

Optimized Placement of Distributed Optic Fiber Sensors for Structural Health Monitoring via the Modified Constitutive Relation Error

José Andrés Pérez Orozco, Ludovic Chamoin, Julien Cortial, Maya de Buhan, Bruno Soulier

Abstract. In Structural Health Monitoring (SHM), sensor integration within materials is crucial. This paper introduces an Optimal Sensor Placement method using the modified Constitutive Relation Error (mCRE) functional. Unlike classical functionals, mCRE searches for structural parameters alongside mechanical fields, leveraging measured data and physical knowledge without further assumptions. The approach integrates mCRE with Bayesian inference to create a new Fisher Information Matrix (FIM) sensitive to updated parameters. The study emphasizes Distributed Optical Fiber Sensors for their embedded sensing capability, high spatial accuracy, and lightweight nature, providing near 1D full-field data. These fibers are modeled with B-Splines, and their placement is optimized using Differential Evolution considering constraints like fiber length and curvature. The methodology is validated using a linear elastic material benchmark, outperforming traditional FIM techniques.

Key words: Optimal Sensor Placement, Fisher Information Matrix, Structural Health Monitoring, Inverse Problems, Modified Constitutive Relation Error, B-Splines Representation

1 INTRODUCTION

Structural Health Monitoring (SHM) combined with data assimilation is increasingly used to assess structural integrity via onboard sensors [1]. This work focuses on a crucial factor in SHM, namely, optimal sensor placement [2, 3, 4, 5], to ensure that the most informative data is collected. Among various sensor types, Distributed Optical Fiber Sensors (DOFS) have gained interest due to their spatial accuracy, lightweight nature, and ability to provide near-continuous strain data [6]. Unlike discrete sensors, DOFS integrate all sensing points along a single fiber, requiring novel placement strategies.

José Andrés Pérez Orozco & Julien Cortial & Maya de Buhan
Safran Tech, Digital Sciences & Technologies Rue des Jeunes Bois, Châteaufort, 78114 Magny-Les-Hameaux, France,
e-mail: jose-andres.perez-oro-zco@safrangroup.com

José Andrés Pérez Orozco & Ludovic Chamoin & Bruno Soulier
Université Paris Saclay, CentraleSupélec, ENS Paris-Saclay, CNRS, LMPS, 4 avenue des Sciences, Gif-sur-Yvette, 91190, France

Two of the popular DOFS technologies include Rayleigh backscattering and Fiber Bragg grating. Both measure the mechanical strain with great accuracy, but Rayleigh naturally provides near-continuous data, whereas Bragg grating requires physical modifications at sensing locations. Rayleigh is limited to $\sim 100\text{m}$, while Bragg grating can span multiple kilometers [7]. Most sensor placement methods involve selecting locations from a candidate set, with differences in cost functions and optimization techniques [6]. However, DOFS pose unique challenges since a single fiber must be optimally routed. Few studies address this challenge; for instance: Li et al. [6] used genetic algorithms to optimize fiber layout using NURBS and maximizing the integration of a weighted functional of strain measures over the optic fiber length. Others works focused on modal shape reconstruction [2] and impact damage detection [8].

Fisher Information Matrix (FIM)-based criteria [9, 10] are widely used for sensor optimization, with common metrics including trace (A-optimality) [5] and determinant (D-optimality) [4, 3]. This work introduces a modified FIM (mFIM) tailored to the modified Constitutive Relation Error (mCRE), a robust inverse problem framework in structural mechanics. The mCRE offers improved convexity [11, 12] and noise resilience [13, 14, 15], yet its use to define various sensor placement metrics remain unexplored, except for recent work by Diaz et al. in the domain of low-frequency dynamics [16].

