

Wave Propagation Analysis of Sigmoid Functionally Graded Square lattice Structures Using Dynamic Stiffness Method and Wittrick-Williams Algorithm

Mudit Mishra, Sandeep Kumar

Abstract. This study investigates in-plane wave propagation in square lattice structures with Sigmoid-Functionally Graded Material (S-FGM) cell elements using Bloch's theorem. Timoshenko Beam theory models the cell elements, with the sigmoid function representing material property variation along the beam thickness. The dynamic stiffness method and Wittrick-Williams algorithm are employed for the analysis. Three approaches are considered: the first assigns a uniform power law index to all elements, the second uses different power law indices for each element, and the third approach maintains a uniform power law index but assigns unique cross-sectional shapes to each element. The results show that the third approach, with distinct cross-sectional shapes, significantly influences the band gap properties of S-FGM SL structures. Findings are validated with finite element method (FEM) simulations and COMSOL Multiphysics, confirming the accuracy of the results. The study emphasizes the importance of heterogeneous material properties for optimizing wave propagation behavior in engineering applications.

Key words: Sigmoid-Functionally graded material, Dynamic stiffness method, Wave propagation, Square Lattice, Wittrick-William algorithm

1 INTRODUCTION

Honeycomb systems are widely used in aerospace, automotive, and structural engineering due to their periodicity and versatile applications. Formed by repeating unit cells of beams or rods, they can use functionally graded materials (FGMs) [1], for tailored mechanical, thermal, and acoustic properties. Common designs include square and hexagonal [2], re-entrant [3] and kagome lattice [4]. Square lattice (SL) structures exhibit high in-plane stiffness and strength due to their regular geometry and uniform distribution of forces. This unique property enhances load-bearing capacity, structural stability, and resistance to deformation, making SL structures highly valuable in various engineering and material science applications.

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Wave propagation in cellular systems has garnered significant attention due to their ability to filter and guide waves. A key feature is the bandgap phenomenon, which blocks wave propagation in specific frequency ranges. Various studies have examined wave behavior in honeycomb lattices. Ruzzenne et al.[5] studied wave beaming and directional effects in re-entrant and hexagonal lattices, while Farzbod et al. [6] compared force treatment methods in lattices, aligning their findings with Langley's work. Phani et al. [4] used finite element methods to analyze band structures, and Gonella et al.[7] investigated dispersion, velocities, and band structures. Leamy [2] included joint inertia in lattice dynamics. However, wave behavior in sigmoid functionally graded (S-FG) lattices remains underexplored, presenting opportunities for tailoring wave propagation properties.

In all the above literature's, Finite Element Method (FEM) was consistently employed. However, a notable drawback of FEM is its reliance on extensive mesh refinement during analysis to achieve precise outcomes. This inevitably prolongs computational time, especially in analyzing dynamic response within mid to high frequency ranges. Moreover, the FE formulation may yield poorly conditioned matrices, potentially leading to numerical singularities.

To overcome the aforementioned challenges, researchers commonly utilize analytical methods to simulate the behavior of unit cells. The dynamic stiffness method (DSM), also known as the spectral element method (SEM), is an analytical modeling technique that allows for the representation of the complete deformation of a system across the entire frequency spectrum using a minimal number of degrees of freedom (DoFs) [8]. Xiang Liu et al. [9] presented the dynamic stiffness (DS) formulation for analyzing wave propagation in re-entrant and hexagonal honeycomb systems. Their study demonstrated how various primitive unit cell configurations affect dispersion curves and iso-frequency contours. However, a primary challenge associated with the DSM is solving the DS matrix. Many researchers opt for the Wittrick-Williams (W-W) [10] algorithm to address this issue, as it is known for its efficiency, reliability, and accuracy in providing eigen-solutions.

Although wave propagation in square lattice (SL) systems has been widely studied, the combination of SL configurations with S-FG materials remains largely unexplored. This study addresses this gap by examining wave propagation behavior in S-FG SL structure using the DSM integrated with the W-W algorithm. To ensure accuracy and reliability, the outcomes are verified against FEM simulations and COMSOL Multiphysics analyses. This approach provides new understanding of the dynamic characteristics of S-FG SL structures.

2 DYNAMIC STIFFNESS FORMULATION OF S-FG SQUARE 2D LATTICE SYSTEMS

The unit cell method is employed to analyze dynamic response in S-FG SL by predicting the behavior of a single unit cell. S-FG Timoshenko beam elements are used, and the DSM is applied to formulate dynamic stiffness matrices, as detailed below.

2.1 Conventions and Principles in Lattice Geometry Notation

This paper thoroughly examines the wave transmission response of S-FG SL systems, illustrated in FIGURE 1 (a). The meticulous assembly of S-FG SL units forms the complete honeycomb structure along predefined directions, represented by lattice basis vectors denoted as e_i (where $i = 1, 2$ as shown in FIGURE 1 (b)). The intersection points of these square unit lattice geometries are referred to as lattice points.

