

Optimization of omnidirectional piezoelectric polymer transducers for high-purity selective excitation of Lamb wave modes

Hao Dong, Pierre Margerit, Marc Rébillat, Mickaël Pruvost, Nazih Mechbal

Abstract. Wave-based structural health monitoring motivates selective excitation of Lamb wave modes by transducer design, which can simplify signal processing and interpretation. While comb array electrodes have been extensively studied for selective excitation, existing tuning methods typically assume uniform width and spacing of the electrodes, thus restricting their performance due to limited tunability. Thanks to advanced printing techniques, the electrodes printed on a flexible P(VDF-co-TrFE) piezoelectric substrate can be arbitrarily shaped, opening up the possibility of unprecedented selective performance. This study investigates the numerical optimization of an annular array of electrodes for high-purity Lamb mode selection. The model is based on the experimentally characterized coupling mechanism between the actuator and the host structure. The results demonstrate that globally optimized annular array electrodes can achieve high-purity excitation of either the S_0 or A_0 mode, with a flexible choice of operating frequency.

Key words: Lamb wave, Structural health monitoring, Piezopolymer transducer, Selective excitation, Global optimization

1 INTRODUCTION

Ultrasonic guided waves, especially Lamb waves, are widely used for inspecting large plate-like structural components in structural health monitoring (SHM) applications. A crucial aspect of designing a wave-based SHM system is the development of transducers capable of effective wave excitation and acquisition. The existence of multiple modes propagating at different velocities motivates special efforts in transducer design to enable the selection of a desired mode, which can reduce the need for sophisticated signal processing and ease the interpretation of measured signals. The transducer can comprise multiple independently driven active components [1, 2] or patterned electrodes [3], such as comb array electrodes patterned on a piezoelectric substrate and interdigital transducers (IDTs). The procedure to choose relevant

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parameters (frequency, geometry, etc.) such that the highest contrast between one desired mode and the other modes is called mode tuning [4, 5, 6]. Conventional tuning methods usually consider periodic arrangements of the electrodes [4, 7], which restricts design flexibility and thus performance. In addition, the design is simply targeted at the best mode contrast at the center frequency of a tone burst input signal, even though it contains rich spectral components. An optimal non-periodic linear IDT for Lamb wave mode-selective acquisition has been recently proposed [8]. An actuator with optimal non-periodic electrodes that eliminates the propagation of a mode wave-packet is promising, which we term as high-purity selective excitation in this work.

The basis of numerical optimization is modeling the structural response of an actuator. Multi-physics models that take full substructure-actuator coupled dynamics into account are accurate but computationally inefficient. Since the actuator applies an external surface traction to the plate, the full-coupled numerical analysis can be simplified into an uncoupled boundary value problem by assuming the surface traction distribution. The latter has analytical solutions and is computationally efficient, which is essential for numerical optimization. Conventional ultrasonic transducers through gel coupling are piston-type and transfer uniform normal traction to the substructure [4, 9]. In comparison, piezoceramic surface-mounted transducers are strain-type, for which a model describing the in-plane traction distribution is proposed based on quasi-static analysis [10, 11]. The model have been further simplified into an ideal bonding solution, also known as the pin-force or line-force models [9, 12]. Another assumption is that the piezoelectric properties of the actuator are constant over the frequency range of interest. This model has been experimentally validated (see e.g. [13, 14]). In comparison, there is not enough discussion and validation in the literature on the simplified models for the piezopolymer actuators, though they have been numerically studied as both piston-type [15, 16] and strain-type transducers [17]. Wilcox et al. [18, 19] model the directional responses of irregular-shaped PVDF IDTs as piston-type transducers based on the superposition of Green's functions, and show excellent agreement with the experiment. Li et al. [7] compare the experimentally measured responses of linear-shaped PVDF IDTs with the normal and shear stress models to conclude the piston-type coupling. Note that these experiments are performed for wave-packets at a few frequencies rather than across a wide frequency range.

In this work, we investigate high-purity selective excitation by an optimized array of concentric annular electrodes patterned on a P(VDF-co-TrFE) piezoelectric substrate. Analytical solutions for the normal stress and line-force shear stress models are first presented and validated. The analytical tuning curves are then compared with experimental results to give insights into the coupling mechanisms. In the two-mode low-frequency range, we use the normal stress model to perform optimization. Geometrically simple designs are achieved for nearly perfect elimination of the target fundamental mode. The flexibility of the approach and the role of irregularity are also discussed.

2 Mathematical model

2.1 Assumptions

The piezoelectric actuator is illustrated in Fig. 1. In this work, we consider axisymmetric electrodes that allow omnidirectional excitation, namely, circular or concentric annular electrodes. A pair of identical electrodes are patterned on the top and bottom surfaces of a P(VDF-TrFE) substrate. The transducer is surface-bonded to a metallic plate through an adhesive layer. The electrodes induce a traction to the plate that generate Lamb waves. Accurate knowl-

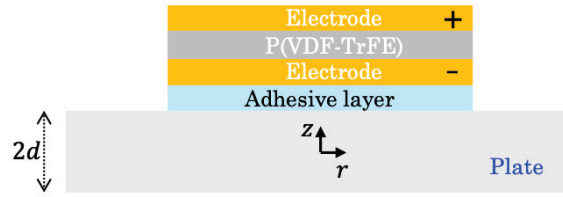


Figure 1: Schematic diagram of the piezoelectric transducer attached to a plate.

