

Effect of topological hybridization on the elastic isotropy of three-dimensional lattice metamaterials

Mohamed M. Naji, Rashid K. Abu Al-Rub, Imad Barsoum, Wael Zaki

Abstract. Three-dimensional (3D) lattice metamaterials, derived from triply periodic minimal surfaces (TPMS), or plate lattices, exhibit tuneable mechanical properties, with their elastic anisotropy and stiffness governed by their underlying topology. In this study, the effect of topological hybridization on the elastic isotropy of TPMS-based lattice structures are explored. Specifically, hybrid lattices formed by combining Diamond and Simple Cubic (DSC) as well as Primitive and Simple Cubic (SPSC) architectures are investigated. Finite element simulations are conducted to analyze the directional elastic moduli and visualize the elastic surfaces. The results indicate that the extent of isotropy is strongly dependent on the parent lattice topologies, with certain hybrid configurations failing to resolve anisotropy while still enhancing stiffness compared to their parent structures. This suggests that hybridization enables a tailored approach to mechanical property optimization. Additionally, we compare the isotropy of hybrid structures to their constituent topologies, providing further insight into the trade-offs and advantages of geometric blending. These findings contribute to the rational design of architected materials for load-bearing and multifunctional applications.

Key words: Metamaterials, Isotropy, Triply Periodic Minimal Surfaces, Hybrid lattices.

1 INTRODUCTION

Three-dimensional (3D) lattice metamaterials, particularly those derived from triply periodic minimal surfaces (TPMS), have gained significant attention due to their tunable mechanical properties and structural efficiency [1–7]. These metamaterials offer a unique combination of high strength-to-weight ratio, isotropic or anisotropic behavior, and energy absorption capabilities, making them ideal for various engineering applications, including aerospace, biomedical implants, and lightweight structural components [8–15].

While TPMS-based architectures exhibit promising mechanical properties, the elastic anisotropy of sheet-based TPMS such as the Diamond (D) and Schwarz-Primitive (SP) remains

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a key limitation in certain applications requiring uniform mechanical response. To address this, topological hybridization, a strategy that blends two distinct lattice topologies, has been proposed as a means of enhancing mechanical performance and achieving improved isotropy. However, the influence of such hybridization on the elastic properties and anisotropy of TPMS lattices remains underexplored.

This study investigates the effect of topological hybridization on the elastic isotropy of TPMS-based lattice structures. In this study, the Simple Cubic (SC) plate lattice was selected for hybridization due to its superior stiffness and significant contribution to the load-bearing capacity of hybrid materials [2,3,16–18]. Two hybrid lattice configurations, Diamond-Simple Cubic (DSC) and Primitive-Simple Cubic (SPSC), are analyzed. Finite element simulations are conducted to evaluate the directional elastic moduli, isotropy, and stiffness of these hybrid structures. The results provide insight into the trade-offs between stiffness enhancement and anisotropy reduction, contributing to the rational design of architected materials for load-bearing and multifunctional applications.

2 MATERIALS AND METHODS

2.1 Topological Design

The TPMS topologies considered in this study, being the Schwarz-Primitive and Diamond minimal surfaces, are generated using their respective mathematical formulations. These surfaces are defined by level-set functions, which describe the implicit surfaces where a scalar field remains constant. These iso-surfaces correspond to specific iso-values known as the level-set parameter c . The mathematical expressions for the selected TPMS topologies are given by:

$$\phi_{SP} = \cos x + \cos y + \cos z = c \quad (1)$$

$$\phi_D = \cos x \cos y \cos z - \sin x \sin y \sin z = c \quad (2)$$

where $x = 2\pi X/L_x$, $y = 2\pi Y/L_y$, and $z = 2\pi Z/L_z$, in which L_x , L_y , L_z represent the unit cell sizes in the X, Y, and Z directions in Cartesian coordinates [1]. The level-set constant c controls the offsetting of the level-set surface from the zero value (i.e. $\phi = 0$), where the zero value results in minimal surfaces that are equally split into subdomains of equal volumes. This formulation ensures that each TPMS unit cell tessellates periodically in three-dimensional space. The level-set expressions in Eqs. (1-2) are incorporated in the MSLattice software [19], which was utilized in this study to generate the TPMS lattices with sheet networks. The generated TPMS unit cells can then be utilized in construction of the hybrid plate-TPMS lattice, achieved by structurally integrating the SC plate lattice with TPMS architectures, portrayed in Figure 1. In this study, the parent and resultant hybrid lattices are fixed at 20% relative density, with a unit cell size of 1 mm. The relative density of the parent SC and TPMS lattices were controlled by adjusting the wall thicknesses, in which the SC plate-lattice features a 0.072 mm wall thickness, adjusted using SolidWorks commercial software, and the sheet-based P and D lattices contain feature sizes of 0.0987 mm and 0.055 mm, respectively.