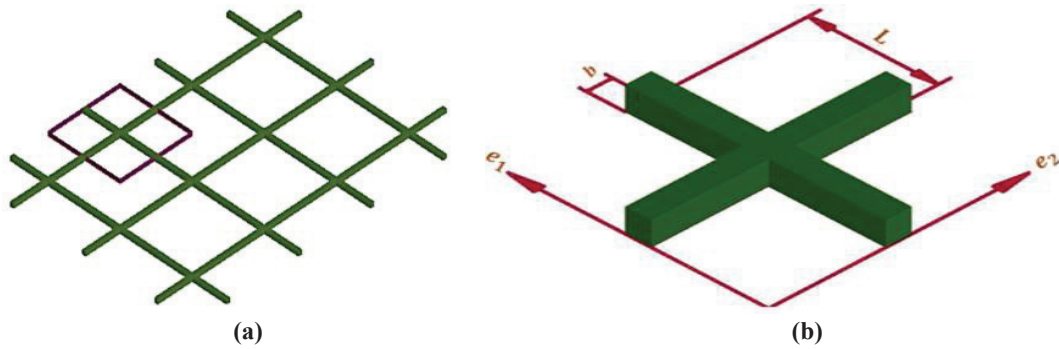


FIGURE 1. (a) S-FG SL structure, (b) Fundamental unit cells for SL structure, accompanied by descriptions of their lattice geometry. Furthermore, the direct lattice basis vectors, indicated as e_1 and e_2 are illustrated to provide a clearer understanding.

The lattice and reciprocal lattice basis vectors for the square structure are illustrated in FIGURE 2. The irreducible Brillouin zone, its sections, and high-symmetry points are highlighted, aiding in the analysis of lattice symmetry and reciprocal relationships.

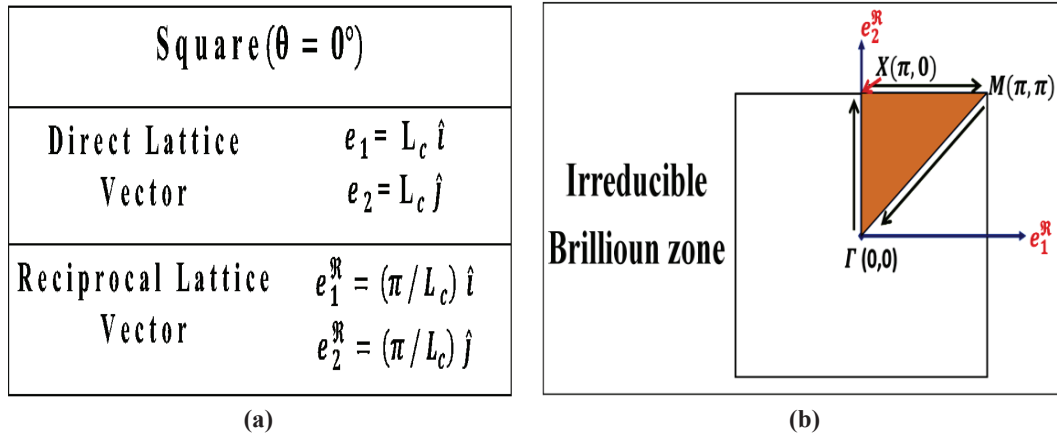


FIGURE 2. (a) Direct and reciprocal lattice vectors of the SL (b) Irreducible Brillouin zone with the coordinates of the high-symmetry points.

2.2 Effective Material Properties of FG Beams

Functionally graded materials (FGMs) feature gradually varying properties, modeled using mathematical functions. This study employs a sigmoid function to analyze volume fraction variations in FG hexagon honeycomb structures.

In the sigmoid law mathematical model, the volume fraction can be represented by a combination of two power-law functions.

$$\begin{aligned} V_f^1(t) &= 1 - \frac{1}{2} \left(\frac{h/2 - z}{h/2} \right)^p \text{ for } 0 \leq z \leq \frac{h}{2}, \\ V_f^2(t) &= \frac{1}{2} \left(\frac{h/2 + z}{h/2} \right)^p \text{ for } -\frac{h}{2} \leq z \leq 0. \end{aligned} \quad (1)$$

Where h is thickness of the S-FG beam and p is exponent of material gradient.