edge of this loading distribution needs solving a model that takes into account the full-coupled dynamics of the plate, bonding layer, electrodes, and piezoelectric layer, which can be computationally inefficient so that numerical optimization becomes challenging. Instead, there are two theoretical assumptions of the stress distribution that can simplify the analysis, associated with different underlying coupling mechanisms, as illustrated in Fig. 2. The first one, termed as normal stress model, assumes that a uniform, frequency-independent, out-of-plane stress is distributed over the area covered by the electrode. The second one, termed as line-force model, assumes that uniform in-plane stress is concentrated along the electrode boundaries and points to the normal direction. A complete shear-lag analysis for a circular piezoceramic actuator is referred to [9]. The ideal-bonding condition when the bonding layer is either very thin or very stiff compared to the plate yields the line-force model, in which all the load transfers take place at the surface electrode boundary. Both assumptions allow us to calculate analytically the excited guides wave fields in a common mathematical framework.

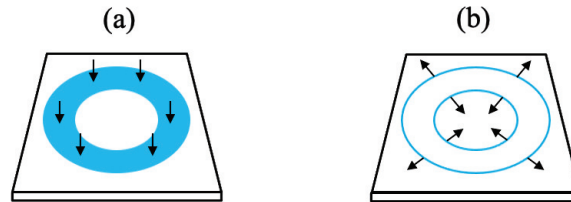


Figure 2: Theoretical assumptions of surface stress distribution: (a) normal stress model (b) pin-force model.

2.2 Integral transform method

The structural response in an isotropic plate induced by each type of the surface loading shown in Fig. 2 can be calculated using the integral transform method. The solving procedure has been detailed in [20]. Here, we summarize the method and solutions. The circular-crested Lamb waves are described in cylindrical coordinates with axisymmetric condition $\partial_\theta = 0$ and vanishing azimuthal displacement component $u_\theta = 0$. Consider the time-harmonic solution $U(r, z, t) = \text{Re}\{u(r, z, \omega)e^{-i\omega t}\}$ for an arbitrary axisymmetric loading distribution on the top surface of the plate:

$$\sigma(r, d) = \tau(r), \sigma(r, -d) = 0, \quad (1)$$

where σ is a general denotation representing either normal stress σ_{zz} or shear stress σ_{rz} , $d = h/2$ is the half-thickness of the plate, and $\tau(r)$ is an arbitrary function. The Lamb waves are described in terms of two scalar potentials, Φ and H , that satisfy the wave equation in the following form

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) + \frac{\partial^2 \Phi}{\partial z^2} + \frac{\omega^2}{c_p^2} \Phi &= 0, \\ \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial (rH)}{\partial r} \right] + \frac{\partial^2 H}{\partial z^2} + \frac{\omega^2}{c_s^2} H &= 0, \end{aligned} \quad (2)$$

where $c_p^2 = (\lambda + 2\mu)/\rho$ and $c_s^2 = \mu/\rho$ are the longitudinal (pressure) and transverse (shear) wave speeds, λ and μ are Lamé constants, and ρ is the mass density. For each type of loading, the solution steps follow Fig. 3.

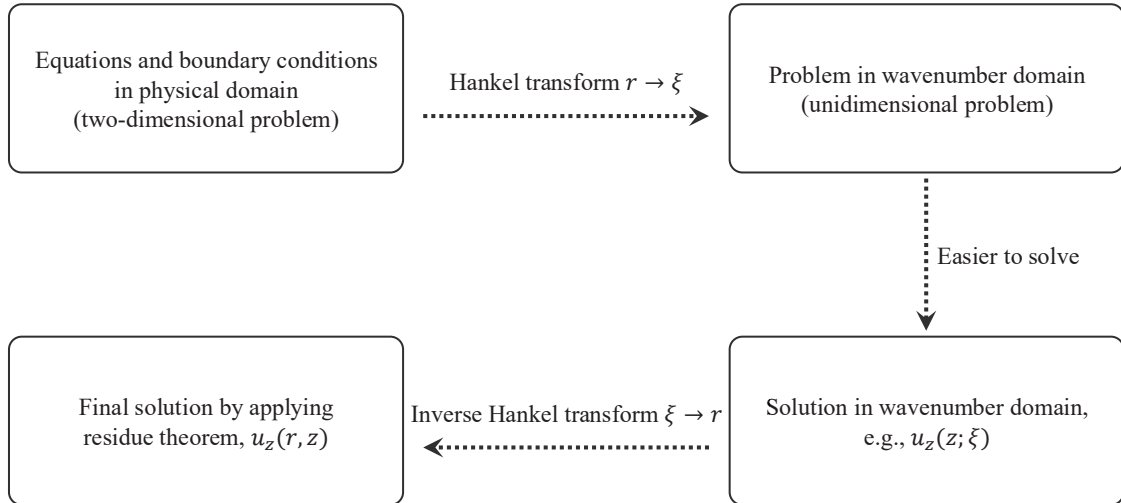


Figure 3: General steps of using the integral transform method for solving the Lamb wave excitation problem.

