

Bidirectional Evolutionary Structural Optimization for Enhanced Design of SMA Superelastic Structures and Actuators

Sefa Oksuz, Darren Hartl

Abstract. The most important engineering behaviors exhibited by shape memory alloys (SMAs) are superelasticity, which is the ability to fully recover considerable and seemingly permanent deformation when a previously applied load is removed, and the shape memory effect, which allows for recovery of the original shape of an SMA component that has seemingly been permanently deformed given the sufficient application of heat. This shape memory allows actuators to be created with high energy density. Evolutionary structural optimization (ESO) is a heuristic method that maximizes the functional performance of a structure by iterative removal of elements that least contribute to the desired functionality until a predefined volume fraction is satisfied. Further developments have given rise to a more efficient approach: Bi-directional ESO (BESO). This iterative scheme adds and removes elements to achieve continuous load paths and even higher functional performance. In previous studies, two independent BESO optimizations of SMA components were conducted for the austenite and martensite phases, each under constant temperatures, without assuming actuator functionality. In contrast, a new approach described herein directly considers the actuator application by incorporating temperature evolution, allowing for the simultaneous optimization of austenite, martensite, and intermediate phases within a continuous thermo-mechanical cycle. This enables the effective utilization of SMAs in actuator design, making them more suitable for practical applications. Here, the BESO method is used for topology optimization of SMA superelastic structures and actuators with the aim of maximizing volume-specific transformation energy density while accepting definite volume constraints. The thermo-mechanical behavior of SMA actuators is calculated using a phenomenological constitutive model, while the sensitivities are evaluated using the real load method. In the more challenging case of actuators, high temperature loading and subsequent cooling comprise the operational regime (i.e., forward transformation of the component) to be optimized, during which the sensitivity is determined. SMA beam structures and simply supported cubic structures are designed, and their displacement and volume-specific actuation energy density values are evaluated throughout the optimization process. The findings show that our novel approach for the optimization of actuator topologies leads to more energy-dense actuators than those designed by prior (i.e., elastic strain energy-based) methods.

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1 INTRODUCTION

Shape memory alloys have attracted considerable attention in recent years owing to rising demands for reliability and functionality across multiple objectives in engineering applications. These unique materials can support significant recoverable inelastic strains associated with a solid-to-solid phase transformation, enabling them to provide both high force and displacements compared to other metals but relatively small compared to mechanical actuators (e.g., hydraulic, pneumatic, screw-driven, etc.) [1]. In parallel with the growing interest in SMAs, topology optimization has become an essential aspect of modern engineering design, driven by the pursuit of superior performance. Over the past several decades, numerous topology optimization approaches have emerged, including the homogenization method [2], the very common Solid Isotropic Material with Penalization (SIMP) method [3, 4], and the level set method [5, 6]. A particularly notable branch of topology optimization is Evolutionary Structural Optimization (ESO), introduced by Xie and Steven [7, 8], which refines a structure toward an optimal layout by iteratively removing low-stress elements. Chu *et al.* [9] subsequently replaced the stress measure with an element-based strain energy criterion, while the BESO framework [10] further extended the purely subtractive ESO method by also permitting the *addition* of previously removed elements in high-stress regions. Later, Huang and Xie [11] showed that soft-kill BESO could be more stable and less sensitive to algorithmic parameters than standard ESO or SIMP, prompting its use in a variety of problems. This has most recently been addressed by Brent *et al.* [12].

Although topology optimization has been extensively studied, the design of SMA-based topologies remains comparatively underexplored. Early work was hindered by limitations in manufacturing methods, but recent progress in additive manufacturing has expanded the possibilities for SMA structures [13]. One of the first studies to focus on SMAs in a topology optimization setting was by Langelaar *et al.* [14], who examined superelastic behavior in an SMA actuator. More recently, Kang and James [15, 16] introduced a density-based approach that includes both the two-way shape memory effects and superelasticity, later extending it to multi-material SMA systems by incorporating transient heat conduction. Zhu *et al.* [17] applied SIMP to develop high-performance SMA structural designs, while Hou *et al.* [18] proposed a pseudoelastic framework intended to maximize energy dissipation. Most recently, Dong *et al.* [19] implemented a BESO-based topology optimization procedure for SMAs, considering their superelastic response at different constant temperatures and applying multiple constraints (e.g., specified volume fraction, displacement, and fundamental frequency). Their results demonstrated higher computational efficiency compared with SIMP-based methods and revealed uniform mechanical properties combined with substantial deformation capacity.