By employing the rule of mixture, one can compute the material properties of the S-FGM.

$$E_{FG} = V_f^1(t)E_c + (1 - V_f^1(t))E_m \text{ for } 0 \leq z \leq \frac{h}{2}, \quad (2)$$

$$E_{FG} = V_f^2(t)E_c + (1 - V_f^2(t))E_m \text{ for } -\frac{h}{2} \leq z \leq 0.$$

And

$$\rho_{FG} = V_f^1(t)\rho_c + (1 - V_f^1(t))\rho_m \text{ for } 0 \leq z \leq \frac{h}{2}, \quad (3)$$

$$\rho_{FG} = V_f^2(t)\rho_c + (1 - V_f^2(t))\rho_m \text{ for } -\frac{h}{2} \leq z \leq 0.$$

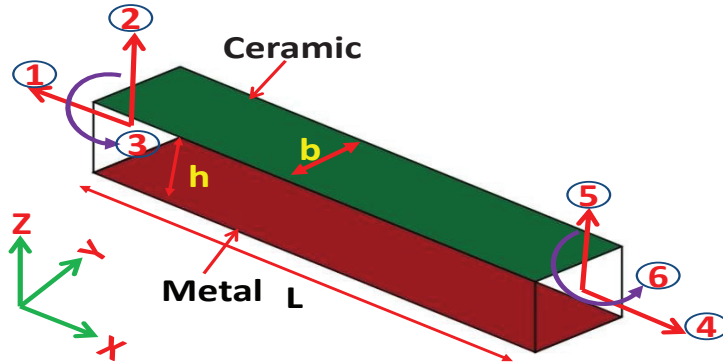


FIGURE 3. Functionally graded material beam element with three degrees of freedom namely axial deformation (1 & 4), transverse deformation (2 & 5) and rotation (3 & 6)

2.3 Dynamic Stiffness Formulation of Unit Cell

The DSM is a reliable approach for precisely determining the eigenvalues and eigenvectors of structures that resemble slender, beam-like elements arranged in two-dimensional frame-like configurations. In this study, instead of examining the entire lattice structure, the focus is on the behavior of an individual unit cell from a 2D periodic lattice. This unit cell is composed of beams, with each node exhibiting three degrees of freedom (DoFs): axial, flexural, and shear

deformations, as shown in FIGURE 3. To create the dynamic stiffness model for the unit cell, classical theory is applied for axial vibrations, while the Timoshenko beam theory (TBT) is used for bending vibrations.

The governing differential equation for axial free vibration in an S-FG beam is supported

$$\rho_z \omega^2 U(x) + E_z \frac{d^2 U(x)}{dx^2} = 0 \quad (4)$$

Where

$$\rho_z = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_{FG} b dz = \frac{bh((2 - 2^p + p)\rho_c + (2^p + p)\rho_m)}{2(1 + p)} \quad (5)$$

$$E_z = \int_{-\frac{h}{2}}^{\frac{h}{2}} E_{FG} b dz = \frac{bh(E_c(2 - 2^p + p) + E_m(2^p + p))}{2(1 + p)} \quad (6)$$

The DS matrix for the axial vibration of the Timoshenko S-FG beam element [9] can be represented by utilizing the above governing differential equation (5)

$$\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_2 & a_1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad (7)$$

Where

$$a_1 = E_e \lambda_a \cot(\lambda_a L); a_2 = -E_e \lambda_a \operatorname{cosec}(\lambda_a L) \quad (8)$$

Where L is length of Beam and $\lambda_a = \sqrt{\frac{\rho_e \omega^2}{E_e}}$

The differential equation that governs the free vibration of bending in a S-FG beam is as follows:

$$\begin{aligned} kG_z \frac{d}{dx} \left(\frac{dW}{dx} - \theta \right) + \omega^2 \rho_z W &= 0 \\ J_z \frac{d^2 \theta}{dx^2} + kG_e \left(\frac{dW}{dx} - \theta \right) + M_z \omega^2 \theta &= 0 \end{aligned} \quad (9)$$

Where

$$\begin{aligned} J_z &= \int_A E(z)(z - z_0)^2 dA \\ &= \frac{1}{96(2 + p)^2(3 + p)(E_c(2 - 2^p + p) + E_m(2^p + p))} b(E_m^2(34^{2+p} \\ &\quad + p^2(7 + p(4 + p)) + 2^{2+p}p(34 + 7p(4 + p))) + 2E_cE_m(-34^{2+p} \\ &\quad + 136p + 149p^2 + 56p^3 + 7p^4 - 32^{2+p}(-8 + p(2 + p(4 + p)))) \\ &\quad + E_c^2(48(-2 + 2^p)^2 + p(176 + 63p + 12p^2 + p^3 - 2^{2+p}(22 + p(4 \\ &\quad + p)))))) \end{aligned} \quad (10)$$