For either type of loading, the solution of out-of-plane displacement on the top surface can

be written in a general form as a series of modes:

$$u_z(r, z = d) = -\frac{\pi i}{2\mu} \left[\sum_{i=0}^{\infty} \tilde{\tau}(\xi_i) \frac{N_S(\xi_i)}{D'_S(\xi_i)} H_0^{(1)}(\xi_i r) + \sum_{j=0}^{\infty} \tilde{\tau}(\xi_j) \frac{N_A(\xi_j)}{D'_A(\xi_j)} H_0^{(1)}(\xi_j r) \right]. \quad (3)$$

Here, each term represents a Lamb waves mode that is propagating or evanescent. $\{\xi_i\}$ and $\{\xi_j\}$ are the radial wavenumbers of symmetric and anti-symmetric modes at the angular frequency ω , respectively. The expressions $N_S(\xi_i)/D'_S(\xi_i)$ and $N_A(\xi_j)/D'_A(\xi_j)$ are termed modal excitability for the symmetric and anti-symmetric modes, respectively. The denominators are given by:

$$\begin{aligned} D_S(\xi) &= 4\xi^2 \zeta_p \zeta_s \sin \zeta_p d \cos \zeta_s d + (\xi^2 - \zeta_s^2)^2 \cos \zeta_p d \sin \zeta_s d, \\ D_A(\xi) &= 4\xi^2 \zeta_p \zeta_s \cos \zeta_p d \sin \zeta_s d + (\xi^2 - \zeta_s^2)^2 \sin \zeta_p d \cos \zeta_s d, \\ \zeta_p^2 &= \frac{\omega^2}{c_p^2} - \xi^2, \\ \zeta_s^2 &= \frac{\omega^2}{c_s^2} - \xi^2. \end{aligned} \quad (4)$$

The symbol ' represents derivative with respect to the argument. In fact, $D_S = 0$ and $D_A = 0$ are Rayleigh-Lamb equations, and their roots give the radial wavenumbers $\{\xi_i\}$ and $\{\xi_j\}$ in Eq. (3), respectively. These dispersion relations can be calculated using the method documented in [21]. The numerators for the normal stress loading condition are

$$\begin{aligned} N_S(\xi) &= \xi \zeta_p (\xi^2 + \zeta_s^2) \sin \zeta_p d \sin \zeta_s d, \\ N_A(\xi) &= -\xi \zeta_p (\xi^2 + \zeta_s^2) \cos \zeta_p d \cos \zeta_s d, \end{aligned} \quad (5)$$

and for the shear stress loading condition are

$$\begin{aligned} N_S &= \xi^2 [2\zeta_p \zeta_s \sin \zeta_p d \cos \zeta_s d + (\xi^2 - \zeta_s^2) \cos \zeta_p d \sin \zeta_s d], \\ N_A &= \xi^2 [2\zeta_p \zeta_s \cos \zeta_p d \sin \zeta_s d + (\xi^2 - \zeta_s^2) \sin \zeta_p d \cos \zeta_s d]. \end{aligned} \quad (6)$$

The function $\tilde{\tau}(\xi_i)$ or $\tilde{\tau}(\xi_j)$ is termed source influence. For the normal loading, $\tilde{\tau}(k)$ is the zeroth-order Hankel transform of the distribution function $\tau(r)$,

$$\tilde{\tau}(k) = \int_{-\infty}^{\infty} \tau(r) J_0(kr) r dr, \quad (7)$$

with J_0 the Bessel function of first type. For the shear loading, $\tilde{\tau}(k)$ is the first-order Hankel transform of $\tau(r)$,

$$\tilde{\tau}(k) = \int_{-\infty}^{\infty} \tau(r) J_1(kr) r dr. \quad (8)$$

$H_0^{(1)}$ is the zeroth-order Hankel function of first type. If the distance r is much larger than the radial wavenumber $\lambda_r = 2\pi/\xi_r$, $r \gg \lambda_r$, the following asymptotic approximation holds:

$$H_0^{(1)}(\xi_j r) \approx \sqrt{\frac{2}{\pi \xi_j r}} e^{i(\xi_j r - \frac{\pi}{4})} \propto \frac{1}{\sqrt{r}}, \quad (9)$$

which indicates cylindrical wavefronts in the far field.

Finally, we present explicit expressions of the source influence for the normal stress and line-force shear stress models from their specific spatial distributions. Consider a circular electrode of radius a and an array of N concentric annular electrodes with outer radii $\{a_i\}$ and inner radii $\{b_i\}$. In the normal stress model, the loading strength takes 1 per unit area, whereas in the line force model, the loading strength takes 1 per unit length. The resulting stress distribution functions $\tau(r)$ and spectra $\tilde{\tau}(k)$ are given in Table 1. It is noteworthy that the source influence spectra of the pressure and line-force models are described by the same order of Bessel functions but differ by an order of wavenumber k .

Table 1: Stress distributions and spectra for axisymmetric electrodes.

