patches in the SC condition. A reduced-order model with a single vibration mode is introduced to describe the targeted vibration mode of the structure. For harmonic motion with the angular frequency ω and the modal coordinate r_s , the governing equations are

$$(-\omega^2 + 2i\omega\zeta_s\omega_s + \omega_s^2)r_s - \omega_s\kappa_s\sqrt{C_s^\infty}V = f_{s,ext} \quad (1)$$

$$i\omega\omega_s\kappa_s\sqrt{C_s^\infty}r_s + i\omega C_s^\infty V = I \quad (2)$$

where V is the voltage across the electrodes of the piezoelectric shunt patches, I is the current at one of the terminals and $f_{s,ext}$ is an external modal load. The modal parameters are defined in accordance to [14], where $\omega_s = 1047$ rad/s and $\zeta_s = 0.0026$ are the natural frequency and damping ratio with SC electrodes, $C_s^\infty = 482$ nF is the modal blocked capacitance, and $\kappa_s = 0.075$ is the effective electromechanical coupling factor (EEMCF). The equivalent electrical circuit to the considered electromechanical system is shown in Fig. 1(c) for $f_{s,ext} = 0$.

Shunting of an electrical circuit with impedance $Z = Z(\omega)$ to the used piezoelectric patches, as illustrated in Fig. 1(c), introduces the relation between voltage and current

$$V = -ZI \quad (3)$$

For a series RL shunt with the resistance R and inductance L , the impedance is

$$Z = i\omega L + R \quad (4)$$

As demonstrated in [14], C_s^∞ and κ_s contain an implicit correction for the existence of residual (non-targeted) vibration modes when used for calibration of a shunt attached to a flexible structure and tuned to target a single vibration mode. In the present paper, the shunt circuit is therefore calibrated directly using the calibration expressions for minimum acceleration amplitude (accelerance) in the frequency response, derived in [16] for a piezoelectric shunt attached to a single-degree-of-freedom (sdof) structure. The optimal inductance and resistance are then

$$L = \frac{1}{C_s^\infty \omega_s^2} \frac{1}{(1 + \kappa_s^2)^2} \quad , \quad R = \frac{1}{C_s^\infty \omega_s} \sqrt{\frac{3}{2} \frac{\kappa_s^2}{(1 + \kappa_s^2)^3}} \quad (5)$$

For the modal parameters given above, the inductance and resistance are evaluated as $L = 1.87$ H and $R = 181 \Omega$.

2.2 Digital shunt hardware

In the digital shunt, the impedance Z is synthesized by a two-terminal device with a digital control unit (DCU) and additional hardware converting the input and output signal of the DCU to the voltage and current at the terminals. The present setup uses the digital shunt hardware in [11] with the same components as in [14]. The hardware is shown in Fig. 2(a) with a schematic of the circuit in Fig. 2(b). The circuit contains the two resistors R_c and R_b , as well as two operational amplifiers (OPA445, Texas Instruments) and one instrumental amplifier (AMP03, Analog Devices) powered by a ± 15 V DC power supply. The circuit is based on a modified Howland current pump and is chosen for its simplicity and as it allows one of the electrodes of the piezoelectric transducer to be grounded.

Assuming ideal operational amplifiers, the digital shunt hardware in Fig. 2 introduces the relation between the input voltage V_{in} and output voltage V_{out} of the DCU to the voltage V across the two electrodes of the digital shunt and the current I_p shown in Fig. 2(b) as