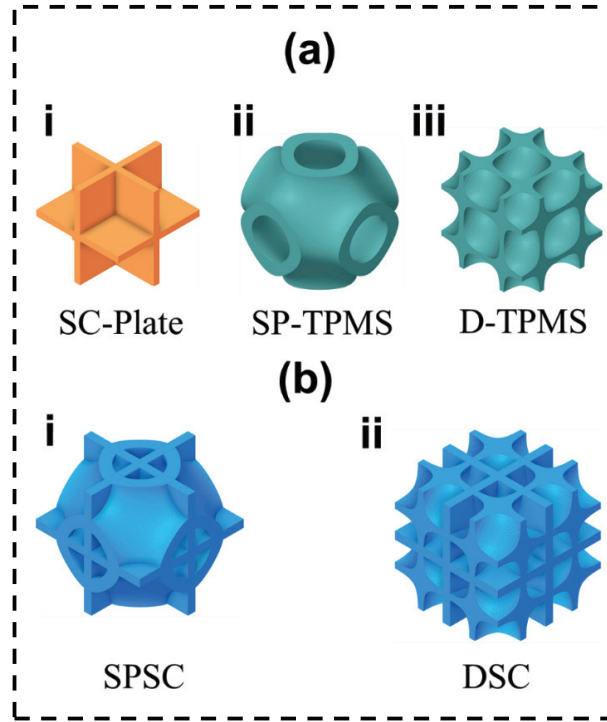


Figure 1 Unit cells of (a) (i) the elementary SC-plate and sheet-based TPMS (ii) SP and (iii) D lattices, and (b) the hybrid SC-TPMS (i) SPSC and (ii) DSC lattice metamaterials

2.2 Finite element methods

To determine the effective elastic properties, appropriate linear elastic constitutive relations that accurately represent the effective properties are necessary to be established first. Considering that the designed sheet-TPMS and hybrid unit cells preserve cubic symmetry, the macroscopic elasticity relation, in Voigt notation form, can be reduced to

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{12} \\ \sigma_{13} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ & C_{11} & C_{12} & 0 & 0 & 0 \\ & & C_{11} & 0 & 0 & 0 \\ \text{sym} & & & C_{44} & 0 & 0 \\ & & & & C_{44} & 0 \\ & & & & & C_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \end{bmatrix} \quad (3)$$

where σ_{ij} , C_{ij} , and ε_{ij} represent the stress, stiffness, and strain components, respectively. This is specifically accomplished using Hill's approximation [20], in which equivalent isotropic material parameters are utilized to represent the macroscopic elastic properties of polycrystalline materials. Hill's approximation serves as a representative modulus, aligning with the modulus value achieved through material testing in any direction when the material exhibits complete isotropy. For the calculation of elastic properties, encompassing

homogenized (or effective) Young's (E^H), shear (G^H), and bulk (B^H) moduli, the following expressions can be obtained using Hill's approximation as

$$B^H = \frac{B^V + B^R}{2} \quad (4)$$

$$G^H = \frac{G^V + G^R}{2} \quad (5)$$

$$E^H = \frac{9B^H G^H}{3B^H + G^H} \quad (6)$$

where superscripts V and R represent the Voigt and Reuss upper and lower bounds, respectively. In the context of a 3D representation of elastic properties, the well-known Voigt and Reuss shear and bulk moduli bounds can be determined using the components of C_{ij} and be reduced, due to cubic symmetry, as illustrated by the following equations [21–23]:

$$B^V = \frac{C_{11} + 2C_{12}}{3} \quad (7)$$

$$G^V = \frac{C_{11} - C_{12} + 3C_{44}}{5} \quad (8)$$

$$B^R = \frac{F}{3C_{11}^2 + 3C_{12}^2 - 6C_{11}C_{12}} \quad (9)$$

$$G^R = 15 / \left[\frac{4(2C_{11}^2 + C_{11}C_{12} + C_{11}^2 + C_{11}C_{12} + C_{11}C_{12} - 2C_{12}^2)}{F} + \frac{9}{C_{44}} \right] \quad (10)$$

$$F = C_{11}^3 + 2C_{12}^3 - 3C_{11}C_{12}^2 \quad (11)$$

The Reuss and Voigt bounds for shear and bulk moduli are also essential for determining the anisotropy of the cellular metamaterial, which can be assessed through using the Ranganathan and Ostoja-Starzewski elastic universal anisotropic index (A^U), expressed as [23,24]:

$$A^U = 5 \frac{G^V}{G^R} + \frac{B^V}{B^R} - 6 \geq 0 \quad (12)$$

It can be inferred from the above equations that in the case of a perfect isotropic behavior, where the Voigt and Reuss bounds for shear and bulk moduli would converge, the anisotropy index A^U values yield to 0.

