

Numerical Investigations of the Asymmetric Energy Landscape in Cantilever Bistable Piezoelectric Energy Harvesters

Danish Bashir, P. M. Anilkumar, Ayan Haldar, Sven Scheffler, B. N. Rao, Raimund Rolfes

Abstract. Bistable piezoelectric energy harvesters are widely investigated due to their broad-band energy harvesting capabilities. These harvesters can be developed using unsymmetrical composite laminates and flexible piezoelectric materials, such as Macro Fiber Composites (MFCs). Depending on the geometrical parameters and boundary conditions of unsymmetrical composite laminate, such harvesters exhibit symmetric or asymmetric properties in their two stable shapes. In this study, a rectangular bistable composite laminate, attached with an MFC patch and clamped on one side, is investigated, and the asymmetry in its static and dynamic characteristics is quantified. It is observed that a rectangular clamped bistable composite laminate has an asymmetric potential energy landscape, which results in asymmetric stable shapes. To study the asymmetry in the properties of the bistable composite laminate bonded with an MFC, a fully nonlinear finite element (FE) model is employed. Asymmetry in static properties, such as strain energy (SE) and out-of-plane displacement, is evaluated for the bistable laminate with the MFC and compared to the case without MFC patch. Similarly, asymmetry in dynamic properties, such as free vibration characteristics, is analyzed for the bistable laminate bonded with the MFC patch and compared to the laminate without MFC patch. The results show that bonding an MFC patch to one surface reduces the asymmetry in the static and dynamic properties of the clamped rectangular bistable laminate, demonstrating its significance in energy harvesting. Furthermore, a detailed parametric study is conducted by varying the size of the MFC patch to identify the optimal configuration with minimal asymmetry. The findings from this study can be applied to design bistable piezoelectric energy harvesters with reduced asymmetry in static and dynamic properties, enhancing vibration-based piezoelectric energy harvesting performance.

Danish Bashir & B. N. Rao
Structural Engineering Laboratory, Indian Institute of Technology Madras, 600 036 Chennai, India
e-mail: ce21d402@smail.iitm.ac.in

Danish Bashir & P. M. Anilkumar & Sven Scheffler & Raimund Rolfes
Institute of Structural Analysis, Leibniz Universität Hannover, Appelstraße 9A 30167, Hannover, Germany

Ayan Haldar
Department of Civil Engineering, Indian Institute of Technology (BHU), 221005 Varanasi, India

Key words: Asymmetry, Bistable, Finite elements, Piezoelectric, MFC

1 INTRODUCTION

Multistable structures, which can undergo shape transformations while simultaneously bearing loads, have attracted significant research interest due to their diverse engineering applications [1]. These structures are ideal candidates for developing morphing and energy-harvesting applications. Among them, bistable structures represent the simplest form of multistable structures. The cured shapes of unsymmetrical composite laminates serve as a classical example of bistable structures. In the cured shapes of unsymmetrical laminates, bistability is induced by the difference in the coefficients of thermal expansion between the different layers of the unsymmetrical layups. These bistable structures possess two equilibrium shapes and can transition continuously from one to another through a snap-through mechanism. This shape change is accompanied by significant nonlinear deformations, which are rich in dynamics and can be utilized for piezoelectric energy harvesting [2].

Bistable composite laminates exhibit a double-well potential energy landscape, with minima separated by an energy threshold [3]. The nature of this potential energy landscape depends on structural geometry and boundary constraints. A square laminate has a symmetric landscape, yielding identical stable states, while a rectangular laminate creates an asymmetric landscape, leading to distinct curvatures [4]. In addition, the boundary conditions (such as center fixed or edge clamped) also influence the potential energy landscape. Integrating piezoelectric components like Macro Fiber Composites (MFCs) for actuation and energy harvesting applications alters the potential energy distribution, affecting bistable response, snap-through behavior, and dynamics [5]. Symmetric potential energy landscapes benefit morphing applications but may cause undesired cross-well oscillations [6]. In contrast, asymmetric potential energy landscapes enhance wave transmission [7], reduce snap-through force [4], and aid vibration mitigation [8]. However asymmetric potential energy landscape hinder piezoelectric energy harvesting because the system tends to get trapped in the deeper well, reducing transition frequency and energy conversion. Additionally, higher activation energy is required for transitions, limiting energy harvesting at low excitation, while asymmetry causes irregular switching, leading to inconsistent voltage output and lower efficiency [9]. While many studies assume perfectly symmetric potential energy distributions, but generally due to integration to another systems or asymmetric boundary conditions, symmetry in the potential energy landscape gets distorted. Extensive research on bistable energy harvesters (BEHs) has examined how asymmetry in potential energy landscapes affects efficiency. Harne and Wang [10] showed that avoiding asymmetry in bistable energy harvesters composed of weakly bistable systems could potentially provide minor performance benefits. He and Daqaq [11] found that an asymmetric potential energy landscape gives rise to weaker restoring forces, due to which bistable systems suffer reduced power output. Wang et al. [9] confirmed that asymmetry hinders bistable energy harvesters by trapping the system in a deeper well, diminishing cross-well oscillations. Giri et al. [12] showed that tuning