Assumption	Normal stress model		Line-force model	
Geometry	Circular	Annular	Circular	Annular
$\tau(r)$	$\begin{cases} 1, r \leq a \\ 0, r > a \end{cases}$	$\begin{cases} 1, b_i \leq r \leq a_i \\ 0, \text{else} \end{cases}$	$\delta(r - a)/r$	$\sum_{i=1}^N [\delta(r - a_i) - \delta(r - b_i)]/r$
$\tilde{\tau}(k)$	$aJ_1(ka)/k$	$\sum_{i=1}^N [a_i J_1(ka_i) - b_i J_1(kb_i)]/k$	$aJ_1(ka)$	$\sum_{i=1}^N [a_i J_1(ka_i) - b_i J_1(kb_i)]$

2.3 FEM

For verification purpose, the analytical solutions for both normal stress model and line-force model are compared with the finite element method (FEM). The wave fields in an aluminum plate generated by an actuator with two annular electrodes are computed at a single frequency. The parameters are as follows: density of the aluminum $\rho = 2.7 \times 10^3 \text{ kg} \cdot \text{m}^{-3}$, Lamé constants of the aluminum $\lambda = 5.74 \times 10^{10}$ and $\mu = 2.61 \times 10^{10}$, thickness of the plate $h = 2 \text{ mm}$, outer radii of the electrodes $a_1 = 0.5 \text{ cm}$ and $a_2 = 1 \text{ cm}$, inner radii $b_1 = 0.25 \text{ cm}$ and $b_2 = 0.75 \text{ cm}$, and frequency $f = 450 \text{ kHz}$. The FEM solution is obtained using the Solid Mechanics Module of COMSOL Multiphysics. An axisymmetric model is built, with its computation domain truncated by a perfectly matched layer to approximate an infinite extent. For the normal

pressure model, a “boundary load” of 1 Pa is applied to the electrode domain. For the line force model, a “ring load” of $1 \text{ N} \cdot \text{m}^{-1}$ is applied to each electrode boundary. Comparative results of the real part of the out-of-plane displacement on the top surface $\text{Re}[u_z(r, z = d)]$ within the range of the FEM computation domain are shown in Fig. 4. In the FEM results, waves are well damped in the artificial layer. At this frequency, only the fundamental modes, namely S0 and A0 modes, are propagating to the far field. Therefore, in both analytical solutions, we only keep these two modes. In the far-field region $r \gg a_2$, excellent agreement between the analytical and finite element solutions can be observed. It should be noted that the small discrepancies in the near field can be eliminated by taking the evanescent modes into account. Compared to the FEM, the analytical solution gives more physical insights and computationally faster.

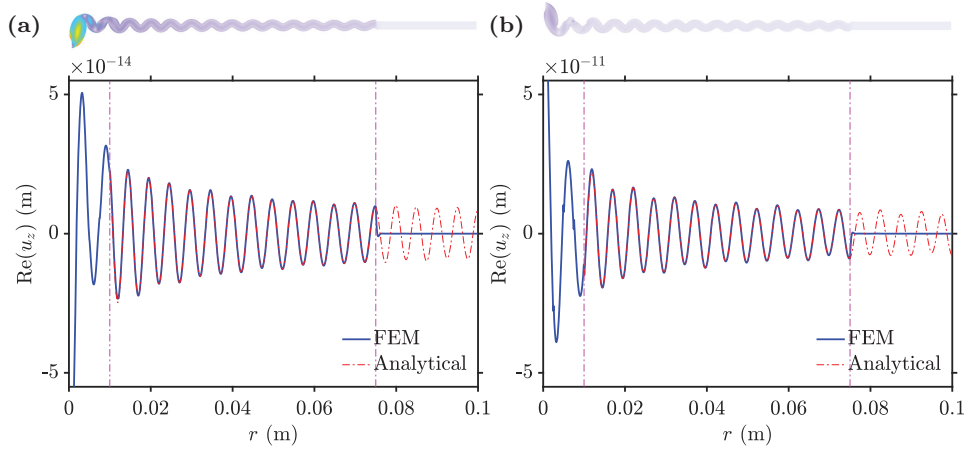


Figure 4: Theoretical displacement fields generated by an actuator with two annular electrodes. A comparison of FEM and analytical results for (a) normal pressure model and (b) line force model. The vertical lines indicate the range of the actuator ($r \leq 0.01 \text{ m}$) and the perfectly matched layer ($0.075 \text{ m} \leq r \leq 0.1 \text{ m}$).

2.4 Model comparison

We now examine the frequency dependence of Eq.(3), namely the behavior of the tuning curves. The displacement magnitude in the far-field can be deduced as

$$|u_z(r, z = d)| = \sqrt{\frac{\pi}{2\xi_r r}} \frac{|\tilde{\tau}(\xi_r)E(\xi_r)|}{d\sqrt{\mu}} \quad (10)$$

where ξ_r is the radial wavenumber for a given mode, $E(\xi_r) = N(\xi_r)d/D'(\xi_r)$ is the corresponding dimensionless modal excitability. The factor

$$A_u(f) = \frac{|\tilde{\tau}(\xi_r)E(\xi_r)|}{\sqrt{\xi_r}} \quad (11)$$

that incorporates all the frequency dependence, is the tuning curve. The Lamb wave dispersion relations $\xi_r(f)$ for the three or four modes are plotted in Fig. 5(a). In this work, we are interested