Here, the determinant of the mFIM is used to optimize DOFS placement via differential evolution, with constraints on length and curvature. B-Splines model the fiber path, and numerical tests show that the mFIM outperforms traditional FIM, enhancing robustness to noise, misalignment, and modeling bias. The paper is structured as follows: Section 2 introduces inverse problems, covering stochastic solution approaches and their connection to Fisher Information-based sensor performance metrics. Section 3 presents the modified Constitutive Relation Error (mCRE) and its associated Fisher Information Matrix. Given the focus on optic fiber sensors, Section 4 outlines key considerations for their optimization. Finally, Section 5 showcases test results for 2D linear orthotropic elasticity cases.

2 Classical sensor performance metrics based on Fisher Information

2.1 Basics on inverse problems

In computational mechanics, inverse problems determine unknown input parameters $\theta \in \mathbb{R}^p$, $p \in \mathbb{N}$ from limited, noisy observations $X_{obs} \in \mathbb{R}^{N_s}$. However, these observations differ from model predictions due to measurement noise (ω_{obs}) or modelling errors (ω_{mod}):

$$X_{obs} = \Pi X(\theta_{real}) + \omega_{obs} + \omega_{mod} \quad (1)$$

where Π is an interpolation matrix to extract information at the sensor locations. Inverse problems can be solved deterministically or stochastically. The latter approach is preferred as it naturally accounts for parameter uncertainties. Bayesian inference provides a probabilistic framework for this objective:

$$\pi(\theta|X_{obs}) \propto \pi(X_{obs}|\theta)\pi_0(\theta) \quad (2)$$

where $\pi_0(\theta)$ is the prior distribution, $\pi(X_{obs}|\theta)$ is the likelihood distribution, and $\pi(\theta|X_{obs})$ is the updated or posterior probability density function on θ after taking observations into account (and the solution to the inverse problem). Considering Gaussian distributions, $\pi_0(\theta)$ and $\pi(X_{obs}|\theta)$ have the form:

$$\pi_0(\theta) \propto \exp \left[-\frac{1}{2}(\theta - \theta_0)^T \Sigma_0^{-1}(\theta - \theta_0) \right] \quad (3)$$

$$\pi(X_{obs}|\theta) \propto \exp \left[-\frac{1}{2}(X_{obs} - \Pi X(\theta))^T [\Sigma_{mod} + \Sigma_{obs}]^{-1}(X_{obs} - \Pi X(\theta)) \right] \quad (4)$$

where θ_0 contains *a priori* parameter values, and Σ_0 , Σ_{mod} and Σ_{obs} respectively denote the prior, model, and observation covariance matrices.

2.2 Fisher Information Matrix and optimal sensor placement

The optimal sensor placement problem aims to minimize uncertainty in identified parameters of an inverse problem. This uncertainty can be directly assessed using the parameter covariance matrix or, more commonly, the Fisher Information Matrix (FIM). The FIM quantifies the amount of information that a random variable contains about a given parameter, and its inverse provides the Cramér-Rao lower bound on the parameter covariance matrix [9]. This implies that a higher Fisher Information leads to lower uncertainty in parameter estimates. As a result, scalar performance metrics derived from the FIM are widely used, particularly the maximization of its determinant (D-optimality) [4, 2, 10, 17].

The FIM is computed as the covariance of the score, the gradient of the log-likelihood distribution with respect to θ [9]:

$$FIM = \mathbb{E} \left[\left(\frac{\partial \log \pi(X_{obs}|\theta)}{\partial \theta} \right)^T \left(\frac{\partial \log \pi(X_{obs}|\theta)}{\partial \theta} \right) \right]_{\theta_{opt}} \quad (5)$$

In this context, $\mathbb{E}$ denotes the mathematical expectation. Alternatively, if the Hessian of the log-likelihood is available, it yields [18]:

$$FIM = -\mathbb{E} \left[\frac{\partial^2 \log \pi(X_{obs}|\theta)}{\partial \theta^2} \right]_{\theta_{opt}} \quad (6)$$

