

Design and Analysis of a Robust Hybrid Tuned Mass Damper System for Structural Vibration Control: Active Synthesis of Multiple TMD

Simon Chesné, Jonathan Rodriguez

Abstract. The integration of feedback control strategies with tuned mass dampers (TMDs) emerges as an effective method for enhancing their capability to mitigate structural vibrations. These devices offer the significant advantage of being fail-safe, particularly in aeronautical applications. Recently, several control laws associated with hybrid mass dampers have been proposed, targeting specific functionalities such as multi-mode control, sky-hook synthesis, or nonlinear controllers. This study introduces an approach to synthesize specific control laws that ensure system stability. We propose leveraging the analysis of mechanical impedance at the interface to design an equivalent mechanical dynamic behavior. The passivity property of the active device is inherently guaranteed by the existence of such equivalent mechanical system. This concept has been successfully demonstrated in recent works. In this paper, we specifically focus on enhancing the robustness of the closed loop on a highly variable system. To achieve this, the concept of Multiple Tuned Mass Dampers (MTMD) is explored. By employing the initial TMD as an inertial actuator, we generate an active force equivalent to seven parallel TMDs, each optimally tuned. Simulations of the resulting device demonstrate excellent vibration attenuation performance, even under significant variations of the primary structure dynamic response, as changes in the modal frequencies. The robustness and stability margins of the proposed system are analyzed and compared to conventional systems.

Key words: Active Tuned Mass Damper, Multiple Tuned Mass Damper, Electromagnetic actuators

1 INTRODUCTION

Structural vibration control is a crucial challenge in various engineering fields, including civil engineering, aerospace, and mechanical systems in general. Among conventional solutions, Tuned Mass Dampers (TMDs) have proven effective in reducing vibrations. However, their performance is often constrained by sensitivity to parameter variations, structural uncertainties, and the additional mass they introduce.

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To overcome these limitations, hybrid systems that integrate both passive and active components have been developed [1], [2], [3], [4]. Active absorbers enable adaptations of TMD properties, enhancing both efficiency and robustness. These approaches aim to mitigate the shortcomings of conventional TMDs while preserving the advantages of an efficient and stable solution. Research in this area highlights the benefits of active control in improving the robustness and effectiveness of vibration mitigation systems. More recently, nonlinearities have been introduced either in the control law [5] or directly into the mechanical system [6].

Another development in TMD technology is Multi-Tuned Mass Dampers (MTMDs). A major drawback of traditional TMDs is their high sensitivity to detuning, which causes a significant drop in performance outside a narrow frequency range. This phenomenon, known as the frequency detuning effect, also affects MTMDs when all resonators are tuned to the same frequency, as well as resonant metamaterials when they are perfectly periodic. In such cases, the attenuation bandgap is deep but narrow and highly susceptible to uncertainties. One effective strategy to address this issue is to introduce a frequency distribution in MTMDs [7], [8], [9], making them more effective over a broader frequency range and more robust to detuning. These different approaches converge toward the concept of a hybrid system capable of dynamically adjusting its properties to achieve optimized behavior inspired by MTMDs. This approach leverages the adaptability of active control while maintaining the inherent stability of a passive TMD.

This study proposes an innovative strategy to actively synthesize a robust vibration mitigation system inspired by MTMDs. By integrating a passive TMD with a synthesized MTMD configuration, the hybrid system aims to enhance performance and robustness in structural vibration control.

2 Mathematical model of hybrid system

A standard linear hybrid system is illustrated in Fig. 1. It consists of a primary system (denoted as index 1) coupled with a tuned mass damper (TMD, denoted as index 2). The hybridization process involves the addition of an active force controlled by a dedicated unit. Disturbances affecting the primary system may originate from ground motion (X_0) or an external force acting on the primary structure (F_1). The control performance remains consistent regardless of the source of disturbance. In the subsequent analysis, F_1 will primarily be considered.