in the frequency range below the cut-on frequency of the first higher-order symmetric mode (S_1 mode, $2.86 \text{ MHz}\cdot\text{mm}$ for aluminum). The excitability as a function of frequency-thickness product for the first three modes is shown in Fig. 5(b). The cut-on frequency of the A_1 mode at $1.55 \text{ MHz}\cdot\text{mm}$ distinguishes the behavior of the excitability functions for the two fundamental modes. Below that frequency, the excitability for the A_0 mode decreases monotonically for both the normal and shear traction models, at a similar rate. In the low-frequency limit, the excitability for the normal stress model is much greater. The excitability for the S_0 mode behaves asymptotically as $O(fh)$ and $O[(fh)^2]$ for the normal stress model and shear stress model, respectively. Above that frequency, the excitability for the A_0 mode becomes quite even. The excitability for the S_0 mode, however, exhibits distinct behavior between the models. The normal stress excitability rapidly increases, whereas the shear stress excitability presents a null at $2.20 \text{ MHz}\cdot\text{mm}$. According to the analytical expressions given in Table, the source influence spectrum $|\tilde{\tau}(\xi_r)|$ has many zero points and local maximum points, through which mode tuning is possible. The spectra deduced from the normal stress model and line-force model differ only by a factor of the radial wavenumber $k_r(f)$. Consequently, their null frequency points are exactly the same. We exemplify the S_0 and A_0 mode tuning curves for the simplest case of a circular electrode of radius $a = 2h$, see Figs. 5(c) and 5(d). For the A_0 mode, the loading will not change the null frequencies and not significantly shift the maximum frequency points. For the S_0 mode, the shear-stress tuning curve has an additional null at $2.2 \text{ MHz}\cdot\text{mm}$, this spectral difference is thus crucial to distinguish the loading properties.

3 Experimental measurement of Lamb wave tuning curves

According to the theoretical analysis in Section 2.4, the low-frequency tuning curves for both A_0 and S_0 modes show spectral resemblances between the normal and line-force shear models, while the tuning curves for S_0 mode show notable differences between the models in a particular frequency range. It is thus of essential importance to experimentally measure the tuning curves over a wide frequency range to validate the models at low frequencies, and to identify the model that would provide a better prediction when it comes to higher frequencies.

The time-harmonic wave field on the surface of the plate has more than one wavelength component corresponding to the multiple modes. To separate them, one may project the field over plane waves, namely, transforming the spatial field into two-dimensional wavenumber field. To do so, the out-of-plane velocity field generated by a circular actuator of diameter 22 mm is measured in a finite region $l_x \times l_y$ using the Laser Doppler Vibrometer. The dimension of the scanned region $\sqrt{l_x l_y}$ and the scanning step Δl respectively restricts the biggest and smallest wavelengths that can be resolved. An aluminum plates with dimensions $1 \text{ m} \times 1 \text{ m} \times 4 \text{ mm}$ are measured without applying any absorption treatment on their boundaries. The effect of the near-perfectly reflecting boundaries is known to introduce densely located high-quality-factor resonances into the free-field frequency responses. The mode density is proportional to the longitudinal dimensions of the plate. The input signal is a linear sweep sine signal. The velocity frequency response at each measurement point is obtained by dividing the frequency spectrum of the measured velocity signal by that of the input signal. The wavenumber spectrum is then

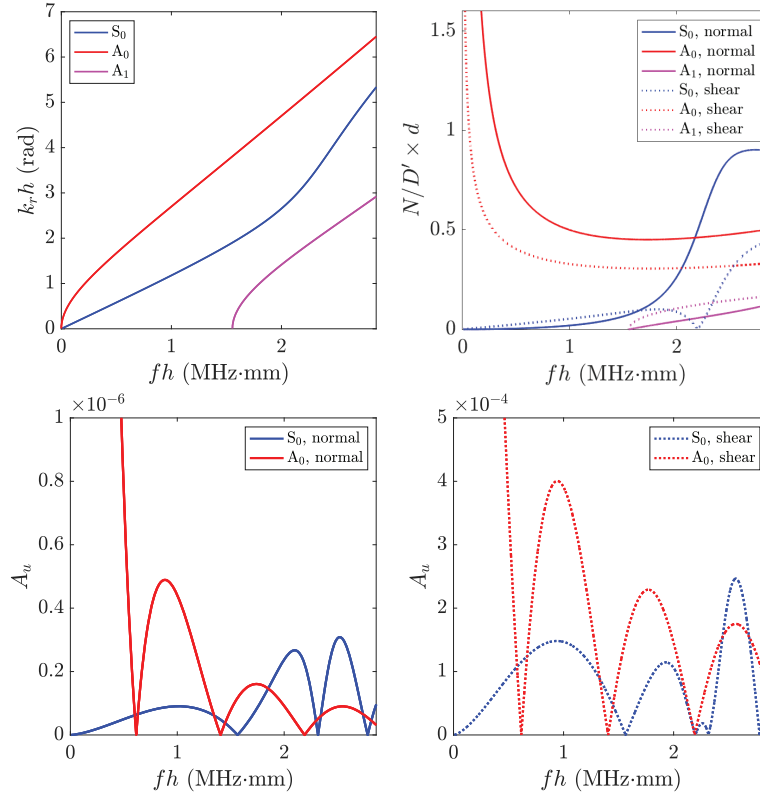


Figure 5: (a) Dispersion curves. (b) Dimensionless excitability. (c) Tuning curves for the normal stress model. (d) Tuning curves for the shear stress model.

calculated by applying two-dimensional spatial Fourier transform to the time-harmonic wave field. The procedure of determining the tuning curves is illustrated in Fig. 6(a). The resonances in frequency domain are first smoothed out by applying moving average to the sequence of two-dimensional spectra. Each individual mode is then selected in the wavenumber domain with a narrow ring-shaped window that is centered about its corresponding wavenumber according to the Rayleigh–Lamb equation. The root-mean-square (rms) value of that mode component within the scanned region is then calculated by averaging the windowed power spectrum. The measured results are compared with the theoretical tuning curve for the velocity $v_z = -i\omega u_z$:

$$A_v(f) = \frac{|\omega \tilde{\tau}(\xi_r) E(\xi_r)|}{\sqrt{\xi_r}}. \quad (12)$$

The results are shown in Figs. 6(b) and 6(c). Both models predict excellently the null frequency points and global variation trends of the tuning curves. The line-force model agrees better than the normal stress model near the null at 2.2 MHz·mm. In the low-frequency range (less than the cut-off frequency of A_1 mode), the models differ only by a smooth weighting function so both are usable when the input signal is narrow-band.