It is then clear that the Hessian of the log likelihood plays a big role in the uncertainty quantification for parameter identification. Intuitively, in the case of one parameter, a high curvature in the distribution around the identified parameter translates to a lower variance. For a Gaussian likelihood (4), if the modelling errors are disregarded, and assuming uncorrelated observation errors with equal variance σ_{obs}^2 , the FIM is given by the following expression:

$$FIM = \frac{1}{\sigma_{obs}^2} \left(\Pi \frac{\partial X(\theta)}{\partial \theta} \right)^T \left(\Pi \frac{\partial X(\theta)}{\partial \theta} \right) \Big|_{\theta_{opt}} \quad (7)$$

It is important to note that this matrix only depends on the sensor locations, as Π extracts information only at the observation points. A sensor configuration that maximizes the FIM's determinant ensures that all principal directions of the FIM have a high information content, making it less sensitive to slight variations in the observation values, and leading to more precise parameter estimate. These variations may result from noise in the measurement instrument or small displacements of the sensors relative to their theoretically optimal positions.

3 The modified Constitutive Relation Error

In structural mechanics, the forward problem determines the displacement field u typically approximated by a finite element solution u_h while the inverse problem estimates constitutive parameters θ . Among various inverse approaches, the modified Constitutive Relation Error (mCRE) stands out for its convexity properties [11, 12] and robustness to noise [13, 14, 15]. Extending the Constitutive Relation Error (CRE) [19], the mCRE balances model confidence with experimental data U_{obs} , forming a hybrid framework for parameter identification.

In the context of linear elasticity and following a discretization with the finite element method, the discretized CRE reads:

$$\mathcal{E}_{CRE}^{2h}(\hat{U}, \hat{V}; \theta) = \frac{1}{2}(\hat{U} - \hat{V})^T \mathbb{K}(\hat{U} - \hat{V}) \quad (8)$$

where $(\hat{U}, \hat{V})$ represents a pair of kinetically and statically admissible solutions. The CRE (8), quantitatively reflects the confidence on modeling, and is commonly referred in the literature to as the distance to model. Extending this, the mCRE [20, 21] introduces a data-driven regularization term referred to as the distance to data:

$$\mathcal{E}_{mCRE}^{2h}(\hat{U}, \hat{V}; \theta) = \underbrace{\mathcal{E}_{CRE}^{2h}(\hat{U}, \hat{V}; \theta)}_{\text{Distance to model}} + \underbrace{\frac{\alpha}{2} \left(\Pi \hat{U} - U_{obs} \right)^T \mathbb{G}_{obs} \left(\Pi \hat{U} - U_{obs} \right)}_{\text{Distance to data}} \quad (9)$$

where $\mathbb{G}_{obs}$ scales measurement errors, and α balances model fidelity against data fit. The optimal θ minimizes this functional:

$$\theta_{opt} = \underset{\theta}{\operatorname{argmin}} \left[\min_{(\hat{U}, \hat{V})} \mathcal{E}_{mCRE}^{2h}(\hat{U}, \hat{V}; \theta) \right] \quad (10)$$

This optimization problem is solved iteratively: first, $(\hat{U}, \hat{V})$ are determined through the following saddle point conditions:

$$\begin{aligned} (\alpha(\Pi^T \mathbb{G}_{obs} \Pi) + \mathbb{K})\hat{U} &= \alpha(\Pi^T \mathbb{G}_{obs})U_{obs} + F \\ \mathbb{K}\hat{V} &= F \\ \hat{V} &= \hat{U} + \Lambda \end{aligned} \quad (11)$$

where Λ is a Lagrange multiplier, and then θ is updated using convex minimization techniques [21]. It is emphasized that $\hat{U}$ is a hybrid quantity, based on the model and the experimental data.