In this study, a topology optimization model for SMAs is introduced to maximize the volume-specific actuation energy density under fixed volume constraints. To assess its effectiveness, two

topology optimization strategies are compared under thermo-mechanical loading. First, a conventional (i.e., elastic strain energy) BESO procedure is performed on a linearized material domain, and the resulting topology is reanalyzed using an SMA user-defined material subroutine (UMAT) applied to the Abaqus finite element solver, producing baseline force–deflection and temperature–deflection data. Subsequently, a second optimization employs a new UMAT-based BESO method directly, which explicitly accounts for temperature-dependent phase transformations throughout the actuation cycle. Contrasting the force–deflection and temperature–deflection outcomes confirms the validity of the proposed approach. In addition, the UMAT-based SMA topology demonstrates a higher volume-averaged martensite fraction than the linear BESO solution, indicating more efficient actuator performance, as indicated by greater volume-specific actuation energy density. The versatility of the framework is illustrated by applying it to a cantilever beam under both constant-temperature superelastic conditions and varying-temperature actuation scenarios—each using the real load strategy—and by extending it to a three-dimensional simply supported cubic domain, demonstrating its suitability for complex geometries.

2 Theoretical Background

A phenomenological SMA constitutive model by Lagoudas *et al.* [20] is used to capture both superelastic and thermally driven phase transformations. It is implemented in the commercial finite element software *Abaqus* through a UMAT, ensuring accurate updates of the stress tensor, the transformation strain tensor and the scalar martensitic volume fraction (MVF) at each load increment. Table 1 summarizes key SMA parameters used in this work, including the elastic moduli of austenite and martensite (E^A , E^M), zero-stress start and finish transformation temperatures (M_s , M_f , A_s , A_f), Poisson’s ratio ν for both phases, and thermal expansion coefficients (α^A , α^M). During each loading step, the UMAT is provided a global estimate for the total strain increment and calculates the transformation strain increment, determines the MVF evolution, and provides the updated stress state to *Abaqus*, all in such a way to expedite convergence of the nonlinear system [20].

Table 1: Representative SMA parameters [19].

Parameter	Value	Parameter	Value
E^A	70 GPa	E^M	60 GPa
M_s	258 K	M_f	233 K
A_s	273 K	A_f	298 K
α^A	10^{-6} K^{-1}	α^M	10^{-6} K^{-1}
ν	0.3		

Objective Function: From Compliance Minimization to Volume-Specific Actuation Energy Maximization. Conventional linear BESO implementations, such as that of Zuo *et al.* [21] use linear strain-energy-based sensitivity measures to minimize compliance under a pre-defined

volume constraint. In contrast, this new work modifies Zuo's original *Abaqus* BESO code using the results from UMAT (e.g., constitutive model) calculations to maximize actuation energy. Drawing inspiration from Wheeler *et al.* [22] and Chemisky *et al.* [23], the local transformation energy $\Phi_{\text{ener},i}^{\text{tr}}$ of the i^{th} element is defined as:

$$\Phi_{\text{ener},i}^{\text{tr}} = \bar{\sigma}_i \bar{\varepsilon}_i^t, \quad (1)$$

where $\bar{\sigma}$ is the Mises stress and $\bar{\varepsilon}^t$ is the energetically conjugate Mises transformation strain, both determined incrementally within the UMAT. The global objective is the ratio of total actuation energy to the current number of elements, subject to the prescribed volume constraint:

$$\max_{\{x_i\}} \frac{\sum_{i=1}^n [\Phi_{\text{ener},i}^{\text{tr}} v_i]}{\sum_{i=1}^n x_i} \quad \text{subject to} \quad \sum_{i=1}^n x_i v_i \leq V^*, \quad (2)$$

where $x_i \in \{0, 1\}$ indicates element removal or retention, v_i is the element volume, and V^* is the goal total volume. A BESO scheme is used to maximize the volume-specific transformation energy within this volume limit. At each iteration, local transformation-energy sensitivities $\Phi_{\text{ener},i}^{\text{tr}}$ are computed and sorted. A threshold $\alpha_{\text{threshold}}$, linked to the target volume V_{k+1} , dictates which elements are removed or added. The target volume for each step is updated by [21]

$$V_{k+1} = V_k (1 - \text{EVR}), \quad (k = 1, 2, 3, \dots), \quad (3)$$

where EVR is the evolution rate that reduces V_k to V_{k+1} . Once the cumulative volume of the first m elements (in ascending sensitivity order) equals V_{k+1} , the threshold is set to the sensitivity of the m -th element. This removal/addition process continues until convergence in the objective, effectively yielding a topology that maximizes recoverable actuation energy. Because the SMA UMAT is integrated into BESO, both superelastic and shape memory (i.e. actuation) responses are directly incorporated into the sensitivity analysis, progressively evolving SMA structures toward topologies with superior martensite phase transformation performance.

