

Vibration-Based Robust Detection of Gear Incipient Faults under Different Speeds and Loads via a Cloud of Functional Models

Dimitrios M. Bourdalos, John S. Sakellariou

Abstract. The detection of incipient faults in a gearbox operating under different speeds and loads is investigated based on a novel unsupervised statistical time series method using a limited set of vibration signals from a single accelerometer. The method employs a cloud of Vector Functionally Pooled AutoRegressive (VFP-AR) models for the representation of the healthy gearbox dynamics at any speed and load within the considered ranges. Their estimation is achieved using vibration signals from the healthy gearbox after a specific angular resampling procedure removing the shaft speed fluctuations. The method's performance is systematically assessed through thousands of experiments on a single-stage spur gearbox, considering two levels of incipient single-tooth pinion faults under 41 distinct rotating speeds and 4 load levels. The results demonstrate near-perfect fault detection, even under speeds and loads not encountered in training, while the method outperforms a conventional multiple-model AR based alternative.

Key words: Spur Gear Faults, Gearbox, Vibration-based Fault Detection, Functional Models, Statistical Time Series Methods, Different Speeds and Loads

1 INTRODUCTION

The reliability of mechanical systems in industrial applications heavily depends on the health of gearboxes, which are essential for torque transmission and speed regulation. Early detection of gear faults is crucial for preventing unexpected and extensive failures, thus minimizing maintenance costs, reducing downtime and ensuring reliability [1]. Vibration-based Condition Monitoring (CM) approaches have become increasingly popular due to their capability to continuously assess machine health in real time without requiring operational downtime. However, in various industrial applications significant challenges arise from the need for variable operating conditions (OCs), such as different rotating speeds and loads, as well as the presence of measurement noise and other uncertainties. These factors may mislead a CM system and degrade diagnostic accuracy, making fault detection in gearboxes a particularly demanding task.

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This necessitates the development of robust methodologies capable of maintaining high detection performance under varying OCs and uncertainty[1].

Vibration-based methods for fault detection under varying conditions in rotating machinery may be categorized as: *non-parametric* and *parametric*. Non-parametric methods are based on features extracted from the vibration signals, such as the signal's RMS, kurtosis, crest factor and so on in the time domain, or spectral energy features in the frequency domain, or time-frequency representations (e.g. spectrogram) to characterize gearbox health [2, 3]. Furthermore, signal decomposition methods, including Wavelet Packet Decomposition (WPD) and Empirical Mode Decomposition (EMD), have been employed to enhance feature extraction by isolating fault-related signal components [2, 4, 5, 6, 7]. The extracted features are often used as inputs to machine learning classifiers such as Support Vector Machines (SVMs) [3] or k-Nearest Neighbors (k-NN) [4], while Deep Neural Networks have also been explored for automating feature extraction [8, 9, 10]. Despite their effectiveness, such approaches generally require extensive labeled datasets encompassing both healthy and faulty conditions, which may not always be available in practice. Furthermore, their performance often depends on prior knowledge of the specific fault types considered and their effects on the vibration characteristics of the signals.

In contrast, *parametric* methods tackle the problem by initially applying filtering and noise reduction techniques, such as the well known Time Synchronous Averaging. Then, parametric AutoRegressive with eXogenous input (ARX) type models are employed to represent the system's underlying dynamics. Fault detection is then performed through residual analysis, where deviations from the expected model behavior are assessed using statistical moments like variance and kurtosis. Various parametric models including AutoRegressive (AR) [11, 12], ARX [13], and Vector AR [14, 15], have been explored for detecting gear faults. However, these studies focus only on load variations while assuming a constant rotating speed and are typically assessed with run-to-failure experiments, with no emphasis on early-stage fault detection.

It is important to highlight that the effectiveness of incipient fault detection using both categories of methods has, so far, been assessed under a restricted set of different OCs. In contrast, the challenge of fault detection across a broad range of speeds and loads that change simultaneously remains largely unexplored. This gap is particularly significant, as variations in both speed and load typically occur concurrently in industrial applications.