$$V = V_{in} \quad , \quad I_p = -\frac{1}{R_c} V_{out} \quad (6)$$

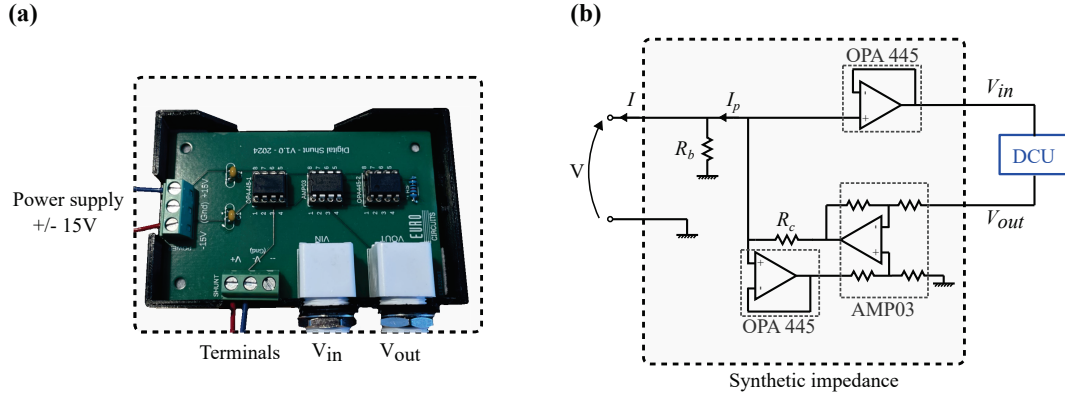


Figure 2: Digital shunt hardware, with an image of the used circuit in (a) and a schematic of the circuit in (b).

The magnitude of the current I_p is limited by the resistor R_c and the maximum output voltage $|V_{DCU,max}| = \max(|V_{out}|)$ of the DCU, such that

$$|I_p| \leq \frac{|V_{DCU,max}|}{R_c} \quad (7)$$

The value of the resistor R_c is then chosen based on the required maximum current $|I_p|$ and the maximum voltage $|V_{DCU,max}|$ of the DCU. For an ideal circuit, the current at the terminal of the digital shunt circuit is only governed by the voltage V_{out} . However, in practice, a resistance R_b must be added to the circuit to remove DC bias current. The current I at the terminal of the digital shunt device is then

$$I = I_p - \frac{1}{R_b}V = -\frac{1}{R_c}V_{out} - \frac{1}{R_b}V_{in} \quad (8)$$

The resistance R_b is chosen to be as large as possible while removing the DC bias current. Using (8), it is possible to account for the influence of R_b when implementing the synthetic impedance by including it in the implemented transfer function. This approach is applied for another digital shunt circuit in [12]. Presently, an alternative approach is used where R_b is considered as part of the inherited resistance of the piezoelectric transducer. The influence of the bias resistance is then accounted for in the calibration of the shunt, see Sec. 4.

Considering R_b as part of the piezoelectric transducer corresponds to letting $I_p = I$, whereby (3) and (6) gives the transfer function of the DCU as

$$V_{out} = \frac{R_c}{Z} V_{in} \quad (9)$$

Using (9), the admittance $1/Z$ of a piezoelectric shunt can then be synthesized with a digital shunt by implementation of the transfer function R_c/Z in the DCU.

In the present experimental setup, a dSpace MicroLab box is used as the DCU, with an input/output voltage range $V_{in} \in [-10, 10]$ V and $V_{out} \in [-10, 10]$ V. The two resistors are