3 NUMERICAL EXAMPLES

In this section, the effectiveness of the proposed UMAT-based BESO framework is illustrated through three representative case studies. In each case, final topologies and martensite distributions are examined for both a conventional (linear) BESO method and a UMAT-based BESO approach. The linear BESO solution considers only compliance minimization under a prescribed volume constraint, with iterative compliance and volume-fraction plots provided to track convergence. By contrast, the UMAT-based approach targets maximum actuation performance, monitored through displacement, volume-specific actuation energy density, and volume fraction. In the first two cases, all these metrics are presented to highlight the design evolution, while the third case serves as a demonstration for more complex 3D structures, focusing on the comparison of pre- and post-optimization topologies. Comparing these scenarios under different loading and temperature conditions underscores the importance of explicitly modeling SMA phase transformations in topology optimization.

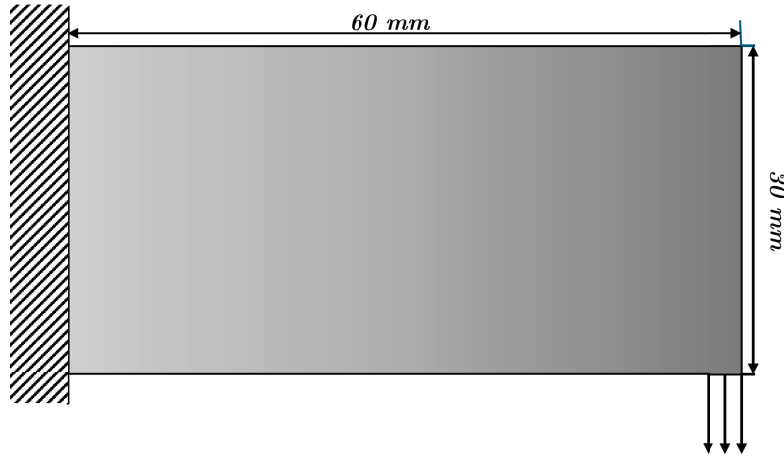


Figure 1: Design domain of the SMA cantilevered structure. This 100% volume domain was reduced to 50% in all examples.

3.1 Case 1: Superelastic Cantilevered Structure

A cantilevered structure measuring $60 \times 30 \times 1$ mm (Figure 1) is modeled using solid elements and subjected to a constant tip load of 1500 N at 273 K to capture superelastic SMA behavior. A 50% volume-fraction target V^* is imposed, and two topology optimizations are carried out:

1. *Linear BESO*, where only the austenitic elastic modulus is used to compute the local elastic strain energy in each, approximating a purely elastic material using that response to select which elements are kept and which are eliminated. The resulting design is later reanalyzed with the full SMA subroutine to assess superelastic effects.
2. *UMAT-Based BESO*, where phase transformation effects at known constant temperature via Eq. (1) are directly incorporated into the optimization process.

Comparisons of force–deflection curves and martensite distributions illustrate how explicitly modeling transformation phenomena via Eq. (1) can better enforce transformation efficiency in this nonlinear problem. In particular, the UMAT-based approach yields more uniform transformation work energy by promoting more uniform martensitic transformations across all regions.

Results for Case 1

As shown in Figure 2, the linear BESO topology reanalyzed under SMA properties exhibits a comparatively lower martensite volume fraction in high-stress regions, and less uniformly overall, whereas the UMAT-based design achieves a more uniform and higher-density distribution. From the force-deflection curves in Figure 3, the UMAT-based approach is shown to create a structure that creates more energy dissipation. Thus, in the context of superelasticity, the objective defined by Eq. (2) generates a better structure.