The equations governing the system's motion are expressed in the Laplace domain as follows:

$$s^2 m_1 X_1 = (k_2 + s c_2)(X_2 - X_1) - F_a + F_1 + (k_1 + s c_1)(X_0 - X_1) \quad (1)$$

$$s^2 m_2 X_2 = (k_2 + s c_2)(X_1 - X_2) + F_a \quad (2)$$

where s is the Laplace variable, m_1 and m_2 are the masses, X_1 and X_2 are their displacements in the Laplace domain, k_1 and k_2 are the stiffness values, and c_1 and c_2 are the mechanical

damping values of the primary system and the TMD. F_a is the active force defined below. The TMDs parameters follow using the equal peak design rules [10]:

$$\omega_2 = \frac{k_2}{m_2} = \frac{k_1}{m_1}\nu \quad (3)$$

with $\nu = 1/(1 + \mu)$ and

$$\xi_2 = \frac{c_2}{2\sqrt{k_2 m_2}} = \sqrt{\frac{3\mu}{8(1 + \mu)}} \quad (4)$$

where ω_2 is the natural frequency of the TMD, ξ_2 is its damping ratio and μ is the mass ratio (m_2/m_1). For this study, μ is set at 1%, which is representative of the concerned applications. The controller $H(s)$, generating the equivalent mechanical behavior will be defined in the next part. Based on the measurement of the primary system's velocity (as a classical velocity feedback), the resulting active force can be written as follows:

$$F_a(s) = sH(s)X_1(s) \quad (5)$$

3 Active damping: MTMD synthesis

Fig. 2 represents the equivalent mechanical system when the controller is properly tuned to synthesize multiple TMDs. This results in the combination of an optimal passive TMD, initially tuned to damp the host structure, with several additional TMDs having slightly different parameters. There are various strategies to optimize the design of these additional degrees of freedom [7], [8], [9]; however, this is not the main focus of this paper.

In this section, the goal is to use a simple formulation based on the resulting impedance at the interface between the host structure and our new device.

For the analysis developed here, and for sake of simplicity, only damping linked to the hybrid device is considered. This means that the system is analyzed without internal mechanical damping $c_1 = 0$. As previously explained, to ensure fail-safe behavior, the actuator consists of the initial TMD itself, combined with a transduction system (typically a voice coil). Consequently, the standard Direct Velocity Feedback control law exhibits poor stability margins [1] and must be modified.

3.1 Mechanical impedance analysis

As depicted in [3], velocity feedback can be interpreted as an impedance synthesizer. To generate an equivalent MTMD, let's examine the mechanical impedance of a general TMD, written below:

$$Z_{tmd}(s) = \frac{s m_{tmd}(k_{tmd} + s c_{tmd})}{s^2 m_{tmd} + s c_{tmd} + k_{tmd}} \quad (6)$$

In order to be able to generate this behavior we have to consider the dynamic of the transducer, in our case the Hybrid device (indexed 2 in figure 1). The resulting impedance on the

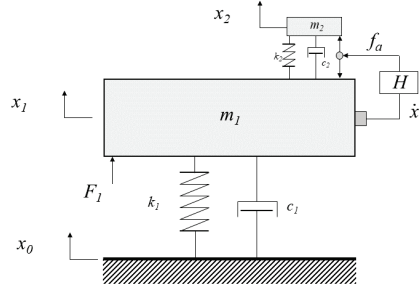


Figure 1: Standard hybrid mass damper using velocity feedback

interface, depending on the control law $H(s)$ can be written:

$$Z_{HMD}(s) = \frac{sm_2(k_2 + sc_2)}{s^2m_2 + sc_2 + k_2} + \frac{s^2m_2}{s^2m_2 + sc_2 + k_2}H(s) \quad (7)$$

3.2 control law design

Using the above equations, it can be understood that to synthesize a new TMD with the properties described in Eq. (6), while using the initial TMD as an actuator, the control law H must satisfy the following expression:

$$H(s) = g \left[1 + \frac{c_2}{m_2s} + \frac{\omega_2^2}{s^2} \right] \frac{sm_{tmd}(k_{tmd} + sc_{tmd})}{s^2m_{tmd} + sc_{tmd} + k_{tmd}} = g \left[1 + \frac{c_2}{m_2s} + \frac{\omega_2^2}{s^2} \right] Z_{tmd}(s) \quad (8)$$

where g is the positive global gain of the control law, which can be interpreted as a mass amplifier. To simplify the following analysis, it will be set to one.

Here, it can be observed that the first part of the control law is intended to cancel the dynamic effect of the initial TMD, which is used as an inertial actuator, to ensure stability.

Considering the linearity of the feedback loop, multiple TMDs can be synthesized in parallel by summing the control laws described above. This leads to:

$$H_{Mtmd}(s) = \left[1 + \frac{c_2}{m_2s} + \frac{\omega_2^2}{s^2} \right] \sum_i Z_{tmd_i}(s) \quad (9)$$

The main challenge now lies in defining the parameters of the various synthesized TMDs, which clearly depend on the context and application. In this study, as a proof of concept, we choose to synthesize six additional TMDs, each with a mass equal to that of the initial real TMD (m_2). The objective is to minimize the H_∞ norm of the frequency response function of the main structure.