$$\begin{aligned}
M_z &= \int_A \rho(z)(z - z_0)^2 dA \\
&= \frac{1}{96(2+p)^2(3+p)(Ec(2-2^p+p) + Em(2^p+p))^2} bh^3(2EcEm((2^p \\
&+ p)(48(-2+2^p)^2 + p(176+63p+12p^2+p^3-2^{2+p}(22+p(4+p))))\rho c - (-2 \\
&+ 2^p-p)(34^{2+p}+p^2(7+p(4+p)) + 2^{2+p}p(34+7p(4+p)))\rho m) \\
&+ Em^2((-34^{2+p}(-2+2^p) + p(4^{1+p}(-14+p(4+p)) + 2^p(192-p(103+37p(4 \\
&+ p)))) + p(258+p(283+p(106+13p))))\rho c + (2^p+p)(34^{2+p}+p^2(7+p(4 \\
&+ p)) + 2^{2+p}p(34+7p(4+p)))\rho m) + Ec^2(-(-2+2^p-p)(48(-2+2^p)^2 + p(176 \\
&+ 63p+12p^2+p^3-2^{2+p}(22+p(4+p))))\rho c + 32^{4+p}(-2+2^p)^2\rho m + p(352 \\
&+ 692p+459p^2+128p^3+13p^4+4^{1+p}(-2+7p(4+p)) - 2^p(160+p(513 \\
&+ 7p(36+5p))))\rho m))
\end{aligned} \quad (11)$$

$$\text{Where } z_0 = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} E_{FG} z b dz}{\int_{-\frac{h}{2}}^{\frac{h}{2}} E_{FG} b dz} = -\frac{(Ec-Em)hp(3-2^{1+p}+p)}{4(2+p)(Ec(2-2^p+p)+Em(2^p+p))} \text{ is the neutral axis of S-FG beam}$$

The DS matrix for the bending vibration of the Timoshenko S-FG beam element [9] can be represented by utilizing the above governing differential equation (9)

$$\begin{bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} d_1 & d_2 & d_4 & d_5 \\ & d_3 & -d_5 & d_6 \\ & \vdots & d_1 & -d_2 \\ \text{sym} & & & d_3 \end{bmatrix} \begin{bmatrix} W_1 \\ \theta_1 \\ W_2 \\ \theta_2 \end{bmatrix} \quad (12)$$

Where

$$\begin{aligned}
d_1 &= R_3 \bar{b}^2 (\lambda_2 + \eta \lambda_1) (cS + \eta sC) / (\lambda_1 \lambda_2 \delta) \\
d_2 &= R_2 k_1 [(\lambda_1 + \eta \lambda_2) sS - (\lambda_2 - \eta \lambda_1) (1 - cC)] / \delta \\
d_4 &= -R_3 \bar{b}^2 (\lambda_2 + \eta \lambda_1) (S + \eta s) / (\lambda_1 \lambda_2 \delta) \\
d_5 &= R_2 k_1 (\lambda_2 + \eta \lambda_1) (C - c) / \delta \\
d_6 &= R_1 (\lambda_2 + \eta \lambda_1) (\eta S - s) / \delta
\end{aligned} \quad (13)$$

With

$$\begin{aligned}
W_j &= E_e / L^j, j = 1, 2, 3 \\
s &= \sin \lambda_1, c = \cos \lambda_1, S = \sinh \lambda_2, C = \cosh \lambda_2 \\
k_1 &= (\lambda_1^2 - \bar{b}^2 s^2) / \lambda_1, k_2 = (\lambda_2^2 + \bar{b}^2 s^2) / \lambda_2
\end{aligned} \quad (14)$$

Where

$$\begin{aligned}
\lambda_1 &\} = \bar{b} \left\{ \pm \Delta / 2 + [\Delta^2 / 4 + (1 - \bar{b}^2 r^2 s^2) / \bar{b}^2]^{1/2} \right\}^{1/2} \\
\lambda_2 &\}
\end{aligned} \quad (15)$$

With

$$\Delta = r^2 + s^2, \bar{b}^2 = \frac{\rho_e \omega^2 L^4}{E_e}, r^2 = \frac{I_e}{AL^2}, s^2 = \frac{J_e}{kG_e L^2} \quad (16)$$

I_z is the moment of inertia of S-FG beam about neutral axis and is given by

$$I_z = \int_A (z - z_0)^2 dA = \frac{1}{48(2+p)^2(E_c(2-2^p+p) + E_m(2^p+p))^2} b h^3 (64E_c^2 + 128E_c^2 p + 64E_c E_m p + 123E_c^2 p^2 + 42E_c E_m p^2 + 43E_m^2 p^2 + 50E_c^2 p^3 + 12E_c E_m p^3 + 34E_m^2 p^3 + 7E_c^2 p^4 + 2E_c E_m p^4 + 7E_m^2 p^4 + 4^{2+p}(E_c - E_m)^2(1+p+p^2) - 2^{2+p}(E_c - E_m)(-E_m p(-8+p+p^2) + E_c(16+p(24+p(21+5p)))))) \quad (17)$$

By combining axial and bending stiffness matrices the elemental DS matrix of S-FG beam can be expressed as

$$K^e(\omega) = \begin{bmatrix} a_1 & 0 & 0 & a_2 & 0 & 0 \\ 0 & d_1 & d_2 & 0 & d_4 & d_5 \\ 0 & d_2 & d_3 & 0 & -d_5 & d_6 \\ a_2 & 0 & 0 & a_1 & 0 & 0 \\ 0 & d_4 & -d_5 & 0 & d_1 & -d_2 \\ 0 & d_5 & d_6 & 0 & -d_2 & d_3 \end{bmatrix} \quad (18)$$

The first step involves converting the above DS matrix into global coordinates. Following this, the overall DS matrix of the unit cell, denoted as $K^O(\omega)$, can be derived by integrating the DS matrices of each element along the lattice directions. The processes of matrix assembly and coordinate transformation in the DSM closely resemble those in the FEM. Ultimately; the overall DS matrix for the entire unit cell is subjected to periodic boundary conditions (PBCs) through the use of Bloch's theorem. The specific procedure for applying these conditions is explained in detail in the following sections.