asymmetry can enhance stability and reduce chaos. Recently, Cai and Harne [13] suggested that lowering asymmetry improves snap-through motion and energy conversion abilities.

Bistable structures have gained significant attention in piezoelectric energy harvesting due to their double-well potential energy landscape [14], which enables them to capture ambient vibrations over a broad frequency spectrum. Their bistable nature can be easily achieved, making bistable unsymmetrical composite laminates particularly well-suited for broadband energy harvesting applications. Bistable unsymmetrical composite laminates bonded with MFC patches have been extensively studied under various geometrical shapes and boundary conditions. Square-shaped, center-fixed bistable composite laminates are commonly investigated due to the symmetry in their stable shapes as developed by Arrieta et al. [15]. Later, Betts et al. [16], conducted in-depth studies by characterising different dynamic regimes and optimized output power for energy harvesting. Harris et al. [17] further explored various bistable unsymmetrical composite laminate geometries, introducing an electromechanical coupling model validated through experimental studies. Lee et al. [18] proposed an innovative square-shaped, center-fixed bistable composite laminate composed entirely of MFCs. These studies mainly focused on the centre fixed boundary condition, which restricts the practical application of these harvesters. Rectangular unsymmetrical composite laminates clamped on one side have also been widely investigated. Arrieta et al. [19] developed a broadband piezoelectric energy harvester using rectangular bistable laminates with a cantilever boundary condition. Li et al. [20] examined clamped cantilever boundary conditions in bistable laminates with hybrid parts, commonly known as bistable hybrid symmetric laminates (BHSLs) for vibration control and energy harvesting applications. Daikun et al. [21] experimentally compared the broadband response of clamped bistable piezoelectric energy harvesters (BPEHs) made of BHSLs with that of their linear counterparts. Liu et al. [22] analyzed the energy harvesting capabilities of clamped BHSLs under wind-induced vibrations. Recently, Boddapati et al. [23] investigated clamped rectangular bistable laminates with MFCs bonded to one side, focusing on energy harvesting under vortex-induced vibrations. These studies primarily examined rectangular bistable laminates or BHSLs with clamped boundary condition, where excitation was applied at the fixed end. While they focused on the energy-harvesting potential of bistable laminates, the influence of the MFC patch on the potential energy landscape of BEH remains unaddressed.

As discussed earlier, a single end-clamped rectangular bistable laminate exhibits an inherently asymmetric potential energy landscape. When such a structure is utilized for energy harvesting, attaching an MFC patch to one side further alters the potential energy distribution, introducing additional asymmetry into an already unbalanced landscape. To the best of the authors' knowledge, no prior study has specifically examined how MFC bonding affects the asymmetric potential energy landscape of a clamped rectangular bistable laminate in the context of energy harvesting. The present study aims to bridge this gap by employing a finite element (FE) modeling approach. A FE framework is developed to predict the stable configurations of a clamped rectangular bistable laminate with an MFC attachment, quantify the induced asymmetry in its

potential energy landscape, and analyze its static and dynamic characteristics. The structure of the manuscript is as follows: Section 2 describes the bistable energy harvester, followed by Section 3, which explains the FE model. A discussion on the verification example is presented in Section 4, followed by a parametric study in Section 5. Finally, concluding remarks are provided in Section 6.