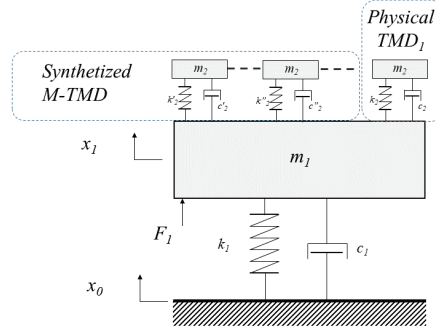


Figure 2: Equivalent mechanical system synthesized by the controller

To achieve this, an optimization function was used to determine the 12 unknown parameters (the six stiffness and six damping values). The *fminsearch* function in MATLAB was employed, to minimize the H_∞ norm of the response, with the algorithm's initial guess simply set to the physical values of the real TMD.

One can underline that one of the main drawback of the proposed system is that the optimization process depends on the gain of the control feedback loop. The optimal parameter distribution may vary depending on this gain. In the illustration provided in this initial study, we remind that the gain is arbitrarily fixed at unity.

3.3 Stability analysis

The introduction of an active system on a tuned damper necessitates a careful analysis of stability as underlined in [11] and [12]. The well-known principle of energy conservation, which holds for all physical systems, states that the variation in stored energy is equal to the sum of external power input and internal power generation [13]. Since the internal mechanical damping values c_1 and c_2 are set to zero, the only internal power generation mechanisms are those associated with the power flow produced by the actuator, which is driven by the controller and its control law $H(s)$. From a stability perspective, the resulting system is passive (or hyperstable) if [13]:

$$\text{Re}[G(s)] > 0 \quad (10)$$

where the function $G(s)$ represents this dissipative mechanism generated by the active device. Using eqs (1)-(5), it can be expressed in the Laplace domain as:

$$G(s) = (sX_1)^2 H(s) \left(1 - \frac{X_2}{X_1} \right) \quad (11)$$

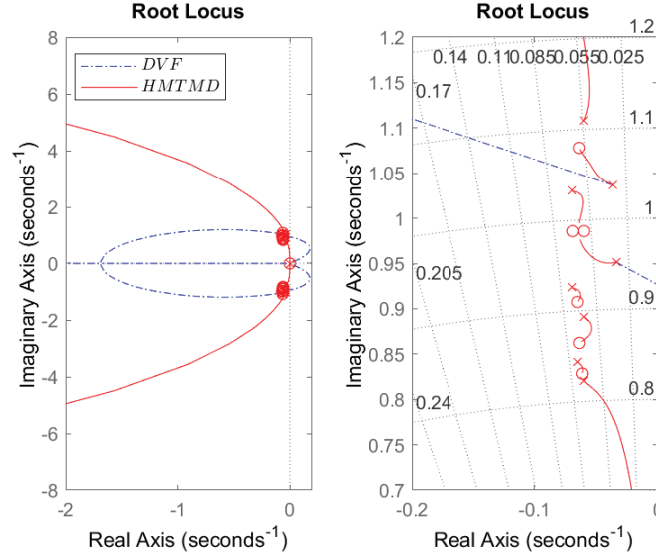


Figure 3: Root Locus for the synthesis of 6 TMDs using the hybrid device (left: Global view, right: Zoom near the TMDs poles)

Using Eqs. (11) and (8), one can easily demonstrate that Eq. (10) is always satisfied. The extension of this principle to multiple control loops for multiple TMDs is straightforward for linear systems. This stability is illustrated in Fig. 3, which presents the root locus representation of a controller emulating 6 others TMDs. It clearly shows that the controlled poles remain on the left side of the imaginary axis, confirming the stability of the system, contrary to classical DVF controller (in blue in this figure) as demonstrated in [1].

4 Performance

The proposed system aims to reduce vibrations while ensuring robust performance despite variations in the parameters of the supporting structure. The following sections briefly illustrate these aspects.

4.1 Vibration attenuation

Figure 4 shows the frequency response of the supporting structure without a TMD, with a passive TMD, and with the proposed hybrid system. A significant improvement in attenuation performance is observed over a wide frequency range. These performances are equivalent to those achieved with seven optimally tuned passive TMDs. However, the hybrid system requires a mass that is seven times lower than the equivalent passive MTMD. The resulting stability margins are infinite for the gain margin (due to the passivity property of the control law) and a phase margin of 72° at 1.2 rad/s .