The *goal* of this study is the examination of the potential for unsupervised detection of incipient single-tooth spur gear fault under different speeds and loads through an extension of a recently presented statistical time series method [16, 17]. The extension includes the use of Vector Functionally Pooled (VFP) models of the AR type, instead of scalar ones, incorporating thus both rotating speed and load. In addition, to address the challenges mentioned above and achieve accurate modeling of the gearbox dynamics under different loads within a continuous and wide range of speeds, the concept of employing a 'cloud' of VFP models representing the healthy dynamics is adopted and combined with the angular resampling of the vibration signals via the Computed Order Tracking approach [18, pp. 148–151]. Each VFP model of the cloud may have a different structure and functional subspace covering a specific sub-range of rotating speeds, while accounting for the effects of all considered loads on the dynamics. The VFP

models are estimated using vibration signals from a single accelerometer, acquired from a sample of the considered speeds (in each specific range) and loads with the healthy gearbox. Fault detection is performed through an optimization framework that explores the consistency of the currently available vibration signal with the VFP models of the ‘healthy cloud’. The method’s performance is systematically assessed through thousands of experiments with a single-stage spur gearbox operating under 41 different rotating speeds and 4 different loads. Two levels of an incipient fault at the base of a single pinion tooth are considered. The results and comparisons with an alternative method, which is based on multiple conventional AR models, are presented via scatter type plots of the fault detection characteristic quantity and a confusion matrix [19].

2 THE EXPERIMENTAL PROCEDURE

2.1 Gearbox, gear fault scenarios, and measurements

The experimental set-up (Fig. 1) consists of a single-stage spur gearbox driven by an AC electric motor and loaded by a DC motor. The gearbox features a pinion with 17 teeth and a gear with 34 teeth. The drive motor operates at 41 distinct speeds, ranging from 15 Hz (1200 rpm) to 25 Hz (1500 rpm) in increments of 0.25 Hz (15 rpm), regulated by a standard variable frequency drive (inverter). The load motor operates as a generator, allowing adjustable loading conditions for the gearbox. Four different load scenarios are implemented: For Load 0, the load motor is detached from the gearbox. For Load 1, the load motor is attached to the gearbox but remains unloaded (no devices are connected to the generator’s outputs). In Load 2, the load motor is attached to the gearbox and a 500-watt device is connected. Finally, in Load 3, the load motor is attached to the gearbox with a 1000-watt device connected. An incipient fault is introduced at the base of a single pinion tooth in the gearbox using a standard Dremel-type cutting tool. The fault scenarios are implemented via two distinct severity levels each defined as a percentage of the total tooth face width (w) affected (see Fig. 1). The first level corresponds

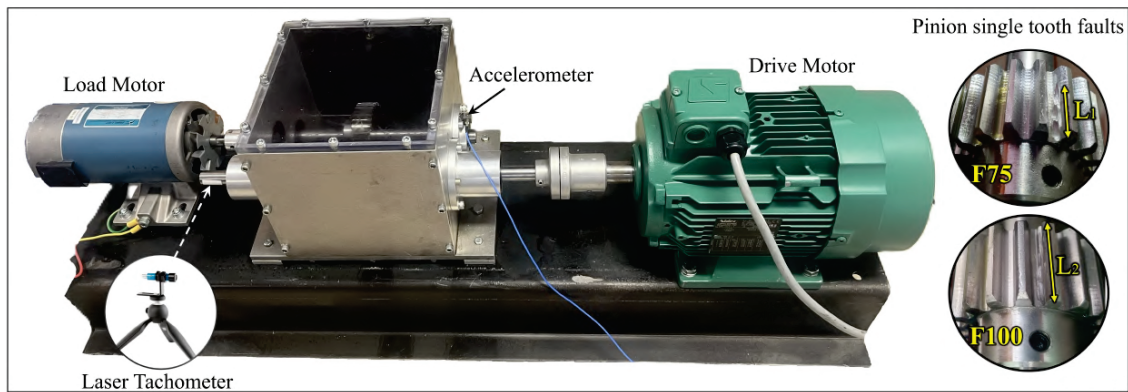


Figure 1: The experimental set-up: Photo of the gearbox including the sensors (accelerometer and tachometer) locations and the pinion single-tooth fault scenarios $F75$ and $F100$.