2 BISTABLE ENERGY HARVESTER

Figure 1(a) illustrates a cantilever bistable piezoelectric energy harvester, where a bistable laminate serves as the host structure under external vibrations. The host structure vibrates within one of its stable shapes and can switch between them, producing significant strains that are transferred to the MFC patch and converted into voltage via the piezoelectric effect. Figure 1(b) shows the clamped rectangular bistable laminate consisting of a symmetric lay-up, $[0/0]$, of length L_s and width W_l , and an unsymmetric lay-up, $[0/90]$, of length L_u and width W_l . An MFC patch of length L_m and width W_m is bonded to the top surface at a distance D_m from the clamped edge. Both lay-ups have a thickness t_l , while the MFC patch has a thickness t_m . The symmetric lay-up is crucial for bistability, as an unsymmetric lay-up alone loses bistability under clamped boundary conditions [23].

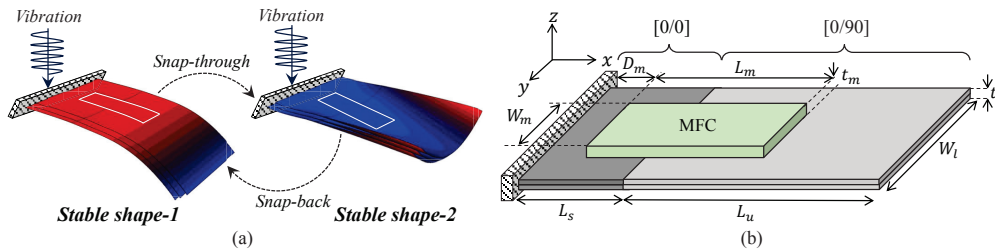


Figure 1: Representation of (a) A cantilever bistable energy harvester under external vibrations (b) Problem definition.

3 FINITE ELEMENT MODEL

A thermomechanically coupled FE model with geometric nonlinearity is formulated using ABAQUS. It is used to determine the equilibrium shapes and assess the static and dynamic characteristics of a clamped bistable rectangular laminate with an integrated MFC patch. Both the laminate and the MFC are modeled as linear elastic materials and are discretized using S4R (four-noded quadrilateral) shell elements due to their computational efficiency. A global mesh size of 2.5 mm is selected after a rigorous mesh refinement study. The connection between the symmetric and asymmetric sections of the laminate, as well as the bonding between the MFC and the laminate, is modeled using the TIE constraint. The assembled laminate-MFC structure is clamped at the symmetric section. To replicate the curing process, a thermal load of 180°C is applied uniformly across the structure. Subsequently, the temperature is gradually reduced to room temperature (20°C), leading the laminate-MFC system to attain its initial sta-

ble configuration. It is crucial to account for geometric nonlinearities in the simulation; thus, the NLGEOM option remains active throughout the analysis. Since the MFC is also represented using S4R shell elements, its piezoelectric constant is correlated with the coefficient of thermal expansion using the concept of free strain per applied voltage [24]. To transition into the second equilibrium configuration, an equivalent thermal load corresponding to an actuation voltage is introduced at the MFC nodes. The actuation voltage must exceed the critical snap-through threshold for the structure to adopt the alternate stable shape. Given the substantial nonlinear deformations associated with the snap-through event, the NLGEOM setting remains enabled, and artificial damping may be incorporated to facilitate numerical stability. Once snap-through occurs, the applied voltage (modeled as a thermal input) is removed to confirm the stability of the second configuration. Following the determination of both stable states, the LANCOS eigenvalue solver in ABAQUS is used to compute the fundamental frequencies and associated mode shapes of the laminate-MFC assembly. A schematic representation of the entire simulation framework is illustrated in Figure 2.