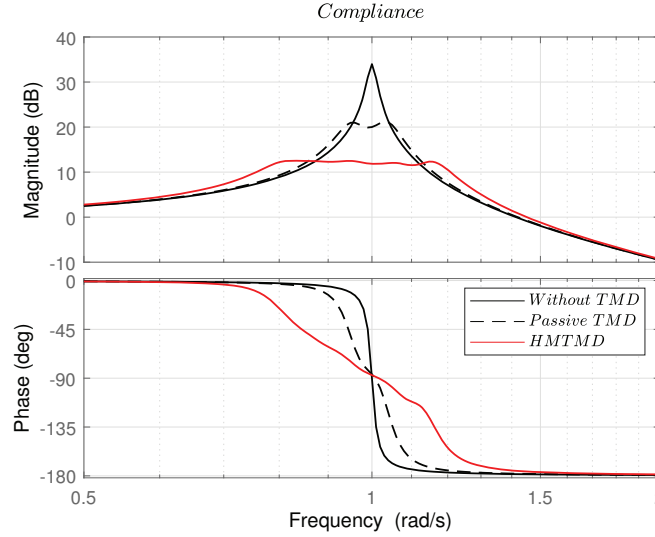


Figure 4: Frequency Response Functions X_1/F_1 of the closed loop system.

4.2 Robustness

Another crucial aspect of control applications, once stability is ensured, is robustness. This refers to the system's ability to maintain equivalent performance even when the environment changes. This is one of the key advantages often highlighted for MTMD. Fundamentally, a change in the environment alters the dynamic properties of the host structure. In such conditions, conventional linear controllers are known to lose performance and, in some cases, even stability. Similarly, passive systems such as TMDs are also susceptible to significant performance degradation. A preliminary approach to assess this aspect is to modify the stiffness of the host structure without updating the controller. A stiffness variation of $\pm 30\%$ is introduced into the model, and the resulting frequency responses are illustrated in Fig. 5.

Without a control device, we can clearly observe a shift in the frequency of the resonance peak. With a passive TMD, performance can be drastically reduced when the initial resonance frequency varies by more than 10%. For the proposed hybrid device, although the nominal performance is naturally reduced, we still achieve an attenuation greater than 10dB . These results are comparable to those obtained with a network of seven properly tuned passive TMDs. However, we remind that the added mass of the hybrid device remains equivalent to that of just a single TMD. In practice, a better understanding of the uncertainty distribution could help define the controller parameters in a more robust manner.

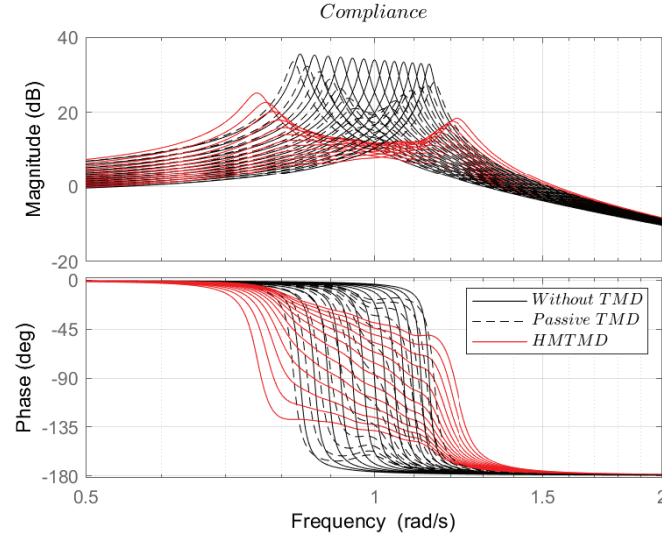


Figure 5: Frequency response X_1/F_1 for stiffness variations ($\pm 30\%$)

5 CONCLUSIONS

This preliminary study illustrates the ability of hybrid mass dampers to control vibrations very efficiently while ensuring fail-safe behavior. It also highlights the relevance of equivalent mechanical impedance analysis in designing controllers to achieve a desired dynamic behavior while ensuring the stability of the resulting system. The synthesis of MTMD appears to be a promising approach to improving the robustness of controllers against system uncertainties or variations. Many aspects are currently under study as tuning optimization or experimental validation. Future research will focus primarily on the optimization step to integrate uncertainty profiles.

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