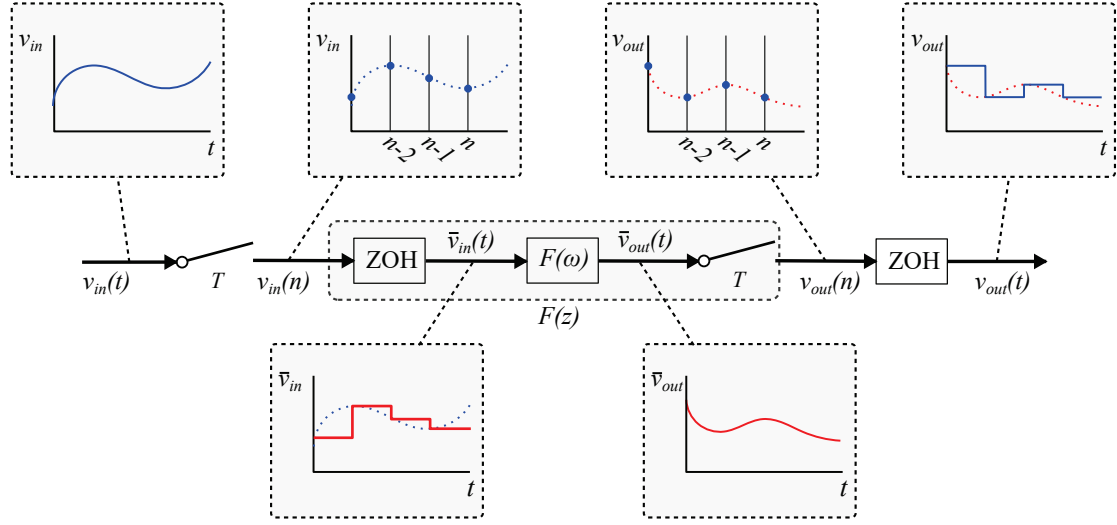


Figure 3: Block diagram of the DCU, with the discrete transfer function $F(z)$ modeled as a ZOH followed by a continuous transfer function $F(\omega)$ and a sampler.

chosen as $R_s = 10 \text{ k}\Omega$ and $R_b = 1 \text{ M}\Omega$. The current I_p is thereby in the range $I_p \in [-1, 1] \text{ mA}$, while the shunt voltage must be in the range $V \in [-10, 10] \text{ V}$. For applications where the magnitude of the shunt voltage V exceeds the operational range of the DCU, two additional resistors can be added to the circuit to reduce the input voltage in the DCU, see for instance [12]. Additionally, some DCUs only allow positive voltage in-/output. In [6], this is accounted for by adding a constant voltage offset in the digital shunt circuit.

3 Discretization of transfer function

In Sec. 2.2, the transfer function $R_c/Z(\omega)$ of the DCU is described in the continuous frequency domain. This section considers the required sampling of the DCU and its influence on the accuracy of the synthesized admittance.

3.1 Effect of sampling

Fig. 3 show a block diagram of the DCU, where the continuous time signals of the parameters defined in previous sections are denoted by the corresponding lower-case symbols as a function of time t . Assuming that the analog-to-digital converter can be considered as an ideal sampler with the fixed sample period T , the continuous input signal $v_{in}(t)$ is converted to the sampled signal $v_{in}(n)$ for the sample n . The corresponding sampled output signal $v_{out}(n)$ is then determined by the difference equation that prescribes the current output from past outputs and past/current inputs, illustrated in Fig. 3 by the discrete transfer function $F(z)$. Finally, as the clock frequency of the DCU is significantly higher than the sampling frequency, the digital-to-analog converter that converts the discrete signal $v_{out}(n)$ to a continuous signal $v_{out}(t)$ is considered as a zero-order hold (ZOH) maintaining a constant voltage between each sample.