Finite element analysis is then employed using the Abaqus/Standard 2023 commercial software. The generated TPMS and hybrid unit cells were first imported into Abaqus, which had previously been meshed using Hypermesh software, incorporating quadratic tetrahedral elements of type C3D10 (per Abaqus element notion), as tetrahedron elements are geometrically versatile and known to adequately define such complex shapes [25], as shown in Figure 2. Subsequently, the material model was defined with the assumption of isotropic behavior of the base material, where the constituent solid part of the cellular metamaterial possesses $E = 70$ GPa and $\nu_s = 0.3$ (i.e., assuming AlSi₁₀Mg as the base material). However, the determined elastic properties will be normalized by the elastic properties of the base material, making the results independent of the selected base solid material. The elastic properties (E_1, G_{12}, ν_{12}, B) were determined individually through the application of the mixed boundary conditions detailed in Table 1 [23], where $x_i, u_i, \mathbf{e}_i$, and $\mathbf{t}_i$ are Cartesian coordinates, displacements, unit direction vectors and traction force vectors, respectively, in $i = 1, 2, 3$ directions, while λ determines the magnitude of the applied displacement. The elastic moduli were obtained by computing the required force to cause the applied displacement. This relationship was converted to a stress-strain response, from which the elastic moduli were determined. These elastic components can then be utilized to obtain the materials constants that constitute the stiffness matrix $[C_{ij}]$, that are essential for Eqs. (3)-(12).

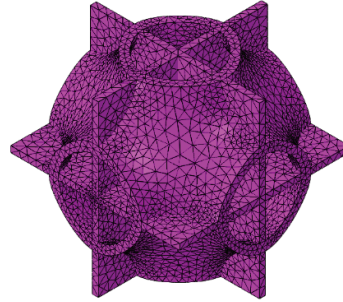


Figure 2 An example of a FEM mesh for the *hybrid SPSC* cellular metamaterial using *quadratic* tetrahedral elements.

Table 1 Applied mixed boundary conditions for elastic moduli calculations of cubic symmetry materials.

Modulus	$x_1=0$	$x_1=L$	$x_2=0$	$x_2=L$	$x_3=0$	$x_3=L$
$E_1=E_2=E_3$	$u_1=0$	$u_1=\lambda \mathbf{e}_1$	$u_2=0$	$\mathbf{t}_2=0$	$u_3=0$	$\mathbf{t}_3=0$
$G_{12}=G_{13}=G_{23}$	$u_2=0$	$u_2=\lambda \mathbf{e}_2$	$u_1=0$	$u_1=0$	$u_3=0$	$u_3=0$
	$u_3=0$	$u_3=0$	$u_3=0$	$u_3=0$		
B	$u_1=0$	$u_1=\lambda \mathbf{e}_1$	$u_2=0$	$u_2=\lambda \mathbf{e}_2$	$u_3=0$	$u_3=\lambda \mathbf{e}_3$

3 RESULTS AND DISCUSSIONS

This analysis involves the investigation of the SPSC and DSC hybrid lattices, aiming to find the normalized Hill's Young's (E/E_s), shear (G/G_s), and bulk (B/B_s) moduli, in addition to the

universal anisotropy index A^U , at a total relative density of 20%. These values are compared with the elastic properties' values of the parent lattices, as presented in Table 2.

Table 2 Normalized elastic moduli values of the elementary and hybrid lattices and their universal anisotropy index values

	Hill's Approximation			A^U
	B/Bs	G/Gs	E/Es	
SC	0.071	0.040	0.101	0.757
SP	0.065	0.028	0.073	1.126
D	0.067	0.030	0.077	0.116
SPSC	0.067	0.036	0.092	0.040
DSC	0.069	0.036	0.091	0.354

Generally, the incorporation of SC onto the TPMS sheets causes an improvement in Young's and bulk moduli. These observations are also supported by the normalized elastic anisotropy polar plots exhibited in Figure 3, in which Figure 3 (a) displays the anisotropy plot of the elementary lattices, while Figure 3 (b) portray the anisotropy polar plot of the hybrid SPSC and DSC lattices.

The plus-like shape of the SC polar plot in Figure 3 (a) indicates a severely anisotropic composition. The D-TPMS sheet lattice displays a less intense anisotropy configuration with similar soft and stiff directions. The polar plot of the SP indicates dominant stiffness in the diagonal directions compared to the axial direction, thereby also depicting an anisotropic composition. It can be denoted the DSC outperforms the SPSC in terms of maximum stiffness, with more anisotropic compositions, however.