Table 1: Details on the vibration signals used for the training and assessment of the fault detection methods.

Gearbox state	Rotating speed (Hz)	Load	No. of signals per speed & load	No. of signals per state
Training Phase				
Healthy (H)	$\{15, 16, \dots, 25\}$	$\{0, 1, 3\}$	1	33
Inspection Phase				
$H / F75 / F100$	$\{15, 15.25, \dots, 25\}$	$\{0, 1, 2, 3\}$	15	7 380
Sampling frequency: $f_s = 10\,240$ Hz; Signal length: $N = 51\,200$ samples (5 s);				
Frequency bandwidth: $BW = [0 - 5\,120]$ Hz; Total No. of signals = 7 413				

to 75% of the tooth face width affected ($L_1 = 0.75w$) while the second to 100% ($L_2 = w$). It is worth noting that the faults have been introduced on the gears inside the gearbox without disassembly, thus avoiding additional uncertainties.

The vibration signals (details in Table 1) are acquired using a single accelerometer placed on the ball bearing housing of the secondary shaft (see Fig. 1), with a sample frequency of $f_s = 10\,240$ Hz. Additionally, a laser tachometer measures the rotating speed of the drive motor. A total of 7 413 signals are recorded, with only 33 of them (0.45% of the complete dataset) from the healthy gearbox to be employed for the training of the fault detection method. The remaining 7 380 signals (99.55%) are exclusively used in the inspection phase for the method's assessment and comparison.

2.2 Effects of different operating conditions and incipient faults on the vibration signals

The effects of the different rotating speeds, loads, and gear faults on the vibration signals are initially studied in the time domain using standard features, commonly employed for fault detection in rotating machinery [4]. These include signal RMS, peak, kurtosis and crest factor and are obtained from 164 vibration signals per health state with each corresponding to a unique combination of rotating speed and load level. Their values are presented in Fig. 2, where distinct colors and markers denote the different health states, while the x-axis indicates the signal number and the y-axis the corresponding feature value. As shown, the feature values for both faulty and healthy states exhibit significant overlap, highlighting the challenge of detecting incipient faults in the considered problem.

Furthermore, the effects of the different rotating speeds, loads, and faults are investigated in the order domain by constructing order spectrum zones via the Fast Fourier Transform (FFT). These zones are obtained using the above mentioned 164 angular vibration signals associated with each health state. The signals angular resampling is performed based on order tracking with a common frequency for all considered rotating speeds; refer to Section 3.1 for further details. The spectrum zones of the healthy state with those of the two fault scenarios 75% ($F75$) and 100% ($F100$) are compared in Figs. 3(a) and (b), respectively. Evidently, fault detection proves to be particularly challenging, as the variations in rotating speeds and loads influence the healthy dynamics in a way that obscures the fault-induced effects, causing the faulty spectrum