3.1 Stochastic interpretation

To relate the mCRE and the Fisher Information Matrix, a Bayesian interpretation of the mCRE is needed, which was developed by Deraemaeker and Ladevèze [22]. Within this framework, the mCRE-based likelihood distribution reads:

$$\pi(U_{obs}|\theta) \propto \exp \left[-\frac{1}{2} \left(\Pi \hat{U} - U_{obs} \right)^T \Sigma_{obs}^{-1} \left(\Pi \hat{U} - U_{obs} \right) - \frac{1}{2\alpha} (\hat{U} - \hat{V})^T \mathbb{K} (\hat{U} - \hat{V}) \right] \quad (12)$$

This distribution is visibly more complex than the one previously mentioned in (4). However, no assumption on the modelling error covariance (Σ_{mod}) has to be made since the modelling error is already integrated in the distribution. When $\mathbb{G}_{obs} = \Sigma_{obs}^{-1}$ and considering a uniform prior defined in the interval $\theta \in [a, b]$, we get:

$$\pi(\theta|U_{obs}) \propto \begin{cases} \pi(U_{obs}|\theta) \propto \exp \left[-\frac{1}{\alpha} \mathcal{E}_{mCRE}^2 \right] & \theta \in [a, b] \\ 0 & \theta \notin [a, b] \end{cases} \quad (13)$$

The interval of the prior can even be $\theta \in [-\infty, \infty]$. In this case, maximum *a posteriori* estimator of (13) equals the maximum *likelihood* estimator of (12) and corresponds to the minimizer of the deterministic cost function of the mCRE (10)[21].

3.2 FIM based on the mCRE

With the theory behind the mCRE established, attention can now be directed to its Fisher Information Matrix, referred to as the modified FIM (mFIM) to distinguish it from earlier developments. Following the classical FIM formulation (5) poses a computational challenge: its expectation involves third and fourth moments (skewness and kurtosis), which are difficult to evaluate. This complexity becomes particularly problematic when maximizing the determinant of the modified FIM using a stochastic optimization algorithm like differential evolution, as it requires numerous matrix computations. To overcome this issue, the second derivative formulation of the FIM (6) is employed, leading to:

$$mFIM = \frac{1}{\alpha} \left(\hat{U}^{0^T} \frac{\partial \mathbb{K}}{\partial \theta} \frac{\partial \hat{U}^0}{\partial \theta} - \hat{V}^T \frac{\partial \mathbb{K}}{\partial \theta} \frac{\partial \hat{V}}{\partial \theta} \right) \bigg|_{\theta_{opt}} \quad (14)$$

Here, the zero superscript in $\hat{U}^0$ refers to the constant portion of $\hat{U}$. Parallels can be drawn between the modified Fisher Information Matrix (mFIM) (14) and the original Fisher Information Matrix (FIM) (7), as both quantify sensitivity with respect to θ . However, while traditional

methods rely on local evaluations at sensor positions using the projection matrix Π , the mFIM instead evaluates global quantities by incorporating the complete fields $\hat{U}$ and $\hat{V}$, along with their sensitivities to θ . This distinction is directly tied to the nature of the mCRE admissible fields, which are data enriched as seen in (11).

4 Optimization problem

FIM based sensor placement strategies can be generalized for various types of sensors, but additional considerations are needed for optic fiber sensors. These sensors measure strain along their longitudinal direction at N_S positions, and while optimizing the coordinates and orientations of each sensing point is possible, it becomes impractical when N_S can reach thousands, especially in Rayleigh Backscattering fibers, which provide near-continuous data [7].

To address this, the fiber's shape is parameterized using B-Splines, significantly reducing the number of optimization variables. Uniform B-Splines require only $N \ll N_S$ control polygon coordinates to define the curve. Considering that CP is a vector containing the control polygon coordinates, the optimization problem thus reads:

$$\begin{aligned} CP_{opt} &= \underset{CP}{\operatorname{argmin}} -\det(\mathbf{M}) \quad M \in [FIM, mFIM] \\ s.t. & \\ \text{Fiber length constraint: } L &\leq L_{Lim} \\ \text{Curvature radius constraint: } C &\geq C_{Lim} \end{aligned} \quad (15)$$

To solve (15) using a differential evolution algorithm, the following cost (or fitness) function is introduced:

$$f = -\det(\mathbf{M}) \times k^{-\max\left(\frac{L}{L_{lim}}, 1\right)+1} \times k^{-\max\left(\frac{C_{lim}}{C}, 1\right)+1} \quad (16)$$

where k is a constant that determines the severity of the penalization, determined empirically to a value of 4 during this work.