3 ENFORCEMENT OF PERIODIC BOUNDARY CONDITION

After getting overall DS matrix the Floquet-Bloch theorem [6] has been utilized to impose PBCs. The procedure to apply the theorem is shown for S-FG SL in FIGURE 4. The image below illustrates the unit cell of SL, including four Timoshenko beams with five nodes (four external and one internal).. The external nodes can be further categorized into output (q_2 and q_4) and input nodes (q_1 and q_3) and. The Floquet-Bloch theorem states that the displacement vector of output nodes may be expressed in relation to the displacements of input nodes as follows:

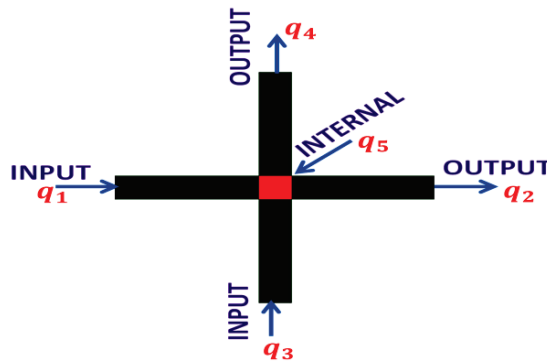


FIGURE 4. S-FG SL unit cell in 2D with designated Output, Input and Internal nodes

$$\begin{aligned} q_2 &= e^{-i\beta_1} q_1 \\ q_4 &= e^{-i\beta_2} q_3 \end{aligned} \quad (19)$$

The wavenumbers β_1 and β_2 correspond to direction along the basis vectors of the SL in reciprocal space. Equation (19) allows us to formulate the generalized displacement vector for the S-FG square unit cell as

$$q_u = T_b(\beta_1, \beta_2) \widetilde{q}_{ui} \quad (20)$$

Where over all displacement vector $q_u = [q_1, q_2, q_3, q_4, q_5]$, independent displacement components in unit cell $\widetilde{q}_{ui} = [q_1, q_3, q_5]$ and matrix $T_b(\beta_1, \beta_2)$ is given by

$$T_b(\beta_1, \beta_2) = \begin{bmatrix} I & 0 & 0 \\ e^{-i\beta_1} I & 0 & 0 \\ 0 & I & 0 \\ 0 & e^{-i\beta_2} I & 0 \\ 0 & 0 & I \end{bmatrix} \quad (21)$$

The overall DS matrix after the application of PBCs is given by

$$K^O(\omega, \beta_1, \beta_2) = T_b^H(\beta_1, \beta_2) \cdot K^O(\omega) \cdot T_b(\beta_1, \beta_2) \quad (22)$$

Where $T_b^H(\beta_1, \beta_2)$ is the harmonic conjugate of the matrix $T_b(\beta_1, \beta_2)$.

3.1 Wittrick-William Algorithm

The overall DS Matrix $K^O(\omega, \beta_1, \beta_2)$ is highly complicated, having transcendental functions of ω , β_1 and β_2 , that make it a challenging nonlinear eigenvalue problem. To overcome this, researchers [8-10] often turn to the Wittrick-Williams algorithm, a sophisticated method that takes use of Sturm sequences. What makes this algorithm impressive is its dual capability: it consistently maintains high accuracy while operating with remarkable efficiency. This combination not only ensures precise solutions but also significantly reduces the time required to tackle intricate problems. For brevity, the algorithm is not presented here; readers are referred to Reference [11] for the full implementation of the W-W algorithm.

4 RESULTS AND DISCUSSIONS

This section utilizes the DSM to explore the wave propagation behavior of S-FG SL. A MATLAB code has been developed to execute the DSM formulation and the previously discussed W-W algorithm. These calculations are carried out on a computer with a CPU running at 8.00 GB of RAM and 3.00 GHz. TABLE 1 details the parameters related to the geometry and material properties of the S-FGM beam.