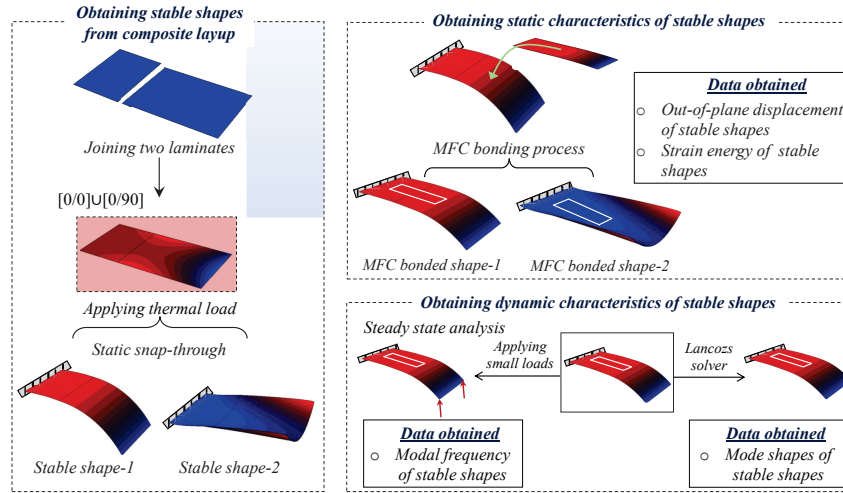


Figure 2: Schematic representation of the steps followed in FE analysis.

4 VERIFICATION EXAMPLE

A rectangular bistable laminate, composed of a symmetric $[0/0]$ and an asymmetric $[0/90]$ layup, with an M8557 MFC patch bonded to its upper surface, is analyzed based on the experimental work of Boddapati et al. [23]. The laminate is clamped along its symmetric section. The MFC patch is positioned on the top surface of the laminate at a distance of $D_m = 27.5$ mm from the clamped boundary. Details regarding the geometric specifications and material properties of both the laminate and MFC patch are provided in Table 1 and 2, respectively.

To examine the influence of MFC patch bonding on the potential energy landscape of a clamped rectangular bistable laminate, along with its static and dynamic behavior, two configurations from the chosen example are considered for further analysis: a rectangular laminate without

Table 1: Geometric properties of lamina and MFC patch [23]

Lamina				MFC			
L_s (mm)	L_u (mm)	W_l (mm)	t_l (mm)	L_m (mm)	W_m (mm)	t_m (mm)	
65	125	100	0.24	85	57	0.29	

an MFC patch, depicted in Figure 3(a), and a laminate with an MFC patch, illustrated in Figure 3(b).

Table 2: Material properties of lamina and MFC patch [23]

	ρ (kg/m ³)	E_{11} (GPa)	E_{22} (GPa)	G_{12} (GPa)	ν_{12}	α_{11} (10 ⁻⁷ /°C)	α_{22} (10 ⁻⁶ /°C)
Lamina (TR50S)	1346	140	9.40	5.00	0.30	-0.18	21.50
MFC (M8557-P1)	5440	27.27	14.76	4.13	0.32	-3.50	0.87

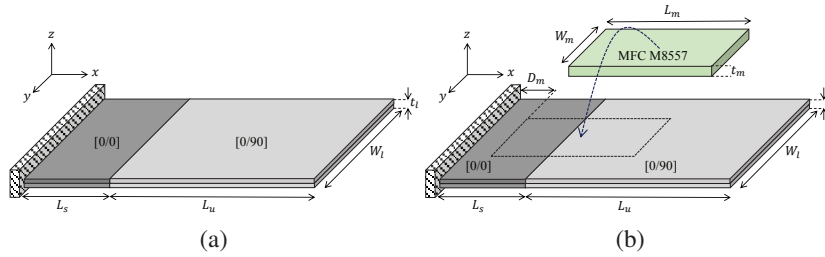


Figure 3: Schematic representation of selected clamped rectangular bistable laminate: (a) Only laminate, (b) With MFC patch.

4.1 Out-of-plane displacement of the bistable shapes

To compare the out-of-plane displacements of the stable configurations, the parameter τ is introduced, which represents the ratio between the area of the bonded MFC patch and the area of the rectangular bistable laminate. The out-of-plane displacements of the stable configurations obtained at the corners of the free end, both with and without the MFC patch, are summarized in Table 3. Also, the out-of-plane displacement from the FE framework for $\tau = 0.255$ is compared with the study conducted by Boddapati et al. [23] and is found to be in good agreement, confirming the validity of the proposed FE framework. It is evident that without the MFC, the rectangular laminate ($\tau = 0$), show unequal out-of-plane displacements, in the two stable configurations with a difference of 46.95 mm, which is attributed to the inherent asymmetry in the stable shapes of the clamped rectangular bistable laminate. On the other hand, when the MFC patch is bonded to the top surface of the laminate ($\tau = 0.255$), there is a substantial reduction in the out-of-plane displacement in both stable shapes, resulting in a reduced absolute difference of 25.33 mm. The decrease in out-of-plane displacement of the stable configurations upon attaching the MFC patch is due to the MFC patch's ability to reduce the natural curvature of the stable shapes. Notably, the reduction in out-of-plane displacement for stable shape 1 after