5 Sensor placement results

The mFIM (14) serves as an uncertainty indicator in inverse problems using the mCRE. Its effectiveness as a sensor performance metric is now evaluated against the classical FIM (7) using a simple mechanical test case, a 2D plate subjected to a tensile load (Figure 1). The plate is made of a linear, homogeneous, orthotropic material. The plate is subjected to a tensile load, with material properties defined as: Young's moduli $E_1 = E_2 = 1$ Pa, Poisson's ratio $\mu = 0.3$. A tensile load of 0.01 Pa is applied as a Neumann boundary condition on Γ_{N1} , while zero displacement is imposed on Γ_D . Measurement noise is set at 5% to compute $\mathbb{G}_{obs}$.

These constitutive parameters lead to a displacement equivalent to that obtained using an isotropic constitutive law, but the sensitivity of the Young modulus in each direction can be evaluated independently. Setting the same value for E_1 and E_2 ensures that the sensitivity fields

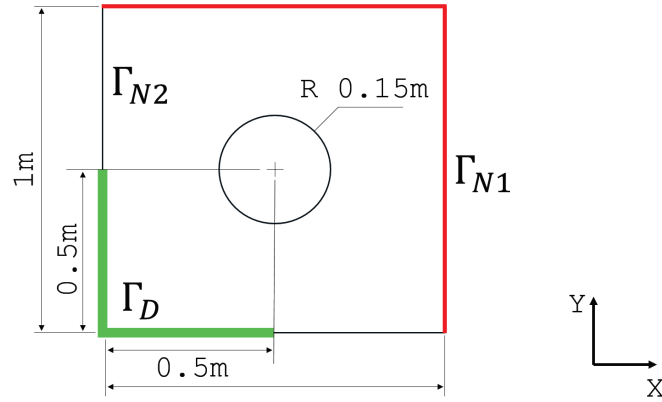


Figure 1: Boundary conditions for the orthotropic case

remain of comparable magnitude, thereby avoiding a bias in the sensor placement towards the identification of a single parameter. The influence of different ratios between E_1 and E_2 on the optimal fiber sensor configuration was analyzed in [23]. The magnitude of the displacement sensibilities ($\partial U / \partial \theta, \theta \in [E_1, E_2]$) can be seen in Figure 2.

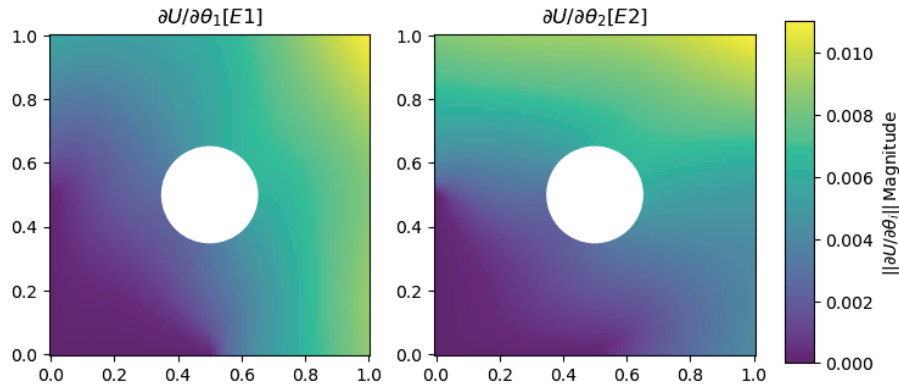


Figure 2: Displacement sensibility with respect to E_1 and E_2

The objective now is to determine the optimal placement of the fiber sensor for the simultaneous identification of E_1 and E_2 using the mCRE approach. The optic fiber is constrained to start on Γ_D , with a maximum length of 3.5m and a minimum curvature radius of 0.01m. The optimization problem is solved using a differential evolution algorithm with a Sobol-distributed initial population, 1000 iterations, a recombination probability of 0.7, and a mutation factor varying between 1.1 and 1.9 (empirically determined, as is typical for global optimization algorithms).