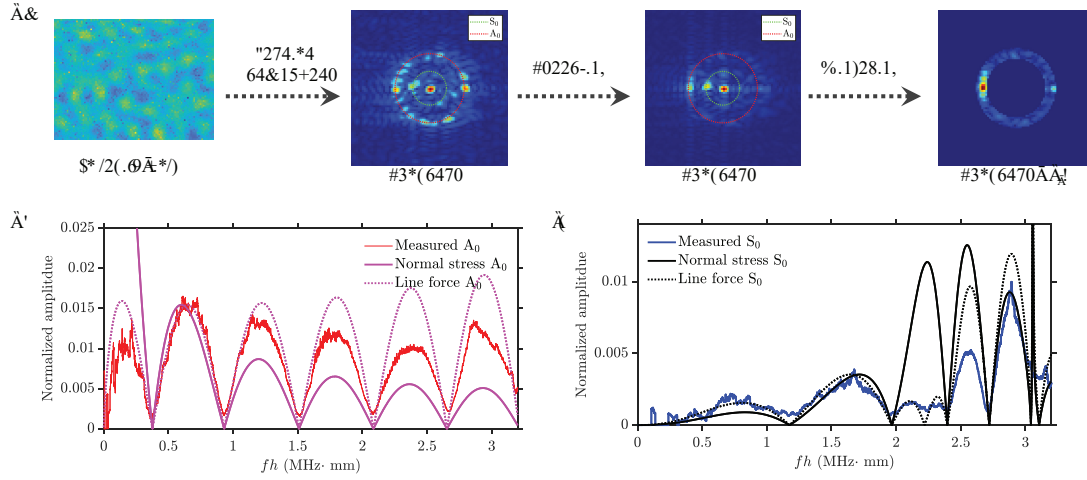


Figure 6: (a) Flowchart of evaluating the tuning curves from measured wave field, exemplified at $f = 360$ kHz. (b)-(c) Comparison of measured and theoretical magnitude responses across a wide frequency range.

4 Optimization

4.1 Formulation

The objective for high-purity selective excitation is to maximize the energy of wave packet of a wanted mode relative to that of an unwanted mode. A tone burst is used as input signal [see Fig. 7(a)]. As shown in Fig. 7(b), the signal has rich frequency components. Therefore, in contrast to the conventional tuning concepts that target simply at the center frequency, in the present work, the objective is to design the annular electrodes such that the continuous wavelength spectral of an unwanted mode is filtered out across the signal frequency band.

To construct the objective function, we begin with the time-domain velocity for a given Lamb wave mode. According to Eq. (3), it writes

$$V_z(r, z = d, t) = \text{Re} \left\{ \int_0^\infty -\frac{\pi}{\mu d} \omega \tilde{\tau}(\xi_r) S(\omega) E(\xi_r) H_0^{(1)}(\xi_r r) e^{i\omega t} d\omega \right\}, \quad (13)$$

where $\xi_r(\omega)$ is the radial wavenumber of the mode, $E(\xi_r) = N(\xi_r)d/D'(\xi_r)$ is the dimensionless modal excitability, and $S(\omega)$ is the Fourier spectrum of the tone burst signal,

$$S(\omega) = \int_0^\infty s(t) e^{-i\omega t} dt. \quad (14)$$

Considering the far-field approximation Eq. (9), the energy of a modal wave packet at distance r can be represented by the following integrated spectral power:

$$W = \frac{\pi}{2\mu^2 d^2 r} \int_0^\infty \left| \omega \tilde{\tau} S E \frac{1}{\sqrt{\xi_r}} \right|^2 d\omega. \quad (15)$$