bonding the MFC patch is 49%, while for stable shape 2, it is 63.64%. The larger reduction in stable shape 2 is attributed to the MFC bending along the shorter side, which provides greater stiffness compared to stable shape 1, where the MFC bends along the longer side, imparting less stiffness. It is also important to note that the absolute difference in the out-of-plane displacement of the stable shapes decreases from 46.95 mm to 25.33 mm with the bonding of the MFC patch. This 46.05% reduction in the out-of-plane displacement difference indicates that the bonding of the MFC patch helps to minimize the asymmetry in the static characteristics of the bistable laminate-MFC assembly.

Table 3: Out-of-plane displacement comparison in two stable shapes of cantilevered bistable laminate with and without MFCs obtained at the corners.

Description	FE Framework		Boddapati et al. [23]	
	Shape-1	Shape-2	Shape-1	Shape-2
$\tau = 0$	59.38	12.43	-	-
$\tau = 0.255$	29.85	4.52	27.00	4.50

4.2 Strain energy of the stable shapes

Table 4 presents the strain energies of the stable shapes for a rectangular bistable laminate without ($\tau = 0$) and with ($\tau = 0.255$) an MFC patch. The strain energy (SE) of stable shapes without the MFC is unequal, confirming the asymmetric potential energy landscape. Bonding the MFC patch to the top surface increases the SE of both shapes due to added stiffness. As the MFC patch alters the curvatures of stable shapes, it makes their stored potential energy unequal, shifting energy levels in the bistable laminate. Without the MFC ($\tau = 0$), the SE difference is 1 mJ, but with the MFC M8557 patch ($\tau = 0.255$), it increases to 3 mJ, amplifying the system's asymmetry.

Table 4: SE comparison of cantilevered bistable laminate with and without MFC patch.

Description	Strain energy (mJ)		
	Shape-1	Shape-2	Difference
$\tau = 0$	72	73	1
$\tau = 0.255$	79	82	3

To gain further insights into what happens to the potential energy landscape of bistable system once MFC is bonded, SE variation during the snap-through process is extracted and plotted for the cases: without MFC patch ($\tau = 0$) and with MFC M8557 patch ($\tau = 0.255$). Figure 4 shows the variation of the SE, where the x axis represents the status of transition from stable shape-1 to stable shape-2 and y axis depicts the SE levels. It is clear that the addition of MFC patch elevates the SE levels of both the stable shapes and also increases the SE difference between them when compared to the case of clamped rectangular bistable laminate with no MFC patch.

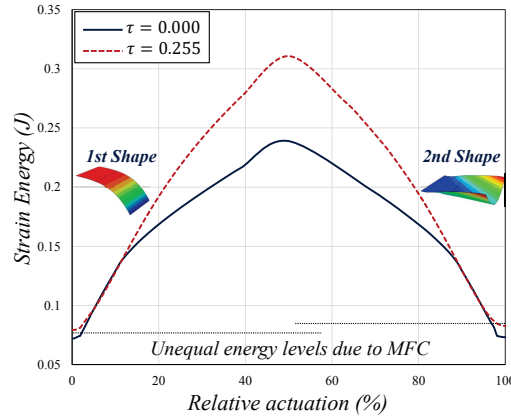


Figure 4: SE versus relative actuation of bistable laminate without MFC ($\tau = 0$) and with MFC M8557 patch ($\tau = 0.255$), showing the induced asymmetry due to attachment of MFC.

4.3 Natural frequency of the stable shapes

To understand the effect of bonding the MFC patch on the dynamic characteristics of the clamped rectangular bistable laminate, a free vibration analysis of the assembly is conducted. Table 5 presents the natural frequencies of the stable shapes for two scenarios: without the MFC patch ($\tau = 0$) and with the MFC M8557 patch ($\tau = 0.255$). From Table 5, it is evident that without the MFC, the two stable shapes of clamped rectangular bistable laminate exhibit unequal frequencies, reflecting the asymmetric nature of their dynamic characteristics. After bonding the MFC patch to one surface of the bistable laminate, the natural frequency of both stable shapes decreases significantly. This is due to the additional inertial effect introduced by the MFC, which reduces the natural frequencies of the stable shapes. However, it is noteworthy that the natural frequency difference between the stable shapes reduces from 98.61 Hz to 64.22 Hz with the addition of the MFC patch. This 34.87% reduction in the frequency difference indicates that the bonding of the MFC patch helps to minimize the asymmetry in the modal characteristics of the bistable laminate-MFC assembly. Reducing the asymmetry in the modal characteristics is highly advantageous for piezoelectric vibrational energy harvesting.