The final shapes that result from the optimization process using the FIM and the mFIM and

the posterior distributions obtained with each sensor configuration are shown in Figure 3. The modified FIM yields a superior sensor configuration for the mCRE, providing lower variance in the identified parameter, which we can verify numerically through the Hessian of the mCRE. The relative improvement is included in Table 1.

As shown in [23], the classical FIM tends to maximize parameter sensitivity along the fiber path, which explains the presence of multiple loops or clusters in regions of high sensitivity. In contrast, the mFIM accounts for global structural quantities and, rather than enhancing local sensitivity, it aims to minimize the overall magnitude of the global sensitivity of the hybrid field associated with the mCRE ($\partial \hat{U} / \partial \theta$).

This intuition can be clarified through two limiting cases. First, when $\partial \hat{U} / \partial \theta \rightarrow 0$, the hybrid field becomes insensitive to the parameter, which corresponds to a scenario where observation values are prescribed throughout the structure, achievable when $\alpha \rightarrow \infty$ and sensors are densely distributed everywhere. Second, when $\partial \hat{U} / \partial \theta \rightarrow \partial U / \partial \theta$, the hybrid field coincides with the physical field due to the absence of observational data, either because no sensors are placed or $\alpha = 0$.

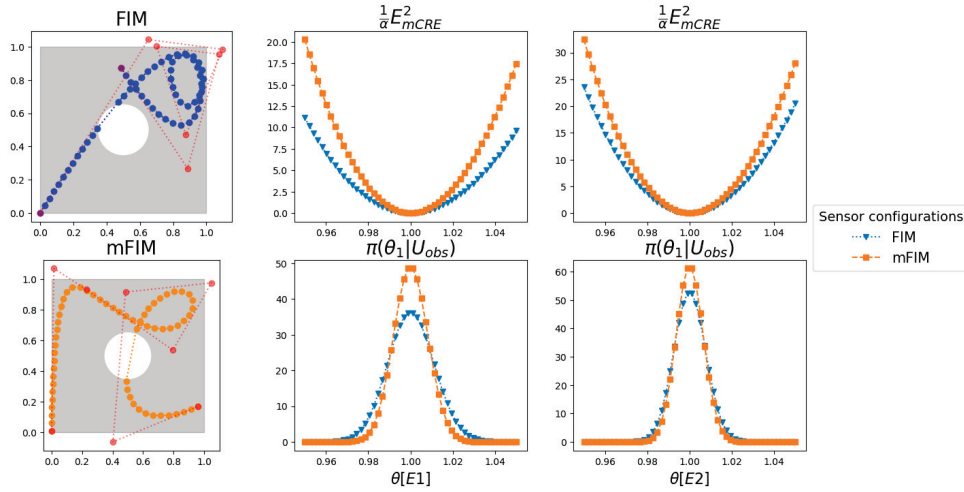


Figure 3: mCRE and posterior distribution curves

Table 1: Improvement in Hessian of the mCRE (compared to the sensor configuration obtained using the FIM)

Improvement using mFIM	
E_1	E_2
45 %	27 %

5.1 Introduction of modeling error

So far, the data has been generated numerically by adding Gaussian noise to the finite element response. However, the mCRE allows incorporating modeling errors, such as discretization errors or deviations in the constitutive law. To account for discretization errors, two different meshes are used: a coarse mesh for the model and a finer mesh, better suited to capture stress concentrations near the hole, for generating the data (Figure 4). The noise level remains at 5%.