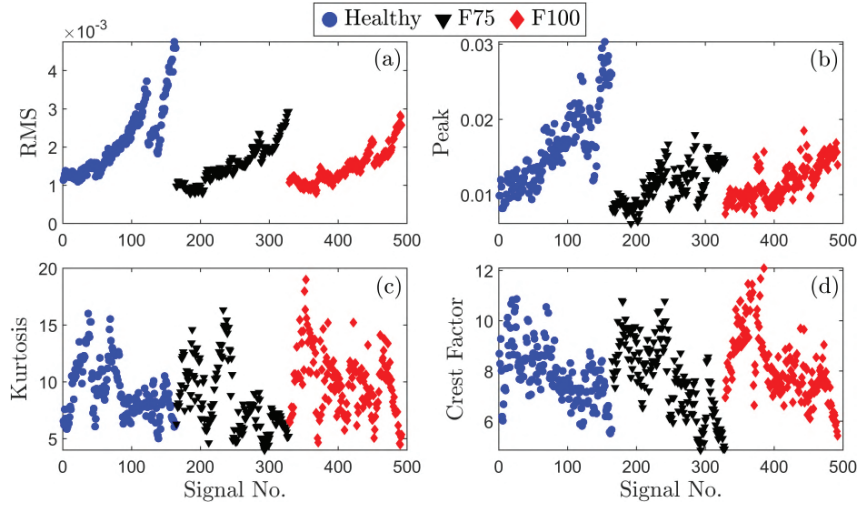


Figure 2: Time domain features corresponding to the Healthy and Faulty ($F75$ and $F100$) states under all 41 speeds and 4 loads: (a) RMS, (b) Peak, (c) Kurtosis, and (d) Crest Factor. Each color indicates a different health state; 492 signals per subplot: 164 per health state, 1 per speed and load combination.

zones to almost entirely overlap with the healthy ones within the considered bandwidth.

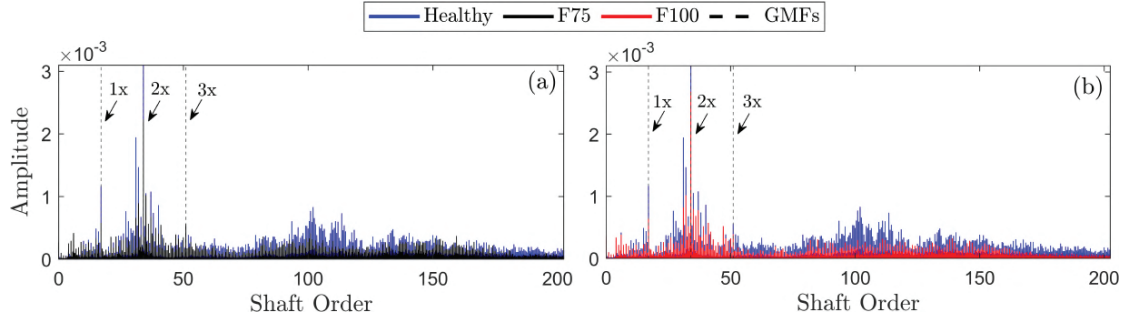


Figure 3: Order spectrum zones based on Fast Fourier Transform (FFT) amplitude estimates using a single angular vibration signal from each rotating speed and load (84 angular signals for each health state): (a) Healthy vs fault $F75$, (b) Healthy vs fault $F100$; The black dashed lines represent Gear Meshing Frequencies (GMFs).

3 THE VFP-AR MODEL CLOUD BASED FAULT DETECTION METHODOLOGY

The fault detection methodology consists of two phases: a *Training phase* and an *Inspection phase*. The *Training phase* operates offline and includes angular resampling of the vibration signals for different nominally constant rotating speeds (Section 3.1), as well as the identification of the VFP-AR ‘cloud’ of models representing the healthy gearbox dynamics (Section 3.2). The *Inspection phase* is performed in real-time and involves fault detection based on the VFP-AR model cloud (Section 3.3).