Table 5: Natural frequency comparison of selected bistable laminate with and without MFCs.

Description	Natural frequency (Hz)		
	Shape-1	Shape-2	Difference
$\tau = 0$	37.91	136.52	98.61
$\tau = 0.255$	35.25	99.47	64.22

5 PARAMETRIC STUDIES

In this section, the effect of changing the area of the MFC patch on the induced asymmetry in the static and dynamic characteristics is examined. The length of the MFC patch remains

constant at 85 mm, while its width is varied until the clamped rectangular bistable laminate loses bistability. Figure 5(a) illustrates the variation in the absolute difference in out-of-plane displacement (Δw) of the stable shapes as the area of the MFC patch increases (increasing τ parameter). As discussed in Section 4, bonding the MFC patch helps reduce the asymmetry in the out-of-plane displacement of the stable shapes. This reduction is more pronounced with smaller MFC patches (lower τ values) compared to larger ones (higher τ values). It is important to note that a smaller MFC patch, combined with minimal out-of-plane displacement of the stable shapes, leads to a significant decrease in the harvester's output power. Therefore, the MFC patch size should balance the induced asymmetry in out-of-plane displacement (Δw) with the harvested output power. Figure 5(a) serves as a design chart for selecting an appropriate MFC patch size, ensuring optimal asymmetry in static characteristics without significantly reducing output power. Figure 5(b) shows the variation in the absolute difference in SE (ΔSE) of the stable shapes as the area of the MFC patch increases (increasing τ parameter). As mentioned earlier, bonding the MFC patch increases the SE levels of the stable shapes due to the additional stiffness provided by the MFC. Larger MFC patches significantly raise the SE levels of the stable shapes compared to smaller patches, leading to an increase in the SE difference as the MFC patch size grows (increasing τ parameter), as shown in Figure 5(b). Beyond a certain value of the τ parameter, the assembly loses its bistability.

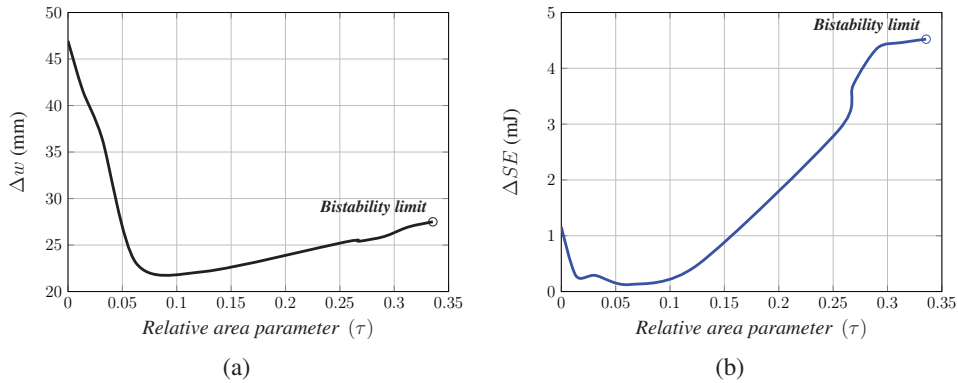


Figure 5: Change in the (a) out-of-plane displacements, (b) SE difference with the area of MFC patches.

Similarly, the effect of varying the area of the MFC patch on the induced asymmetry in the modal characteristics of the laminate-MFC assembly is analyzed. Figure 6 shows how the absolute difference in natural frequency (Δf) between the stable shapes changes as the area of the MFC patch increases. As discussed in Section 4, bonding the MFC patch lowers the natural frequency of both stable shapes, with this reduction more noticeable with larger MFC patches (higher τ values) compared to smaller patches (lower τ values). As a result, the absolute difference in natural frequency (Δf) between the stable shapes decreases as the MFC patch size increases. This can be used as a design guideline for selecting the optimal size of the MFC patch in developing BEH, ensuring minimal asymmetry in their modal characteristics.