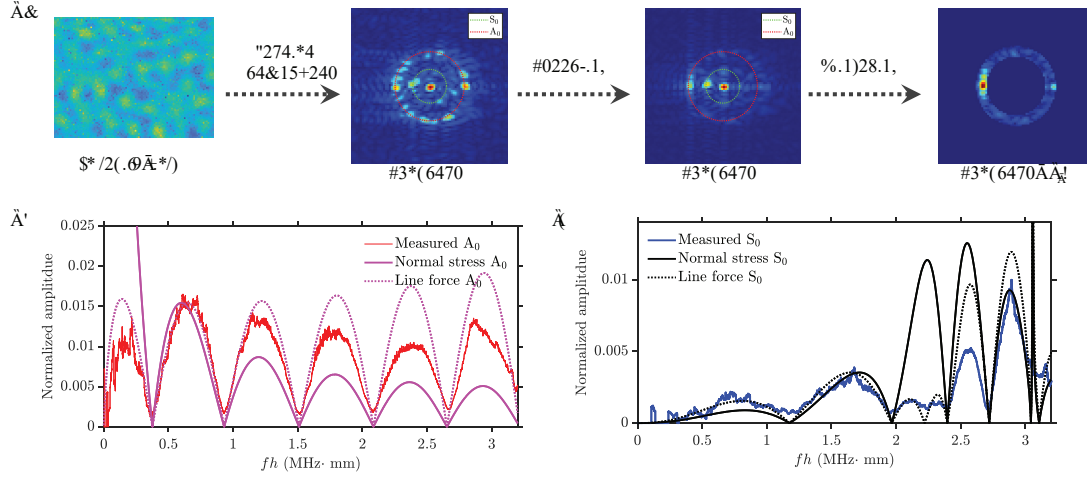


Figure 7: (a) Tone burst signal with 7 cycles and a center frequency of 450 kHz. (b) Spectrum of (a) and -3 dB bandwidth. (c) Schematic diagram of the annular electrodes and design variables.

In the low frequency range where only the two fundamental modes are propagating, the objective function can be defined as

$$J = \frac{W_{\text{mode1}}}{W_{\text{mode2}}} = \frac{\int_0^\infty |\tilde{\tau}_1 S E_1|^2 \omega^2 \xi_1^{-1} d\omega}{\int_0^\infty |\tilde{\tau}_2 S E_2|^2 \omega^2 \xi_2^{-1} d\omega}, \quad (16)$$

which is the energy ratio between an unwanted mode 1 and a wanted mode 2, termed as the modal rejection ratio of mode 1. Note that Eq. (16) is independent of the distance r .

As shown in Fig. 7(c), the design variables (represented by the parameter vector $\mathbf{q}$) are the widths of and spacings between the annular electrodes, $q_0, q_1, \dots, q_M$. During the optimization process, the dimension of the actuator (represented by its outer radius a) and the number of electrodes N_e are fixed. There are $M = 2N_e - 1$ free variables. These parameters only affect the source influence spectra $\tilde{\tau}$ for mode 1 and mode 2. Therefore, Equation (16) can be regarded as a ratio between two weighted integrals with weight functions $|SE|^2 \omega^2 \xi^{-1}$.

The optimization problem is formulated as a constrained non-linear minimization problem:

$$\mathbf{q}^* = \arg \min_{\mathbf{q}} J(\mathbf{q}), \text{ subject to } \sum_{i=1}^M q_i \leq a \text{ and } q_i \geq 0 \ (i = 1, 2, \dots, M), \quad (17)$$

The optimization algorithm used to find a global optimal solution is the Global Search approach and is implemented with MATLAB solvers `ga` and `fmincon`. The algorithm starts a local solver from multiple start points sampling multiple basins of attraction and searches thoroughly for a global minimum.

4.2 Numerical results

Optimization experiments are carried out for an actuator of radius $a = 10$ mm mounted on an 2-mm-thickness aluminum plate. The input tone burst signal has seven cycles and a center frequency well below the cut-on frequency of A_1 mode (7.8×10^5 kHz). The design target is selective excitation of either S0 mode or A0 mode. The objective function Eq. (16) is computed using the analytical model presented in Sec. 2.2 and the numerical integral. The weight functions, that is, signal spectrum, dispersion relation, and excitability spectrum for each mode at the quadrature frequency points, are tabulated before the optimization process to accelerate the computation. A periodic electrode distribution with uniform width and spacing $q_0 = q_1 = \dots = q_M = a/2N_e$ serves as a reference or conventional design. Optimization experiments are carried out by increasing the number of electrodes from $N_e = 1$ to find simple arrangements that allow high-purity mode selection.

Optimization results for selective excitation of S0 or A0 mode are shown in Table 2 and Fig. 9. The optimal designs are compared with the reference periodic designs. The optimal

Table 2: Optimization results. The second column represents the mode to be excited. f_c is the center frequency of the input tone burst. J_{Ref} and J_{Opt} are the modal rejection ratios of the reference periodic designs and optimal designs, respectively.

No.	Mode	f_c (kHz)	N_e	J_{Ref}	J_{Opt}
1	S0	450	2	2×10^3	6×10^{-3}
2	S0	500	2	9×10^2	1×10^{-1}
3	A0	450	2	6×10^{-4}	1×10^{-6}
4	A0	450	4	1×10^{-2}	2×10^{-6}

solutions with only one electrode fail to exhibit satisfactory performance. When the number of electrodes increases to $N_e = 2$, the optimal solutions exhibit excellent modal selection performance. First, two-electrode optimal designs for selective excitation of S0 mode are discussed. The modal rejection ratios of these two optimized actuators working at different frequencies, 450 kHz and 500 kHz respectively, are significantly reduced compared to the reference design. Their electrode distributions and time-domain modal wave packets at a distance $r = 0.2$ m are compared in Figs. 9(a) and 9(b). For each optimal design, high-purity excitation of S0 mode is manifested in the time-domain signal, accompanied by a significant increase in the wave amplitude compared to the initial design. In addition, these two optimal designs excite S0 wave packet in similar amplitude, despite of their distinct geometry and operating frequency. Next, at the same center frequency as design No. 1, a two-electrode A0-mode-selective design is realized, as shown in the third rows in Table 2 and Fig. 9. Note that the initial electrode distribution is the same as design No. 1, whereas the target of design No. 3 is to eliminate S0 mode. That's