3.1 Angular resampling for different nominally constant rotating speeds

The first step of the methodology involves angular resampling of vibration signals based on Computed Order Tracking [20] using the tachometer signal in order to eliminate non synchronous frequency modulation in the time domain signals. This modulation arises primarily due to fluctuations in shaft speed, even when the machinery inverter is set at a nominally constant speed [20]. Based on this and to retain all frequency content up to the Nyquist limit as obtained in the time domain, the sampling frequency in the angular domain (or θ -domain) is selected as $f_s^\theta = f_s/\omega$, where f_s denotes the sampling frequency in the time domain and ω the nominal rotating speed measured in revolutions per second (rev/s). However, the different f_s^θ per rotating speed prevents the gearbox dynamics modelling in a unified way across all different rotating speeds, as a single, common, f_s^θ should be adopted. The selection of this frequency is dictated by the maximum rotating speed ω_{max} of the machinery, resulting in an operational bandwidth of $[0, \frac{f_s}{2\omega_{max}}]$ in θ -domain. Based on this, it is evident that for any speed $\omega < \omega_{max}$, the resulting frequency bandwidth in time domain should include frequencies in the range $[0, \frac{f_s^\theta}{2} \times \omega]$ Hz, which is narrower than the original measurement bandwidth that extends up to $f_s/2$ Hz. This indicates the need for low-pass filtering of all vibration signals at any $\omega < \omega_{max}$ before angular resampling, using a cut-off frequency of $\frac{f_s^\theta}{2} \times \omega$, in order to avoid signal distortion due to aliasing.

As part of this filtering process, higher-frequency components are lost, especially as rotating speed decreases. To limit this loss of information, the ratio between the maximum and minimum speeds should be constrained within a practical range [18, pp. 156–159]. Thus, n adjacent sub-ranges that cover the entire considered range of rotating speeds are selected in this study, determined based on the highest frequency component that must be retained in each range. This step is crucial to the presented methodology, as it allows the modelling of the gearbox dynamics through a ‘cloud’ of Functionally Pooled models, as described in the following.

3.2 VFP-AR model based gearbox dynamics identification

Once the n adjacent speed sub-ranges have been determined, the objective of this step of the methodology is the modelling of the healthy gearbox dynamics at any speed $\omega \in [\omega_{min}^i, \omega_{max}^i]$ ($i = 1, \dots, n$) and load $l \in [l_{min}, l_{max}]$ for each range using a Vector Functionally Pooled AutoRegressive (VFP-AR) model [21]. Thus, a ‘cloud’ of n VFP-AR models, collectively representing the healthy gearbox dynamics for the entire considered speed range and all loads, is estimated, with each model expressed in θ -domain as follows (a functional argument in brackets $[\cdot]$ designates function of an integer variable; a lowercase boldface symbol represents vector):

$$y_{\mathbf{k}}[\theta] + \sum_{i=1}^{n_a} a_i(\mathbf{k}) \cdot y_{\mathbf{k}}[\theta - i] = e_{\mathbf{k}}[\theta] \quad (1)$$

$$a_i(\mathbf{k}) = \sum_{j=1}^p a_{i,j} \cdot G_j(\mathbf{k}), \quad e_{\mathbf{k}}[\theta] \sim \text{iid } \mathcal{N}(0, \sigma_e^2(\mathbf{k})), \quad \mathbf{k} \in \mathbb{R}^2 \quad (2)$$

with $y_k[\theta]$ designating the angular vibration signal, $\theta (= 1, \dots, N)$ equispaced samples after angular resampling, n_a the AR model order, $e_k[\theta]$ model residual signal, while iid stands for independent and identically distributed, and $\mathcal{N}(\cdot, \cdot)$ for normal (Gaussian) distribution with the indicated mean and variance. The operating parameter vector $\mathbf{k}$ is defined as $\mathbf{k} = [\omega_v \ l_m]^T \equiv \mathbf{k}_{v,m}$, where $v = 1, \dots, M_1$ and $m = 1, \dots, M_2$, with M_1 the number of different speeds within a sub-range and M_2 the number of different loads.

Based on Eq. (2), the AR parameters $a_i(\mathbf{k})$ are functions of $\mathbf{k}$ belonging to a p -dimensional functional subspace spanned by the (mutually independent) functions $G_1(\mathbf{k}), G_2(\mathbf{k}), \dots, G_p(\mathbf{k})$ (functional basis). The latter are bivariate (two variables) orthogonal polynomials forming a functional basis. VFP-AR model identification includes: (i) determination of the VFP-AR model order n_a and its functional subspace dimensionality p for a given basis function family (orthogonal polynomials such Legendre, Chebyshev may be equivalently used) using a Genetic Algorithm (GA) for the minimization of the Bayesian Information Criterion (BIC), (ii) model validation via testing of the obtained model residual signals whiteness hypothesis and, (iii) model estimation based on Ordinary Least Squares (OLS) with data pooling [21] using $M_3 = M_1 \times M_2$ vibration signals from the healthy gearbox. It is noted that each VFP-AR model of the ‘Healthy Cloud’ may have a different AR order and functional subspace.