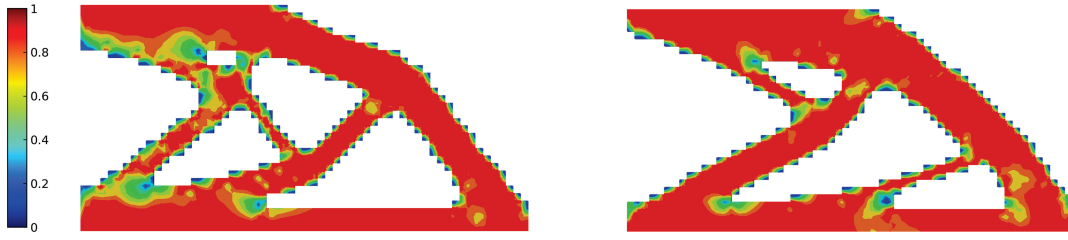


Figure 2: MVF distributions for the linear strain energy-based BESO design (left) and the UMAT-based BESO design (right).

Figure 4 compares the iterative evolution for both methods. Although using a UMAT inevitably introduces additional computational overhead, the number of iterations required for convergence remains similar between the two approaches, which is promising. Notably, the average martensite volume fraction for the linear BESO solution is 0.869, whereas the UMAT-based BESO reaches 0.90, reflecting the more extensive phase transformation afforded by our proposed topological optimization approach. Correspondingly, the average volume-specific actuation energy density for the linear BESO design measures 121.75 N mm per element, while the UMAT-based approach attains 127.21 N mm per element, underscoring the enhanced phase transformation efficiency offered by the UMAT-based design tool.

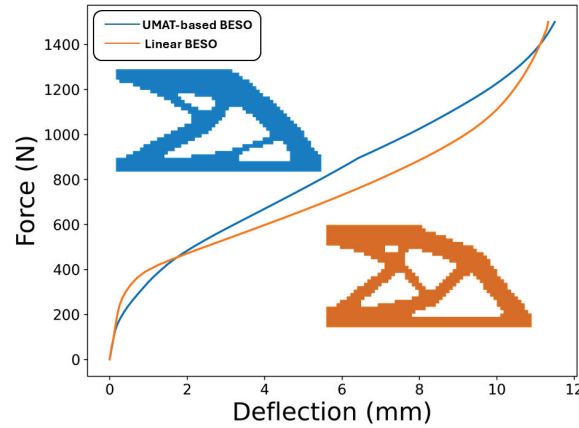


Figure 3: Force-deflection curves for the linear BESO and UMAT-based BESO.

3.2 Case 2: Thermally Driven Cantilever Actuator

Using the same cantilever geometry and a 900 N distributed load, an actuator scenario is modeled by initially loading the structure at 328 K, followed by cooling to 258 K. A 50% volume-fraction target is imposed on the design. A UMAT-based BESO routine optimizes the topology for this temperature-dependent actuation cycle, generating temperature-deflection

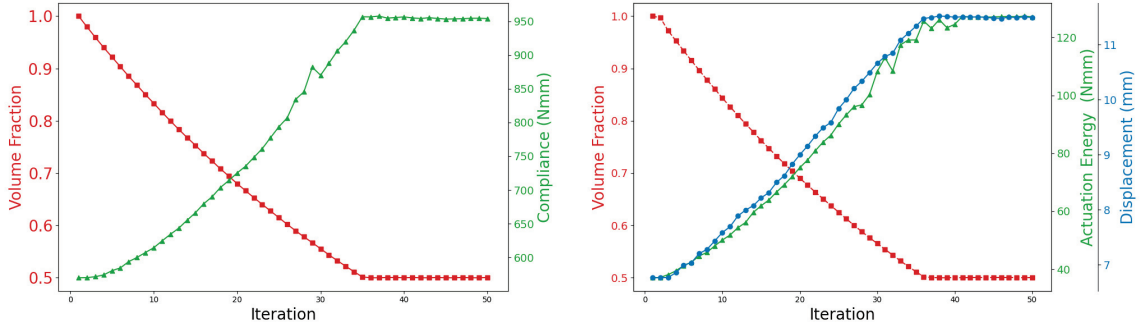


Figure 4: BESO convergence histories for the linear BESO approach (left) and the UMAT-based approach (right).

graphs and martensite distributions. To validate these outcomes, a linear BESO solution is also computed; the resulting form is then reanalyzed within the SMA framework under the same load and temperature conditions used in the UMAT-based optimization case, allowing direct comparisons of transformation-driven performance.