TABLE 1. Material and geometric characteristics of the S-FG beam

Material Properties			
	<i>Density</i> (kg/m^3)	<i>Poisson's Ratio</i>	<i>Young's Modulus</i> (GPa)
<i>Aluminium</i>	2702	0.3	72
<i>Alumina</i>	3960	0.3	380
Geometric Properties			
	Length	Depth	Width
	0.25 m	0.0125 m	0.0125 m

4.1 Band Structure Validation of the S-FG Square Lattice

To verify the MATLAB code and DSM formulation, a comparative study is performed, focusing specifically on the dispersion curves of the S-FG SL structure. The dispersion curve of the S-FG SL is also calculated using COMSOL Multiphysics software and FEM. In the FEM model, each beam in the S-FG SL unit cell is modeled with 30 elements, totaling 90 components for the entire unit cell. The COMSOL Multiphysics model employs a solid representation with total 4 components and a fine mesh to depict the S-FG SL unit cell. FIGURE 5 illustrates the results obtained from all three methods.

In FIGURE 5 it is evident that the dispersion branches from all three approaches completely align for the lower modes. However, for higher modes, the dispersion curves calculated by the FEM consistently show higher values compared to those obtained by the DSM approach, which might be because of fundamental discretization inaccuracies in FEM. In contrast, the curves from COMSOL sometimes exceed those from DSM and sometimes fall below. This variability can be attributed to COMSOL's use of a FE solid model, which introduces combined effects of discretization errors and differences between solid and beam models. Moreover, the COMSOL model considers the geometric neutral surface, whereas FEM and DSM use the physical neutral surface, potentially leading to higher errors in the COMSOL results.

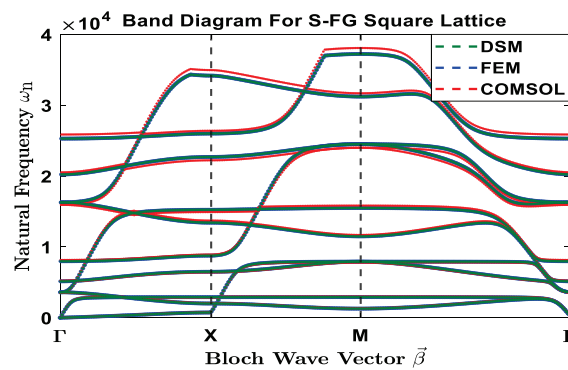


FIGURE 5. The first ten dispersion curve for S-FG SL structure calculated by DSM, FEM, COMSOL ($h = L/15$ and $p = 5$)

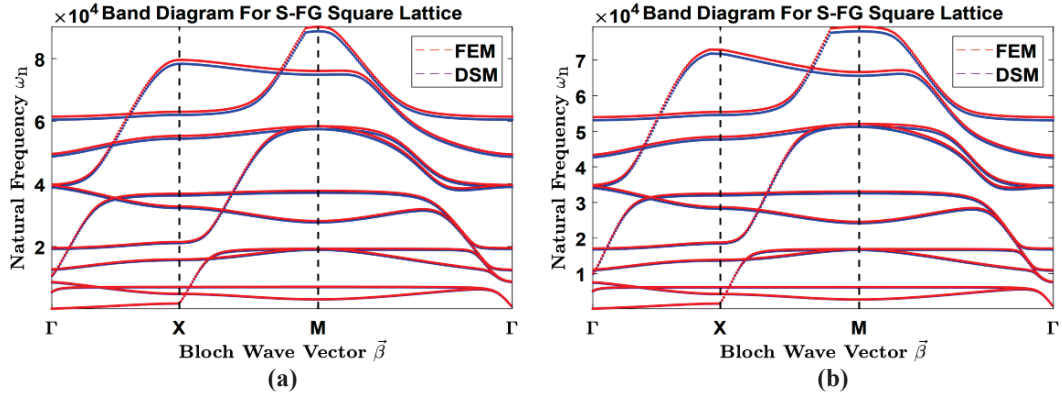


FIGURE 6. Band diagram of S-FG SL with $h = L/15$ and (a) $p = 0$ (b) $p = 2$

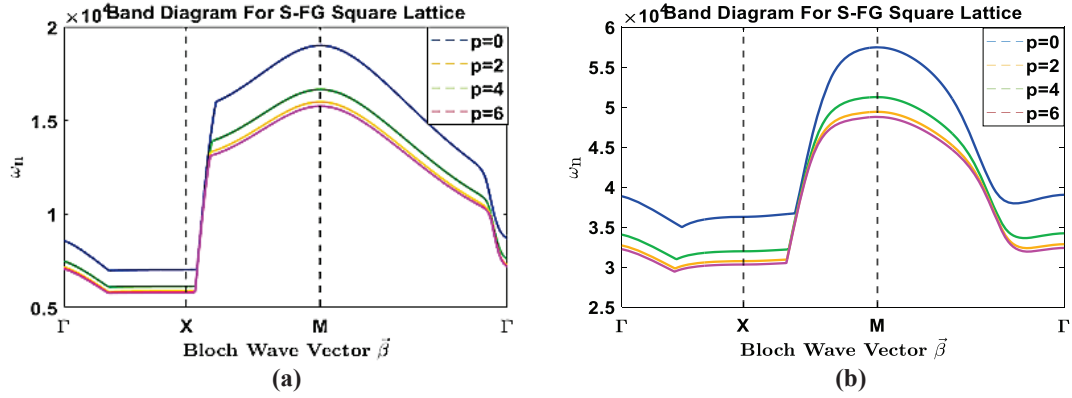


FIGURE 7. Impact of variation in power law gradient index on (a) the 3rd mode and (b) the 7th mode band diagrams of the S-FG SL with $h = L/15$.