3.3 VFP-AR model based fault detection

Once a new vibration signal $y_u[t]$ is acquired from the gearbox under unknown health state and load, it is initially angularly resampled as previously described using the corresponding tachometer signal, yielding $y_u[\theta]$. Subsequently and based on the machinery rotating speed, $y_u[\theta]$ is driven through the VFP-AR(n_a) model from the ‘Healthy Cloud’ corresponding to the speed range in which the current speed belongs, and Eq. (1) is written as:

$$y_u[\theta] + \sum_{i=1}^{n_a} a_i(\mathbf{k}_{\omega,l}) \cdot y_u[\theta - i] = e_u[\theta, \mathbf{k}_{\omega,l}] \quad (3)$$

where the load in $\mathbf{k}_{\omega,l}$ is unknown. If $y_u[\theta]$ originates from the healthy machinery under any of the considered loads, this signal will be consistent with the employed VFP-AR model. This consistency is examined through the estimation of the load in $\mathbf{k}_{\omega,l}$ based on the following Non-linear Least Squares (NLS) estimator:

$$\hat{l}_u = \arg \min_{l \in K} \sum_{\theta=1}^N e_u^2[\theta, \mathbf{k}_{\omega,l}], \quad \hat{\sigma}_{e_u}^2 = \frac{1}{N} \sum_{\theta=1}^N e_u^2[\theta, \mathbf{k}_{\omega,\hat{l}_u}] \quad (4)$$

with $e_u[\theta, \mathbf{k}_{\omega,l}]$ provided by Eq. (3) and $K = [l_{\min}, l_{\max}]$ the range of the examined loads into which the machinery normally operates. If the current dynamics originates from the healthy gearbox then $e_u[\theta, \mathbf{k}_{\omega,\hat{l}_u}]$ should be a white (uncorrelated) signal which may be examined via any typical whiteness testing procedure; herein the Pena-Rodriguez D statistic which follows a standard normal distribution for a white sequence [22] is employed. If $e_u[\theta, \mathbf{k}_{\omega,\hat{l}_u}]$ is white this means that $y_u[\theta]$ originates from the healthy gearbox operating under $\mathbf{k} = [\omega \ \hat{l}_u]^T$ speed and

load, otherwise the gearbox is declared as faulty. It is noted that in case of a healthy declaration, an interval estimate of the current operating load may also be provided to the user [23].

4 EXPERIMENTAL RESULTS

The considered rotating speed range of $[15, 25]$ rev/s is divided into $n = 2$ adjacent sub-ranges, $[15, 20]$ rev/s and $[20, 25]$ rev/s, as described in section 3.1. Based on the maximum speeds of these ranges, the corresponding angular resampling frequencies are $f_s^\theta = 512$ samples/rev for the first sub-range and $f_s^\theta = 409$ samples/rev for the second. Furthermore, a Chebyshev type II low-pass filter is applied to the vibration signals of all considered $\omega < 20$ in the first range and $\omega < 25$ in the second, before angular resampling, avoiding signals aliasing.

Consequently, the healthy gearbox dynamics within the continuous rotating speed $[15, 25]$ rev/s and load level $[0, 4]$ ranges, is represented by a ‘Healthy Cloud’ consisting of $n = 2$ VFP-AR models, estimated using the 33 acceleration signals of the baseline that correspond to a sample of rotating speeds and load levels from the considered ranges (see Table 1). All estimation details are shown in Table 2.