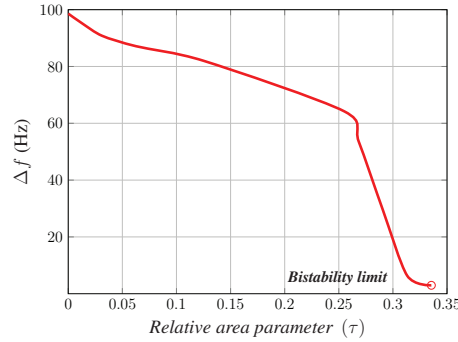


Figure 6: Change in the natural frequency difference with the area of MFC layers.

6 CONCLUSIONS

Bistable rectangular composite laminates exhibit an asymmetric double-well potential energy landscape, leading to asymmetric stable shapes. Consequently, their static and dynamic characteristics also exhibit induced asymmetry. The nature of this potential energy landscape is significantly influenced by the bonding of additional elements like Macro Fiber Composites (MFC) for shape-changing and energy-harvesting applications. This study investigates the effect of MFC bonding on the asymmetric potential energy landscape of a bistable piezoelectric energy harvester (BPEH). A fully nonlinear FE model is employed to quantify asymmetry in the potential energy landscape and the static and dynamic characteristics of BPEH. Results show that bonding an MFC patch alters the asymmetric potential energy levels of the stable shapes in a clamped rectangular bistable laminate-MFC assembly, further increasing the existing asymmetry. However, MFC bonding positively impacts static and dynamic characteristics by reducing asymmetry in out-of-plane displacement (Δw) and natural frequency (Δf), thereby enhancing piezoelectric vibrational energy harvesting. The symmetric static and modal characteristics of the two stable shapes facilitate continuous snap-through transitions under external ambient vibrations. A detailed parametric study explores the effect of MFC patch size on induced asymmetry in the potential energy landscape and static and dynamic characteristics. Increasing the MFC patch size increases the asymmetry in the potential energy landscape, whereas smaller MFC patches more effectively reduce asymmetry in the out-of-plane displacement. Additionally, larger MFC patches decrease asymmetry in modal characteristics. These findings provide a design guideline for selecting MFC patch sizes that minimize asymmetry in the potential energy landscape and static and dynamic characteristics, optimizing piezoelectric vibrational energy harvesting.

REFERENCES

- [1] X. Lachenal, S. Daynes, and P. M. Weaver, “Review of morphing concepts and materials for wind turbine blade applications,” *Wind energy*, vol. 16, no. 2, pp. 283–307, 2013.
- [2] D. Bashir, P. M. Anilkumar, S. Scheffler, A. Haldar, B. N. Rao, and R. Rolfes, “A review