Results for Case 2

Figure 5 compares the linear BESO design (reanalyzed with SMA properties) against the UMAT-optimized topology. The UMAT-based approach again achieves more uniform and thus more volume-efficient martensitic transformation, supporting the finding that direct incorporation of phase-transformation behavior during the optimization leads to improved actuator performance.

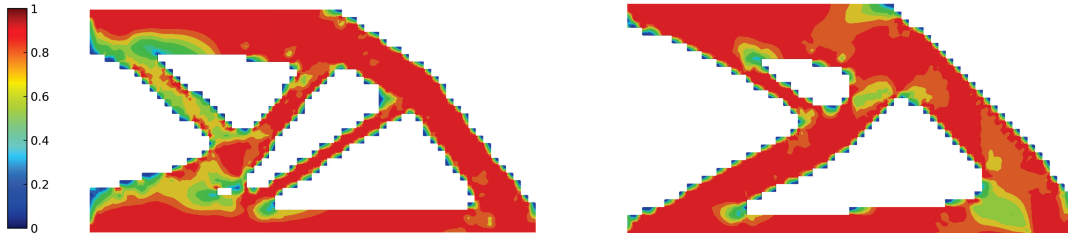


Figure 5: MVF distributions for the linear strain energy-based BESO design (left) and the UMAT-based BESO design (right).

In Figure 6, the linear method converges more rapidly but does not capture displacement or phase fraction accurately, thus offering lower actuation potential when reexamined using SMA properties. In contrast, the UMAT-based actuator optimization shows higher iteration counts to converge, driven by the additional complexity of temperature-dependent phase transitions.

Finally, Figure 7 shows that, although cooling was ended at 259 K due to convergence concerns for some models, further cooling would yield additional actuation deflection for the UMAT-based design. Although the deflection difference appears modest at this intermediate

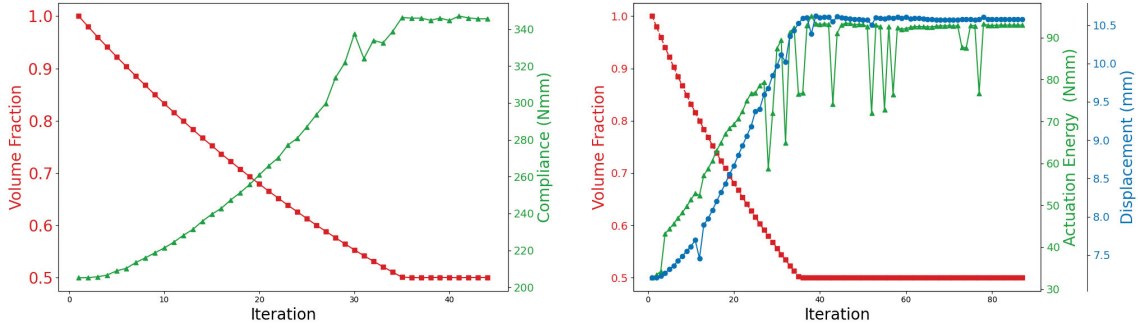


Figure 6: BESO convergence histories for the linear BESO (left) and the UMAT-based (right).

temperature, the UMAT-based approach achieves superior material utilization: the higher and more uniform martensite distribution translates to 14% greater stored transformation energy. In terms of martensite content, the linear BESO solution averages 0.85, whereas the UMAT-based BESO reaches 0.88, indicating a greater phase transformation under the proposed method. Correspondingly, the average volume-specific actuation energy density for the linear BESO design is measured at 81.54 N mm, while the UMAT-based approach achieves 93.01 N mm, underscoring the higher actuation energy potential of the UMAT-based model.

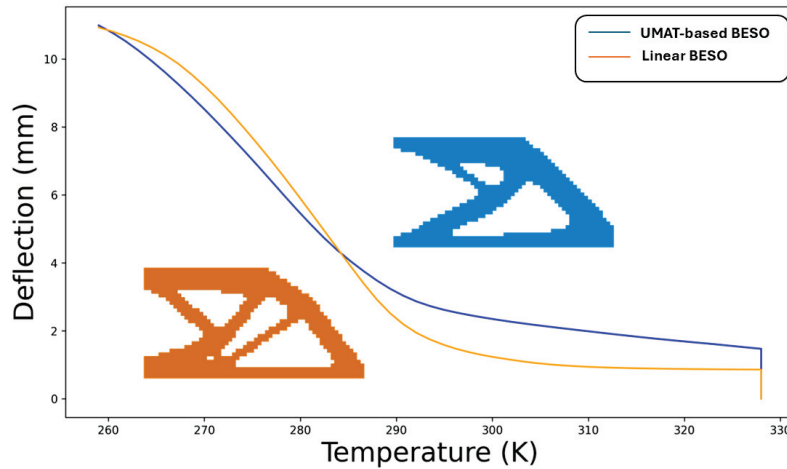


Figure 7: Temperature-deflection curves for the linear BESO and UMAT-based BESO.