The performance of the VFP-AR model cloud fault detection method is assessed in the inspection phase based on 2 460 signals from the healthy gearbox and 2 460 from each fault scenario, with the gearbox operating under 41 different speeds and 4 different load levels (see Table 1). The tachometer signal is also acquired during each vibration measurement in this phase, while the health state of the gearbox and the load level is considered as completely unknown. The D statistic values of the method for all considered inspection phase signals (7 380) are presented in Fig. 4. Based on these, it is evident that signals corresponding to the healthy gearbox are almost fully separated from those obtained from the considered faults indicating

Table 2: Estimation details for the cloud of VFP-AR models.

Model No.	Speed range (rev/s)	f_s^θ (samples/rev)	Model	Samples Per Parameter	Condition Number
1	$[15, 20]$	512	VFP-AR(250) ₁₃	211.85	$1.44 \cdot 10^5$
2	$[20, 25]$	409	VFP-AR(250) ₁₁	261.07	$4.87 \cdot 10^5$

Estimation method: Ordinary Least Squares

Functional subspace dimensionality determination based on Genetic Algorithm: population = 100, elite count = 20, crossover fraction = 0.7, maximum number of generations = 1000, Tolerance of the objective function = 10^{-10} .

Functional basis: bivariate Shifted Legendre polynomials:

$$\mathcal{G}^1 = [G_{0,0}(\mathbf{k}), G_{0,1}(\mathbf{k}), G_{0,2}(\mathbf{k}), G_{0,3}(\mathbf{k}), G_{0,4}(\mathbf{k}), G_{1,0}(\mathbf{k}), G_{1,1}(\mathbf{k}), G_{1,2}(\mathbf{k}), G_{1,3}(\mathbf{k}), G_{2,0}(\mathbf{k}), G_{2,2}(\mathbf{k}), G_{3,2}(\mathbf{k}), G_{3,3}(\mathbf{k})]$$

$$\mathcal{G}^2 = [G_{0,0}(\mathbf{k}), G_{0,1}(\mathbf{k}), G_{0,2}(\mathbf{k}), G_{0,3}(\mathbf{k}), G_{0,4}(\mathbf{k}), G_{1,0}(\mathbf{k}), G_{1,1}(\mathbf{k}), G_{1,2}(\mathbf{k}), G_{1,4}(\mathbf{k}), G_{2,1}(\mathbf{k}), G_{3,3}(\mathbf{k})]$$

$G_{i,j}(\mathbf{k})$: bivariate orthogonal polynomial of total degree $i + j$, obtained as tensor product from two univariate polynomials $G_i(x_1)$, $G_j(x_2)$ of degrees i , j , respectively.

nearly-perfect fault detection performance.

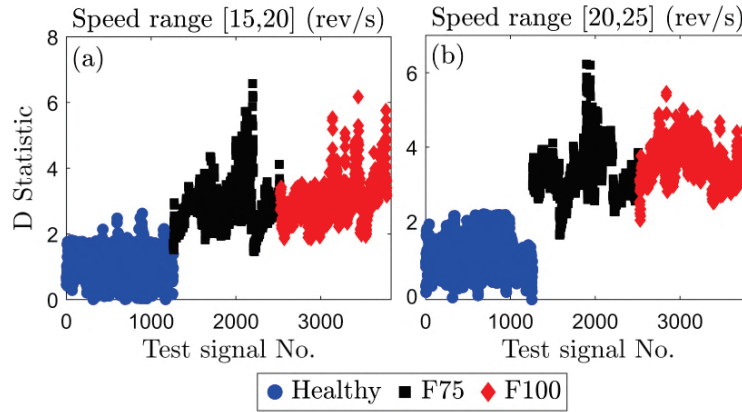


Figure 4: Fault detection results via the Pena-Rodriguez D statistic of the VFP-AR model cloud methodology: (a) 1st sub-range of speeds [15, 20] (rev/s) and (b) 2nd sub-range of speeds [20, 25] (rev/s); 7 380 signals including 41 rotating speeds and 4 load levels (inspection phase).