- of the dynamic behavior of thermally induced bistable configurations of unsymmetrical composite laminates and their applications,” *Archives of Computational Methods in Engineering*, pp. 1–43, 2024.
- [3] A. M. Giri, S. F. Ali, and A. Arockiarajan, “Dynamics of symmetric and asymmetric potential well-based piezoelectric harvesters: A comprehensive review,” *Journal of Intelligent Material Systems and Structures*, vol. 32, no. 17, pp. 1881–1947, 2021.
 - [4] A. P. Kumar, P. M. Anilkumar, A. Haldar, S. Scheffler, E. L. Jansen, B. N. Rao, and R. Rolfes, “Tailoring bistability in unsymmetrical laminates using an additional composite strip,” *Thin-Walled Structures*, vol. 168, p. 108212, 2021.
 - [5] P. M. Anilkumar, S. Scheffler, A. Haldar, E. L. Jansen, B. N. Rao, and R. Rolfes, “Investigations on the dynamic snap-through of MFC bonded self-resetting bistable laminates,” *Composite Structures*, vol. 332, p. 117906, 2024.
 - [6] A. J. Lee, A. Xie, and D. J. Inman, “Suppression of cross-well oscillations for bistable composites through potential well elimination,” *Journal of Vibration and Acoustics*, vol. 142, no. 3, p. 031003, 2020.
 - [7] J. R. Raney, N. Nadkarni, C. Daraio, D. M. Kochmann, J. A. Lewis, and K. Bertoldi, “Stable propagation of mechanical signals in soft media using stored elastic energy,” *Proceedings of the National Academy of Sciences*, vol. 113, no. 35, pp. 9722–9727, 2016.
 - [8] B. Danish, P. M. Anilkumar, and B. N. Rao, “Suppression of cross-well vibrations of a bistable square cross-ply laminate using an additional composite strip,” *International Journal of Dynamics and Control*, pp. 1–11, 2023.
 - [9] W. Wang, J. Cao, C. R. Bowen, and G. Litak, “Multiple solutions of asymmetric potential bistable energy harvesters: numerical simulation and experimental validation,” *The European Physical Journal B*, vol. 91, pp. 1–9, 2018.
 - [10] R. Harne and K. Wang, “Prospects for nonlinear energy harvesting systems designed near the elastic stability limit when driven by colored noise,” *Journal of Vibration and Acoustics*, vol. 136, no. 2, p. 021009, 2014.
 - [11] Q. He and M. F. Daqaq, “Influence of potential function asymmetries on the performance of nonlinear energy harvesters under white noise,” in *International Design Engineering Technical Conference*, vol. 46391. ASME, 2014, p. V006T10A060.
 - [12] A. M. Giri, S. Ali, and A. Arockiarajan, “Influence of asymmetric potential on multiple solutions of the bi-stable piezoelectric harvester,” *The European Physical Journal Special Topics*, vol. 231, no. 8, pp. 1443–1464, 2022.

- [13] W. Cai and R. L. Harne, “Characterization of challenges in asymmetric nonlinear vibration energy harvesters subjected to realistic excitation,” *Journal of Sound and Vibration*, vol. 482, p. 115460, 2020.
- [14] R. L. Harne and K. Wang, “A review of the recent research on vibration energy harvesting via bistable systems,” *Smart materials and structures*, vol. 22, no. 2, p. 023001, 2013.
- [15] A. Arrieta, P. Hagedorn, A. Erturk, and D. J. Inman, “A piezoelectric bistable plate for nonlinear broadband energy harvesting,” *Applied Physics Letters*, vol. 97, no. 10, 2010.
- [16] D. N. Betts, A. I. Salo, C. R. Bowen, and H. A. Kim, “Characterisation and modelling of the cured shapes of arbitrary layup bistable composite laminates,” *Composite Structures*, vol. 92, no. 7, pp. 1694–1700, 2010.
- [17] P. Harris, “Bistable laminates for energy harvesting,” Ph.D. dissertation, University of Bath, 2017.
- [18] A. J. Lee, A. Moosavian, and D. J. Inman, “A piezoelectrically generated bistable laminate for morphing,” *Materials Letters*, vol. 190, pp. 123–126, 2017.
- [19] A. Arrieta, T. Delpero, A. Bergamini, and P. Ermanni, “Broadband vibration energy harvesting based on cantilevered piezoelectric bi-stable composites,” *Applied Physics Letters*, vol. 102, no. 17, 2013.
- [20] M. Li, D. Yu, Y. Li, X. Liu, and F. Dai, “A bi-stable device for simultaneous vibration absorption and energy harvesting using bi-stable piezoelectric composite laminate,” *Composite Structures*, vol. 314, p. 116971, 2023.
- [21] D. Pan, B. Ma, and F. Dai, “Experimental investigation of broadband energy harvesting of a bi-stable composite piezoelectric plate,” *Smart Materials and Structures*, vol. 26, no. 3, p. 035045, 2017.
- [22] X. Liu, H. Jia, M. Li, Y. Li, Y. Tao, and F. Dai, “Piezoelectric wind energy harvester of bi-stable hybrid symmetric laminates,” *Composites Science and Technology*, vol. 242, p. 110198, 2023.
- [23] K. Boddapati and A. F. Arrieta, “Structural multistability for multi-speed wind energy harvesting from vortex-induced vibrations,” *Smart Materials and Structures*, vol. 33, no. 10, p. 105048, 2024.
- [24] P. Portela, P. Camanho, P. Weaver, and I. Bond, “Analysis of morphing, multi stable structures actuated by piezoelectric patches,” *Computers & Structures*, vol. 86, no. 3-5, pp. 347–356, 2008.