The dispersion curves of the first 10 modes of the S-FG SL are shown in FIGURE 6 for material gradient indices (p) of $p = 1$ and $p = 10$. The dispersion branches are calculated using the DSM (shown in blue) and verified with FEM (shown in red). From FIGURE 6, it is evident that while the general shape of the dispersion branches retains uniformity across both power law indices, there is a clear reduction in the frequency magnitude of all modes in FIGURE 6 (b) compared to FIGURE 6 (a). This suggests that varying the index (p) impacts the scaling of the dispersion curve of the S-FG SL.

In FIGURE 7, the dispersion curves for the 3rd and 7th branch of dispersion diagram of the S-FG SL are depicted across a range of power law indices. This figure illustrates that with higher power law indices, the curves for both branches shift downwards, suggesting a reduction in the frequency magnitude. The decrease in frequency magnitude can be attributed to the greater proportion of metallic components present within the S-FG beams at higher power law indices. Increase in the metallic volume fraction results in reduced density and stiffness, which consequently lowers the frequency magnitude. Furthermore, it becomes apparent that the distance between the curves narrows with higher values of p .

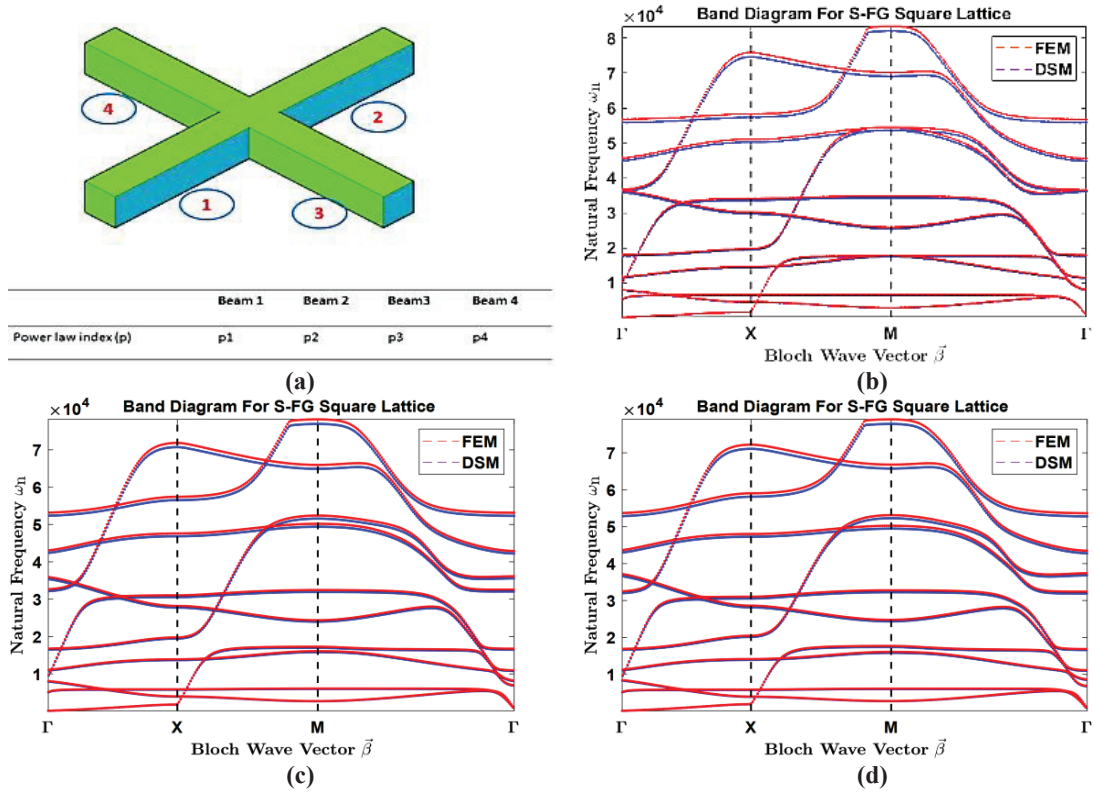


FIGURE 8. (a) Geometry of the FG SL with a table specifying the material gradient index (p) for each beam element within the unit cell. (b) Band diagram for a uniformly graded lattice with $p_1 = p_2 = p_3 = p_4 = 1$ (c) Band diagram for a lattice with graded indices $p_1 = 1, p_2 = 2, p_3 = 4, p_4 = 8$ (d) Band diagram for a lattice with graded indices $p_1 = 1, p_2 = 1, p_3 = 4, p_4 = 10$

The dispersion curves for the S-FG SL structure have been calculated and are shown in FIGURE 8. In this lattice, each element within the unit cell has a distinct power law index. Interestingly, despite this variation in the power law indices, there is no noticeable impact on the dispersion curves. They remain unchanged, indicating that the distinct power law indices do not affect the overall dispersion characteristics. However, only a few variations are shown here for the sake of brevity. It is possible that other variations, not included in this presentation, could lead to the opening of band gaps in the dispersion curves.