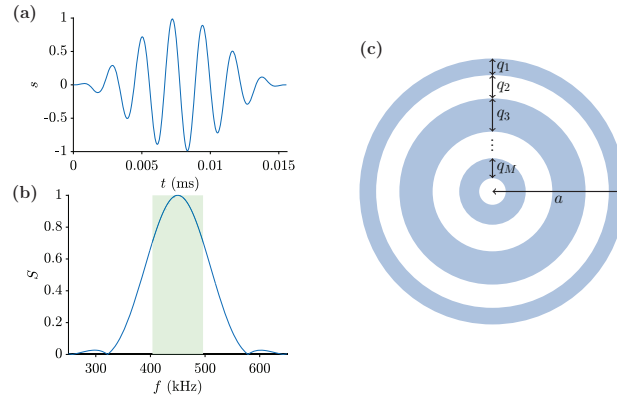


Figure 8: High-purity selective excitation. Comparison of the electrode distribution and time-domain waveform at $r = 0.2$ m of the relevant modes between the reference periodic designs and optimal designs. (a)–(d) correspond to Nos. 1–4 in Table 2, respectively. The velocity has been scaled by an identical factor in these figures.

why the initial objective function value of design No. 3 is reciprocal to that of No. 1. After optimization, this value is decreased by more than two orders of magnitude. In terms of the time domain signal, the S0 mode wave packet vanishes, exhibiting good modal contrast. It should be noted that the wave amplitude for A0 mode for the optimal design is drastically smaller than that for the initial design. A possible cause is the thin annular electrodes in the optimal design and thus relatively small excitation area, which indicates a small magnitude of the source influence spectrum. Nevertheless, this loss in amplitude can be greatly compensated when the number of electrodes increases to $N_e = 4$, as shown in the last rows in Table 2 and Fig. 9. The optimized modal rejection ratio is comparable to that of the two-electrode design No. 3, while the amplitude of A0 mode increases to a level similar as the S0 mode amplitude in design No. 1 or 2.

The present optimal electrode distributions show noticeable irregularity. In the following, we shed some insights on how the irregularity improves the performance of modal selection and the design flexibility. In conventional linear IDTs, the period of electrodes should be half of the selected wavelength [4], which indicates that given the dimension of the actuator and the number of electrodes, the operating frequency cannot be flexibly chosen. For example, Figure 8(a) shows the source influence spectra $\tilde{\tau}$ for the reference two-electrode design. According to the conventional approach, the nodal frequencies in the S0 and A0 mode spectra are the operating frequencies to which the center frequency of the input signal should be aligned. However, in the present optimization method, the target center frequency needs not to be consistent with the nodal frequencies. In the optimization experiment No. 1, the design objective is to excite S0 mode with a 450-kHz tone burst, whereas this target frequency is distant from any A0 nodal frequencies for the reference two-electrode design. In fact, A0 and S0 modes have similar source influence magnitude at the center frequency of the input signal. After optimization [see Fig. 8(b)], the A0 spectrum exhibits a rejection band owing to gathering of three adjacent nodes, resulting in high-purity excitation of S0 mode. In the optimization experiment No. 4, the design

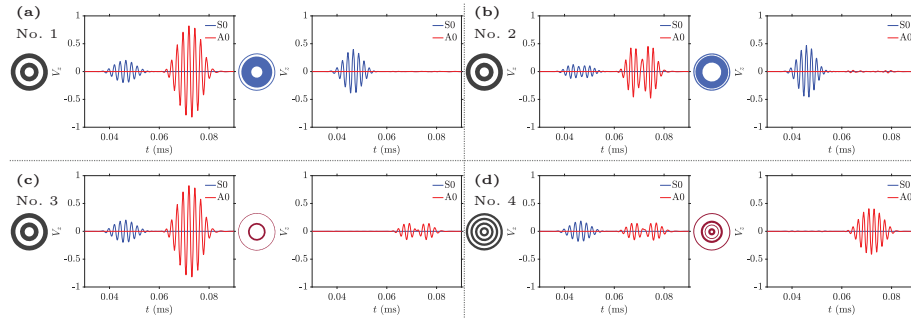


Figure 9: Source influence spectra for (a) reference two-electrode design, (b) optimal design No. 1, (c) reference four-electrode design, and (d) optimal design No. 4. The colored background labels -3 dB bandwidth of a tone burst signal centered at 450 kHz [cf. Fig. 7]. The magnitude has been scaled by an identical factor in these figures.

objective is to excite A0 mode with a 450-kHz tone burst. Although the center frequency is located around an A0 spectral node of the reference four-electrode design [see Fig. 8(c)], the optimization process is still able to gather two adjacent S0 nodes into the signal band to emit A0 mode with high mode contrast [see Fig. 8(d)]. These results indicate remarkable flexibility of the proposed design approach.

5 Conclusion

The effectiveness of analytical models have been validated in terms of predicting the response of surface-mounted P(VDF-co-TrFE) piezopolymer transducers. Numerical optimization of annular-array electrodes enables a high modal rejection ratio within compact dimensions. The optimized designs feature a small number of electrodes, resulting in a simple geometry. The introduced irregularity in the electrode distribution leads to the clustering of nodes in the source influence spectrum, effectively forming a rejection band. Additionally, the design frequency of the tone burst signal can be freely selected, independent of the geometric constraints of conventional IDTs of the same size. These findings can be used for high-performance mode-selective transducers in structural health monitoring applications.

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