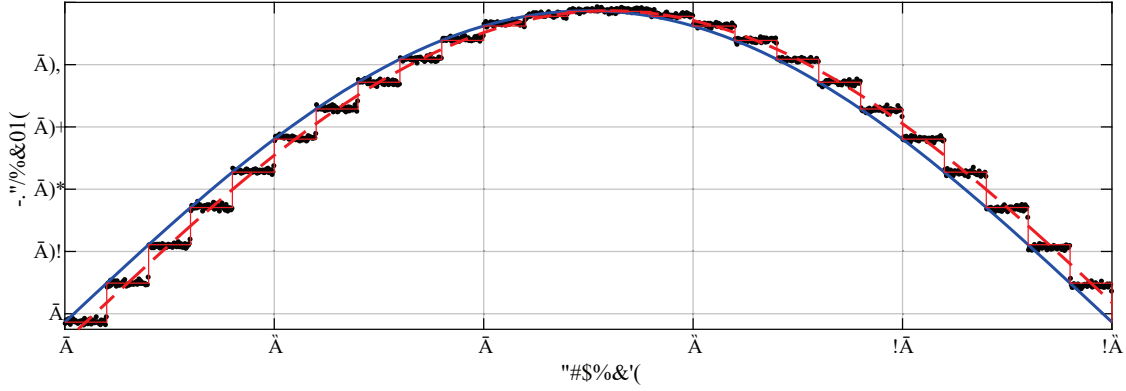


Figure 4: Sine current generated by the digital shunt hardware (black dots) with an ideal sine curve manually fitted to the signal (solid blue), including the ZOH (solid red), and with an ideal phase delay $\tau = T/2$ (dashed red).

As described in [17], the delay associated with the hold is the most important parameter to account for in the implementation of digital control systems. The influence of the ZOH on the output of the digital shunt is demonstrated in Figure 4 showing the current $i(t)$ measured at the terminal of the digital shunt unit (black dots) when a sine function with the amplitude 1 mA and frequency 200 Hz is prescribed with a sampling frequency of 10 kHz. A continuous sine function with a frequency of 200 Hz (solid blue) is manually fitted to the first data point of each sample. It is seen that applying a ZOH (solid red) to the continuous sine function gives a good description of the measured data.

Following [15, 17], the transfer function of a ZOH can be written for a unit impulse as

$$G(\omega) = \frac{1 - e^{-2i\omega\tau}}{2i\omega\tau} = e^{-i\omega\tau} \left(\frac{e^{i\omega\tau} - e^{-i\omega\tau}}{2i\omega\tau} \right) \approx e^{-i\omega\tau} \quad (10)$$

where $\tau = T/2$ and a Taylor series of the parenthesis in the second expression is applied to simplify the expression. The Taylor series of the parenthesis is $1 + \frac{1}{6}(i\omega\tau)^2 + \mathcal{O}((i\omega\tau)^4)$, whereby the error of the approximation is seen to be small for $\omega\tau \ll 1$. The ZOH is then considered as an addition of an ideal delay τ to the continuous signal. For the example in Fig. 4, the delayed signal (dashed red) is seen to give a better description of the measured data compared to the original continuous signal (solid blue).

3.2 Difference equation

The difference equations represented as the discrete transfer function $F(z)$ in Fig. 3, governs the output voltage $v_{out}(n)$ with $v_{in}(n)$ as the input voltage and is used to synthesize the required admittance of the digital shunt. The discrete filter $F(z)$ can be represented as a hold approximating the continuous signal $v_{in}(t)$ from the discrete signal $v_{in}(n)$, i.e. representing the associated continuous filter $F(\omega)$. When the approximating hold is a zero-order hold as in the

full expression in (10), the discrete and continuous filters are related by

$$F(z) = (1 - z^{-1}) \mathcal{Z} \left(\frac{F(\omega)}{i\omega} \right) \quad (11)$$

If the continuous filter $F(\omega)$ is defined to synthesize the admittance of a series RL-shunt with the inductance $\hat{L}$ and resistance $\hat{R}$, this leads to

$$F(z) = (1 - z^{-1}) \mathcal{Z} \left(\frac{R_c}{i\omega \hat{L} + \hat{R}} \right) = \frac{\frac{R_c}{\hat{R}} - \frac{R_c}{\hat{R}} e^{-\frac{\hat{R}}{\hat{L}} T}}{z - e^{-\frac{\hat{R}}{\hat{L}} T}} \approx \frac{\frac{R_c}{\hat{L}} T}{z - (1 - \frac{\hat{R}}{\hat{L}} T)} \quad (12)$$

where the final expression follows from the Taylor series approximation of the exponential function: $e^{-\hat{R}T/\hat{L}} = 1 - \hat{R}T/\hat{L} + \mathcal{O}((\hat{R}T/\hat{L})^2)$. This simplified discrete transfer function corresponds to applying the forward Euler rule in numerical integration, for which the difference equation implemented in the DCU can be expressed as

$$v_{out}(n) = \left(1 - \frac{\hat{R}}{\hat{L}} T \right) v_{out}(n-1) + \frac{R_c}{\hat{L}} T v_{in}(n-1) \quad (13)$$

While other more advanced integration methods such as Tustin's method can be applied for the discretization of the continuous transfer function, the forward Euler rule is considered here due to its simplicity. Implementing the difference equation from the full expression in (12) corresponds to using a ZOH discretization.