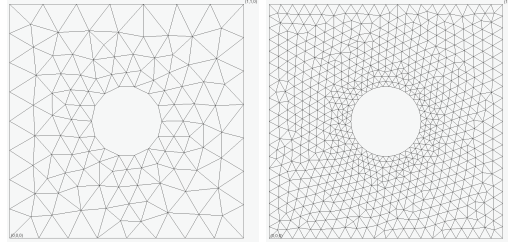


Figure 4: Coarse mesh used for the model and fine mesh used to generate the data

Figure 5 compares the optimized sensor placements using FIM and mFIM, along with their corresponding posterior distributions. The fiber configuration optimized using the classical FIM remains unchanged, as it does not account for modeling errors. In contrast, the modified FIM once again yields a superior sensor placement for the mCRE, effectively reducing the variance in the identified parameters. This improvement, quantified via the Hessian of the mCRE, is detailed in Table 2. Notably, the optimal parameter estimate is no longer centered at 1, as the model is now fitted using a coarser mesh. However, the mFIM successfully identifies a sensor configuration that shifts the estimate closer to the true value of 1.

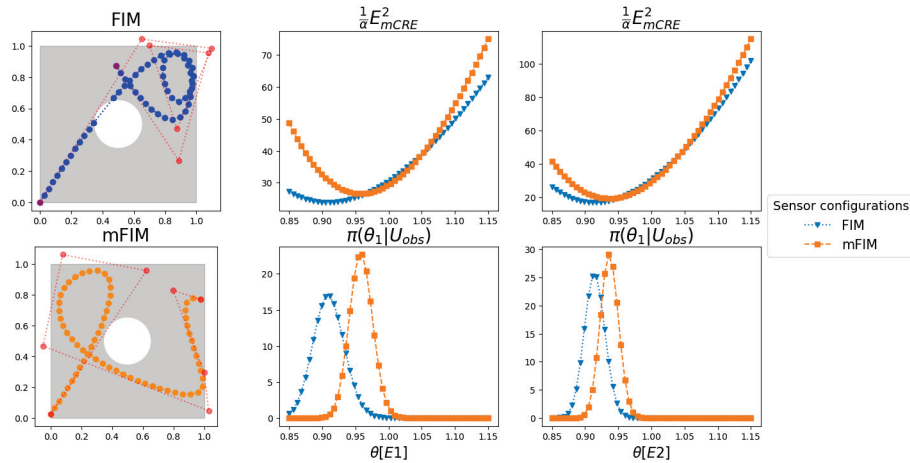


Figure 5: mCRE and posterior distribution curves

Table 2: Improvement in Hessian of the mCRE using different meshes (compared to the sensor configuration obtained using the FIM)

Improvement using mFIM	
E_1	E_2
34 %	17 %

6 Conclusions

This work leads to several key conclusions. For parameterizing the optical fiber in a 2D plane, B-Splines prove efficient, requiring only control point coordinates—unlike other methods [8, 2], which also demand derivative values for interpolation.

For optimal sensor placement, metrics based on the Fisher Information Matrix (FIM) are reliable, as they directly relate to the variance of identified parameters and depend on sensor configuration. The modified FIM (mFIM), introduced in this work (see Eq. 14), serves as a tailored performance indicator for the modified Constitutive Relation Error (mCRE). It also allows direct integration of modeling errors and measurement noise into the placement strategy. While the classical FIM is a local indicator, the mFIM captures the global sensitivity of the mCRE fields.

The main drawback of the mFIM is the computational cost of evaluating the admissible fields $\hat{U}$ for each sensor configuration. However, this can be mitigated through parallelizable optimization algorithms like differential evolution, model order reduction techniques, and improvements in solving the underlying linear systems. For instance, isolating sensor-dependent terms via the Sherman-Morrison-Woodbury formula [24] offers a promising direction. These aspects are subjects of ongoing research.

While this study focused on the theoretical and numerical foundations of the mFIM for sensor placement, future work will explore its application to real-world composite monitoring for damage detection. This includes addressing physical constraints such as manufacturing variability and sensor integration during layup, as well as defining operational limits, e.g., maximum detectable damage size, strain resolution, and sensor survivability, to enable a full SHM solution for advanced composite structures.

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