FIGURE 9 illustrates the band structure of an S-FG SL with elements of varying cross-sectional shapes. The diagram reveals the emergence of an additional bandgap (yellow region), likely resulting from increased reflection and interference at the junctions where the cross-sectional dimensions of the elements differ. These variations in cross-section seem to create disturbances that contribute to the formation of the new bandgap.

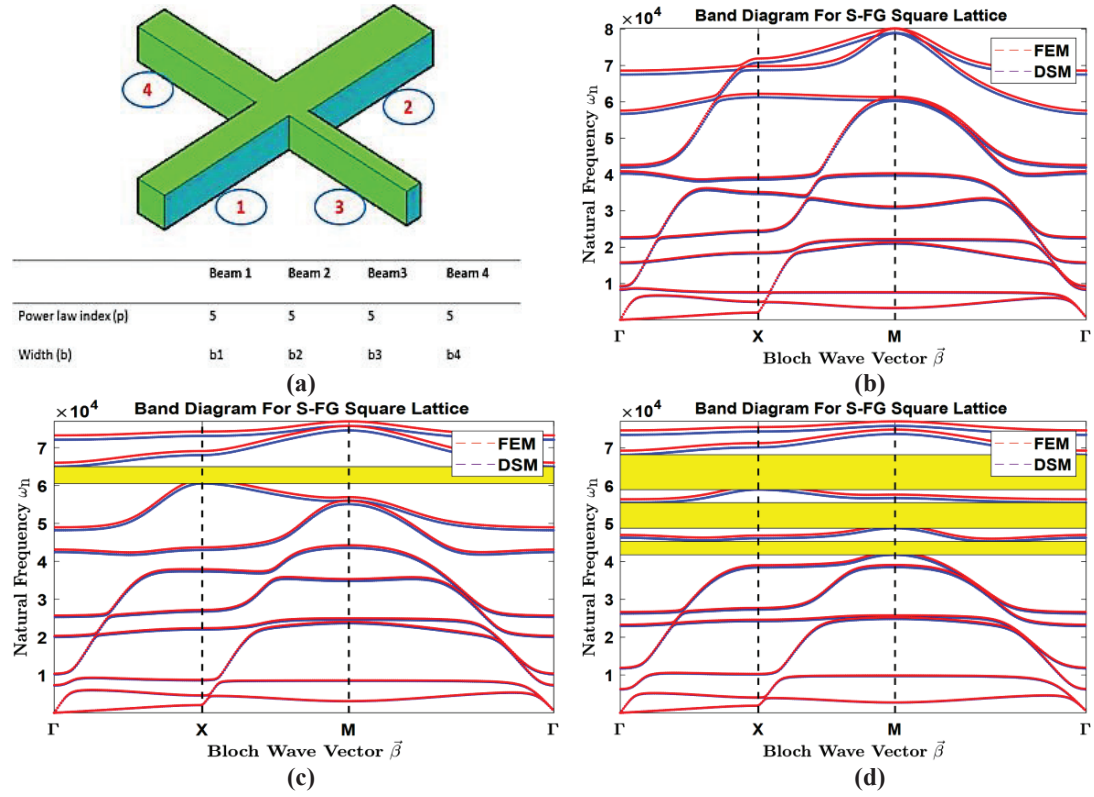


FIGURE 9. (a) Geometry of the FG SL with a table listing the material gradient index (p) and beam widths within the unit cell. (b) Band diagram for the lattice with beam widths $b_1 = k$, $b_2 = 1.5k$, $b_3 = 1k$, $b_4 = 1.5k$ (c) Band diagram for the lattice with $b_1 = k$, $b_2 = 2k$, $b_3 = 1k$, $b_4 = 2k$ (d) Band diagram for the lattice with $b_1 = 1k$, $b_2 = 2.5k$, $b_3 = k$, $b_4 = 2.5k$, where $k = L/10$ and $L = 0.125$ m.

5 CONCLUSION

Wave propagation in S-FG SL structures is investigated using the exact DSM. The impact of the power law index (p) and cross-sectional geometry on dispersion behavior are analyzed. It is observed that increasing p reduces wave frequency, emphasizing the connection between material composition and wave dynamics. Graphical analysis is used to demonstrate how adjusting p tailors dispersion properties for specific applications. Band gaps are formed by introducing variations in p or cross-sectional geometries within unit cells, highlighting the importance of material and geometric design in controlling wave-guiding and filtering functionalities in honeycomb structures.

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