3.3 Correction for phase delay

This section presents a simplified correction for the delay introduced by the DCU when synthesizing the impedance of a shunt targeting a single vibration mode with the frequency ω_s . The correction has previously been applied in [14]. When the transfer function in (10) for a ZOH is considered as an ideal delay, the Fourier transforms of the continuous signals in Fig. 3 are related by

$$\bar{V}_{in} = V_{in} e^{-i\omega\tau}, \quad V_{out} = \bar{V}_{out} e^{-i\omega\tau} \quad (14)$$

From these representations, the relation between the continuous transfer function $F(\omega)$ implemented in the DCU and the ideal transfer function of the shunt $R_c/Z(\omega)$ in (9) is defined as

$$F(\omega) = \frac{\bar{V}_{out}}{\bar{V}_{in}} = \frac{V_{out} e^{i\omega\tau}}{V_{in} e^{-i\omega\tau}} = \frac{R_c}{Z(\omega)} e^{2i\omega\tau} \quad (15)$$

When the transfer function $F(\omega)$ is written on the same form as the ideal transfer function $R_c/Z(\omega)$, the expression in (15) can be used to account for the delay introduced in the digital shunt by letting $\omega = \omega_s$ and defining the parameters in $F(\omega_s)$ such that (15) is satisfied.

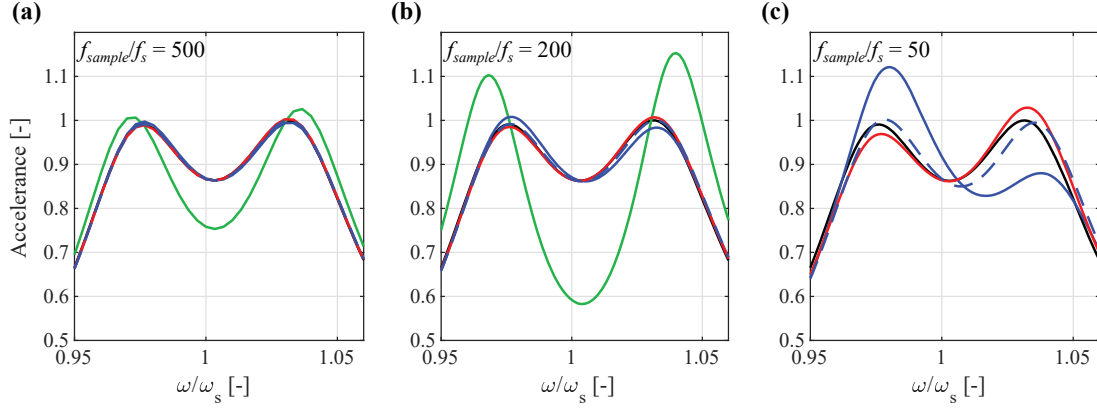


Figure 5: Frequency response with varying ratios between target frequency f_s and sampling frequency f_{sample} in (a), (b), and (c), with a series RL shunt calibrated without correction for delay (solid green), with the proposed correction using forward Euler integration (solid blue) and full ZOH integration (dashed blue), with the correction in [15] using Tustin's method (solid red), and with the analytical continuous transfer function (solid black).