3.3 Case 3: Demonstration of a 3D Actuator

A three-dimensional simply supported cubic domain (Figure 8) is subjected to an 8000 N distributed load, with the same thermal profile of loading at 328 K followed by cooling to 258 K. A

25% volume-fraction target V^* is specified to demonstrate the adaptability of the UMAT-based BESO framework to more complex (i.e., lower density) geometries. The resulting topology, temperature-deflection response, and martensite distribution show how transformation-based optimization can be applied effectively to intricate three-dimensional domains, providing a pathway to advanced SMA actuator designs.

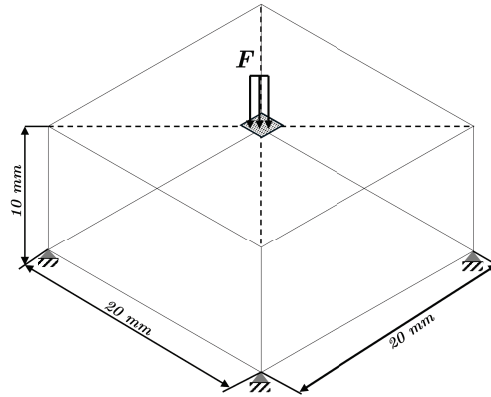


Figure 8: 3D simply supported cube design domain.

Results for Case 3

Figure 9 illustrates the final topology once the volume fraction is reduced to 25% of the initial design. A substantial increase in transformation (indicated by a higher MVF) is observed in critical load-bearing regions, underlining the effectiveness of the UMAT-based BESO method in navigating complex three-dimensional domains. Even at lower volume fractions, the proposed framework exploits thermally driven transformations to maximize actuation performance.

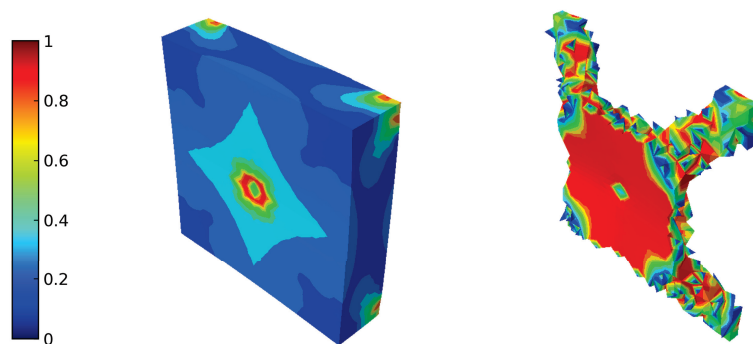


Figure 9: MVF distributions of initial topology (left) and optimized topology (right).

4 CONCLUSIONS

A UMAT-based BESO scheme has been presented for the topology design of SMAs, effectively accounting for both superelastic and thermally driven phase transformations. Numerical examples considering cantilever structures confirm that directly embedding transformation mechanics into the BESO process yields significantly enhanced force-deflection characteristics under constant-temperature loading and superior temperature-deflection responses for actuator scenarios. Extending the framework to a three-dimensional simply supported cubic structure further demonstrates its suitability for complex geometries. By maximizing volume-specific actuation energy within a prescribed volume fraction, the proposed method enables SMAs that achieve uniform, high-density martensite distributions under both constant-temperature superelastic conditions and temperature-varying actuation scenarios, ultimately producing higher actuation energy. This methodology is particularly relevant for aerospace, biomedical, and robotics applications where compact, high-performance actuators are essential. Recent advances in additive manufacturing of NiTi alloys [13] have enabled the fabrication of such complex optimized geometries, further enhancing the practical applicability of this framework.

Future work should investigate the impact of topology optimization on maximum stress distributions, as stress concentrations may affect fatigue life under cyclic loading. Incorporating stress constraints into the BESO framework, extending to multi-objective optimization considering actuation energy, weight, and thermal response time, and experimental validation through additive manufacturing of NiTi structures represent promising research directions.

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