4 CONCLUSIONS

This study explores the role of topological hybridization in improving the elastic isotropy of TPMS-based lattice metamaterials. Through finite element simulations, the hybrid Diamond-Simple Cubic (DSC) and Primitive-Simple Cubic (SPSC) lattices were evaluated for their elastic properties. Key findings include:

- Hybridization significantly influences both stiffness and anisotropy, with SPSC lattices achieving elastic isotropy, while DSC lattice demonstrates superior stiffness but retain higher anisotropy.
- The addition of SC elements to TPMS lattices generally enhances Young's modulus and bulk modulus, with the extent of improvement depending on the parent lattice configurations.

These results demonstrate that strategic hybridization of lattice topologies enables tailored mechanical properties, balancing stiffness and isotropy for various engineering applications. Future work may extend this study by incorporating experimental validation and further topologies to refine the design of hybrid metamaterials.

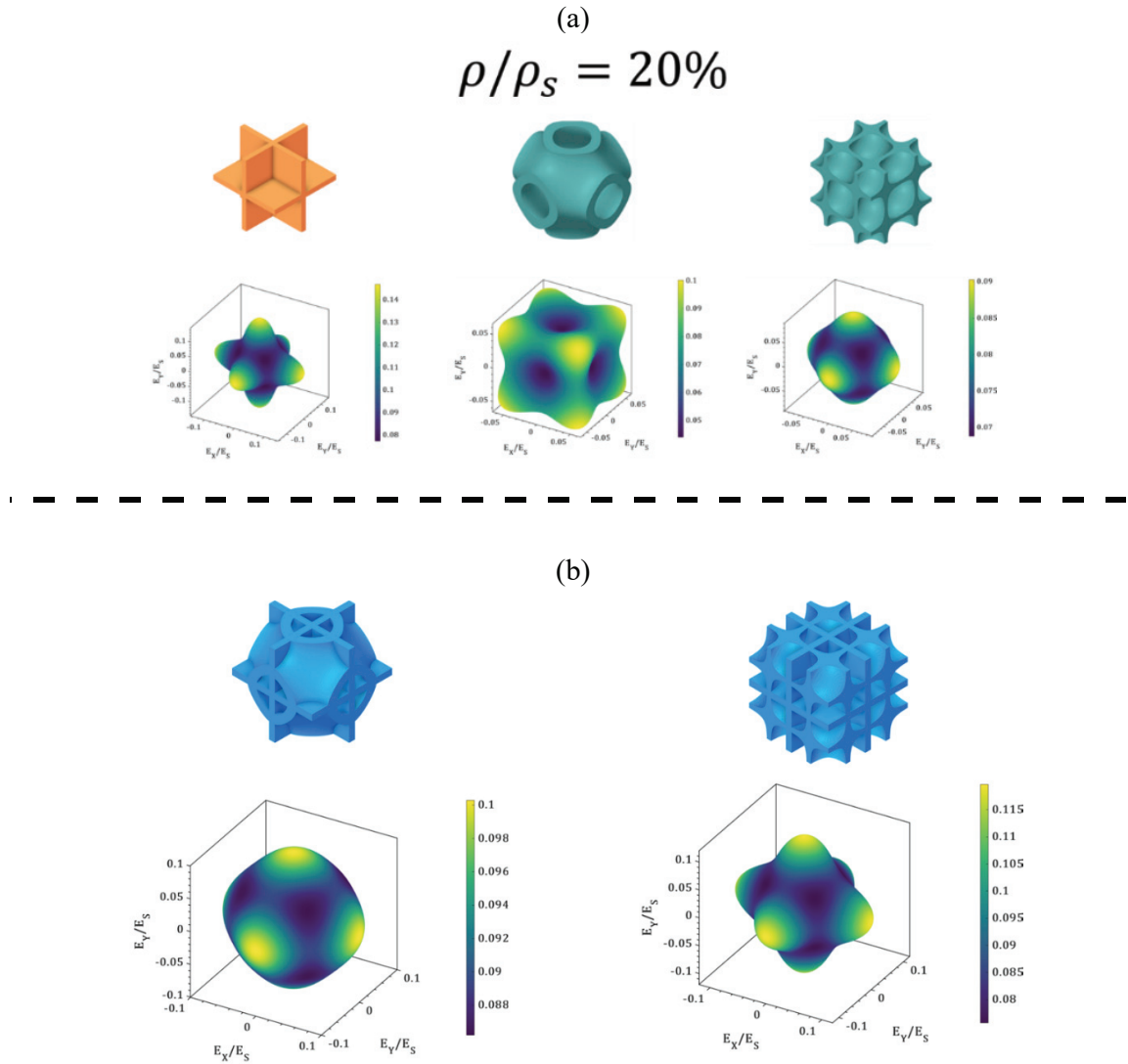


Figure 3 Polar anisotropy plots with normalized modulus values at 20% relative density of (a) the elementary lattices, (b) SPSC, and DSC.

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