For the series RL-shunt with the inductance L and resistor R , the corrected parameters implemented in the DCU ($\hat{L}$ and $\hat{R}$) are found as

$$\hat{L} = L \cos(2\omega_s \tau) - \frac{R}{\omega_s} \sin(2\omega_s \tau) \quad (16)$$

$$\hat{R} = R \cos(2\omega_s \tau) + \omega_s L \sin(2\omega_s \tau) \quad (17)$$

The effect of the correction in (16)-(17) is illustrated in Fig. 5 for a numerical time simulation of a digital shunt implemented for the reduced order model described in Sec. 2.1. The time simulation is performed at a sampling frequency of 50 MHz, using a digital shunt modelled with a sampling rate of 50 kHz. The natural frequency ω_s of the structure has been modified to give the ratio between the shunt sampling frequency f_{sample} and the targeted frequency $f_s = \omega_s / (2\pi)$ as 500, 200 and 50 in (a), (b) and (c), respectively. The response is shown for a shunt calibration without correction for delay (solid green), for the correction in (16)-(17) with both forward Euler integration (solid blue) and the full ZOH integration in (12) (dashed blue). Finally, a more detailed correction for resonant shunts in [15] is also included (solid red), based on pole matching between the Laplace and discrete domain, and Tustin's method for the discretization of the the transfer function.

While the proposed correction, using either ZOH or Euler integration, is not as accurate as the shunt implementation from [15], a significant improvement compared to the case without correction is still obtained by the proposed correction when the ratio between the sampling and targeted frequency is at least 200. In the conducted experiments for the composite blade, the target frequency was 166 Hz with a sampling frequency of 50 kHz, corresponding to $f_{sample}/f_s \approx 300$ for which the simplified correction with Euler integration is considered sufficiently accurate and thus implemented in [14] because of its simplicity.

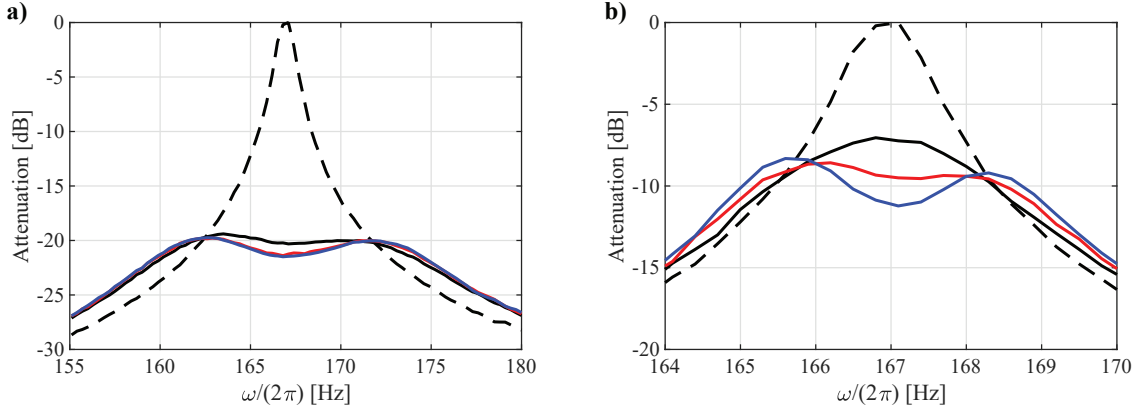


Figure 6: Attenuation of acceleration at the top of the composite blade with an EEMCF of $\kappa_s = 0.074$ in (a) and $\kappa_s = 0.016$ in (b), with response for a shunt in OC condition (dashed black), and with a series RL-shunt calibrated without correction (solid black), with partial correction (solid red) and full correction for losses (solid blue).

4 Bias resistance and dielectric losses

In Sec. 2.2, the bias resistance R_b of the digital shunt hardware is neglected by considering it to be part of the inherited resistance in the piezoelectric transducer. This section briefly describes how the influence of the resistance can be accounted for in the shunt calibration.