Finally, a comparison with the Multiple Model AutoRegressive (MM-AR) method [24], is presented. This method represents the healthy gearbox dynamics through a set of multiple AutoRegressive (AR) models, each associated with a distinct rotating speed and load level. During the training phase, AR models of the same order $n_a = 250$, are individually identified based on the 33 signals of the healthy gearbox (Table 1). In the inspection phase, a new AR model of the same order is estimated based on a fresh vibration signal from the gearbox under unknown health state and load. This model's similarity to the healthy reference models is quantified via the Mahalanobis distance. If it remains below a predetermined (in training phase) threshold, the gearbox condition is classified as healthy otherwise as faulty.

The fault detection results are comparatively presented in Fig. 5 via a typical Confusion Matrix. In the upper left 2×2 submatrix, each column represents a true health state and each row an estimated state. The i, j -th cell shows the ratio of the actual i -th state signals that were misclassified as the j -th state. Correct classifications appear on the diagonal, while errors are off-diagonal. The rightmost column reports the percentages of correctly (green) and incorrectly (red) classified signals within each predicted class, whereas the bottom row gives these percentages relative to the actual health state. Finally, the bottom-right cell summarizes the overall correct detection (green) and false alarm (red) rates; for a more detailed description, see [25].

Evidently, the VFP-AR model cloud method's nearly-perfect results are verified, underlying the detection of the healthy state (H) for 99.0% of the considered test cases, with an impressively low false alarm ratio of 1.0%. Furthermore, it accurately detects the considered incipient faults (F: stands for both fault scenarios) at 98.6%, resulting in an overall detection rate of 98.7% (rightmost bottom cell). Moreover, its superiority over the MM-AR based method is evident, which achieves a lower rate of 86.5%. This pinpoints the limitation of the MM-AR based method to operate at different values of rotating speeds and loads other than those used for its

		(a) VFP-AR model cloud			(b) MM-AR		
		H	F		H	F	
Estimated State	H	2436/2460 33.0%	71/4920 1.0%	97.2% 2.8%	1488/2460 20.2%	22/4920 0.3%	98.5% 1.5%
	F	24/2460 0.3%	4849/4920 65.7%	99.5% 0.5%	972/2460 13.2%	4898/4920 66.4%	83.4% 16.6%
		99.0% 1.0%	98.6% 1.4%	98.7% 1.3%	60.5% 39.5%	99.6% 0.4%	86.5% 13.5%
		True State			True State		

Figure 5: Fault detection comparative assessment via confusion matrices: (a) the VFP-AR model cloud method and (b) the MM-AR method (correct classifications indicated by green – misclassifications by red; 7 380 inspection signals).

training. This happens as the MM-AR based method approximates the healthy dynamics using distinct AR models in contrast to the VFP-AR model cloud based method where the healthy dynamics is modelled for any value within the continuous ranges of the considered OCs.

5 CONCLUSIONS

The robust detection of two severity levels of incipient single-tooth gear fault in a gearbox operating under 41 distinct rotating speeds and 4 distinct load levels has been explored using an unsupervised vibration-based statistical time series methodology. This methodology relies on angular vibration signals and a cloud of Vector Functionally Pooled AR (VFP-AR) models that represent the healthy gearbox dynamics at any rotating speed and load within the range of interest. Fault detection is performed through an optimization framework that assesses the consistency of the current vibration signal with the models in the cloud.

Based on the fault detection results obtained from 7 380 experiments, it is concluded that:

- (i) The method achieves nearly perfect fault detection, with a 98.7% detection rate and only 1.3% false alarms, outperforming an alternative MM-AR based method.
- (ii) The method can detect incipient faults without prior knowledge of their nature or their effects on specific vibration characteristics. This is achieved using only 33 vibration signals (0.45% of the entire database) for its training, collected from the healthy gearbox for a sample of speeds and loads from the considered ranges.

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