The influence of dielectric losses in the piezoelectric transducer on the accuracy of shunt calibration is considered. The dielectric loss is modeled as a hysteresis loss by the introduction of a complex capacitance $\tilde{C}_s^\infty = C_s^\infty(1 - i\eta_\epsilon)$ in the governing equations in (1)-(2). By impedance matching at the targeted frequency ω_s , it can be shown that the inductance L and resistance R of a series RL-shunt can be corrected for the influence of dielectric losses by

$$\frac{L}{\bar{L}} = \frac{1}{\eta_\epsilon^2 \bar{\lambda}^2 + \eta_\epsilon^2 \bar{\rho}^2 - 2\eta_\epsilon \bar{\rho} + 1} \quad (18)$$

$$\frac{R}{\bar{R}} = \frac{1 - \frac{\eta_\epsilon}{\bar{\rho}}(\bar{\lambda}^2 + \bar{\rho}^2)}{\eta_\epsilon^2 \bar{\lambda}^2 + \eta_\epsilon^2 \bar{\rho}^2 - 2\eta_\epsilon \bar{\rho} + 1} \quad (19)$$

where $\bar{L}$ and $\bar{R}$ are the optimal inductance and resistance when dielectric losses are neglected, and $\bar{\lambda} = \omega_s C_s^\infty \bar{L}$, $\bar{\rho} = \omega_s C_s^\infty \bar{R}$. Further papers are in preparation for a deeper insight of the problem, showing that the influence of the corrections in (18)-(19) increases for larger dielectric losses η_ϵ and small EEMCFs κ_s .

The dielectric loss η_ϵ can be considered as a frequency-dependent resistor $1/(\omega\eta_\epsilon C_s^\infty)$ in parallel with the capacitor C_s^∞ . The bias resistance of the shunt circuit R_b can then be combined with the inherited dielectric loss of the piezoelectric transducer. When evaluated at the target frequency ω_s , the combined loss is given as

$$\eta = \frac{1}{\omega_s R_b C_s^\infty} + \eta_\epsilon \quad (20)$$

By letting $\eta_\varepsilon = \eta$ in (18) and (19), the influence of the bias resistance R_b can thus be directly included in the correction for dielectric losses.

The influence of the dielectric losses η_ε and the bias resistance R_b on the tuning of a series RL-shunt attached to the composite blade is demonstrated experimentally in Fig. 6 for the piezoelectric patches in rows 4-6 from the bottom of the blade with $\kappa_s = 0.074$ in (a) and for the patches in row 8 with $\kappa_s = 0.016$ in (b). The frequency response at the top of the blade is shown in Fig. 6 for the shunt in the OC condition (dashed black), for an RL-shunt calibrated without correction for dielectric loss (solid black) and with the correction for dielectric loss in (18)-(19) (solid blue). The ideal response of the shunt is characterized by two peaks of equal magnitude in the frequency response curve. While the magnitudes of the two peaks in Fig. 6 are not exactly identical, the corrected calibration response is substantially more accurate than the calibration neglecting dielectric losses, especially for small EEMCFs.

In the present example, the modal parameters are estimated by modal analysis from impedance measurements using the digital shunt. The bias resistance R_b is thereby directly applied as an inherited parallel resistance and included in the correction for dielectric losses (solid blue). When the bias resistance is removed from the transfer function of the measured impedance, the correction for dielectric losses does not account for the bias resistance (solid red). For low EEMCFs, it is seen that including the bias resistance R_b in the correction for dielectric losses gives a more accurate tuning compared to the case where the bias resistance is neglected.

5 Conclusion

The present paper considers practical design considerations for digital piezoelectric shunts. The importance of accounting for the delay introduced by the digital control unit is illustrated, and a simplified correction based on an ideal delay assumption is shown to be accurate when the ratio between the sample and target frequency is at least 200. Finally, it is demonstrated experimentally that the bias resistance in the digital shunt hardware can be included in the proposed correction for dielectric losses, improving the accuracy of the shunt implementation.

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