

A stationary return mapping scheme for the simulation of elastic-plastic bending of axially moving structures with non-material finite elements

Jakob Scheidl, Stefan Ramsauer

Abstract. Finite element simulations of elastic-plastic forming processes typically rely on some variant of the established return mapping scheme, to obtain a both statically and plastically admissible state for a given load increment. Arbitrary-Lagrangian-Eulerian (ALE) methods make use of a non-material kinematic parametrisation of finite element variables to facilitate the simulation of continuous forming processes that feature axially moving continua. The resolution of variables at spatially fixed points gives rise to advection problems for the plastic variables in account for their downstream transport with the material particles through the non-material mesh. An enhanced return mapping scheme is devised for the time-saving computation of steady-state operation. It is based on a consistent inclusion of the plastic state advection into the corrector step of the iterative algorithm. Plasticity in the numerical examples, which mimic elastic-plastic bending of axially moving thin-walled structures, is treated with condensed models of structural plasticity. Performance and utility of the proposed scheme are verified in comparison to an established time-driven (transient) algorithm.

Key words: Metal plasticity, Axially moving structures, ALE, Finite elements, Predictor corrector method

1 Introduction

Advanced numerical techniques are required to allow for a both accurate and time-efficient simulation of continuous metal forming processes. Here, we consider bending problems of axially moving structures. Such problems feature moving thin continua that are transported through an open control domain and sustain persistent plastic deformations along the way. Finite elements in mixed Eulerian–Lagrangian kinematic formulation allow for a more efficient numerical treatment of problems featuring axially moving structures, see [1] or [2] in the context of sheet metal roll forming. Such models utilize spatial (Eulerian) coordinates for the parametrisation of primary fields, which leads to a partial decoupling of the material particle motion from the deformation of the finite element mesh. The major benefits in comparison

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to purely Lagrangian methods with respect to forming processes are the ability to use appropriately refined meshes in the elastic-plastic regions as well as the more accurate and simpler contact resolution [3]. Plastic deformations on the other hand travel with the material particles, which gives rise to advection problems in the mixed parametrisation. The classic way of treating elastic-plastic problems of moving continua is known as *operator-split method* [4], which features a quasistatic Lagrangian step to obtain a statically and plastically admissible equilibrium state followed by a Eulerian step to account for the material transport in downstream direction.

The test cases considered here are geometrically linear continuous elastic-plastic bending of an axially moving beam and an axially moving plate. Mixed Eulerian–Lagrangian beam and plate finite element models are devised for the numerical solution using different variants of elastic-plastic return mapping schemes [5]. The beam problem has already been considered in [6], where the basics of a novel solution algorithm to compute stationary solutions of bending-dominant forming processes are discussed for the first time. The algorithm shall be referred to as *Streamline Integration in Space*, or SIS in brief, in contrast to the established *Streamline Integration in Time* (henceforth SIT) that originates from the classic operator-split method. The names of both algorithms relate to the adopted solution strategy of integrating the plastic advection problem along the trajectories of material particles (“streamlines”). Provided that plastic states are sampled at finite element integration points of a regular mesh, the method of streamline integration is particularly convenient to use, as the integration points naturally align with the particle trajectories.

Often times in simulation practice the stationary state of operation is of primary interest. For these cases, the SIS is both more time-efficient and convenient to use than the SIT, because it employs an incremental strategy that yields a sequence of practically relevant steady-state solutions. The SIS originates from a “stationary” reformulation of plastic evolution laws that allows for a modification of the corrector step as part of an enhanced elastic-plastic return mapping scheme. Consequently, the Eulerian transport step and the condition of stationary operation can be fully incorporated in the predictor-corrector scheme. Numerical results obtained for both the SIS and the SIT demonstrate correspondence of the two methods as well as the efficiency of the SIS for the simulation of steady states. Simplified structural plasticity models, which employ modified laws of plasticity in terms of stress resultants and structural strain measures, are taken from [6] for the beam and [7] for the plate to further increase computational efficiency.

2 Problem statement

The strong similarity of the beam bending and plate bending test cases considered in Section 4 and Section 5 facilitates a generic problem statement that applies to each of the benchmark examples alike. As depicted in Figure 1, the basic setup features a thin structure that is transported through an open control domain $0 \leq x \leq \ell$ at some constant material transport rate v and subjected to transverse forces that induce elastic-plastic bending deformations. The rate of transport v is assumed to be low such that inertia effects may be disregarded (quasistatic treatment). Linear beam or plate theory govern the kinematics of the structure under the usual prerequisites: Kirchhoff’s kinematic assumption and small transverse deflections (ge-

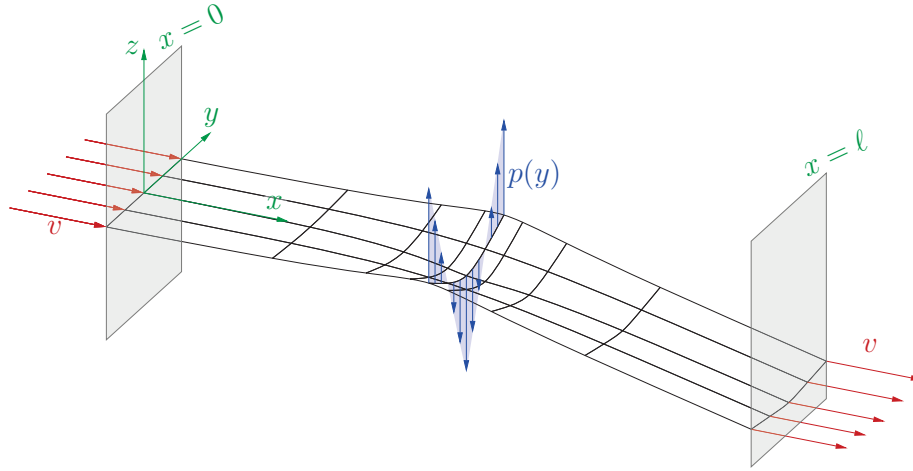


Figure 1: Problem setup for elastic-plastic bending of a thin, axially moving structure; geometrically linear beam and plate models will be considered.

omometrically linear theory). The material behaviour of the underlying continuum is linear elastic ideal plastic, but simplified stress-resultant plasticity models are employed to resolve persistent plastic deformations entirely on the structural level. Such structural plasticity models rest on an appropriate re-formulation of the governing laws of plasticity in terms of structural field variables, namely: bending moments and curvature strains, see [7].

The governing system for static, linear elastic bending of the structure constitutes the basis of the mechanical models. It suffices to compute deformed configurations in purely elastic loading scenarios. Higher load magnitudes induce plastic deformations that travel downstream with the material particles. In these cases, the system of equations is complemented by the plastic constitutive laws and the kinematic relations of material transport.

The field of transverse deflections w in z -direction of the Cartesian reference frame, depicted in Figure 1, fully describes the deformed configuration of the structure. The actual spatial parametrisation depends on the geometric dimension of the structure at hand: $w = w(x, t)$ for the one-dimensional beam or $w = w(x, y, t)$ for the two-dimensional plate. The variational problem to determine the deformed configuration in terms of the primary field w reads:

$$\delta V^\Sigma[w] = \int_{\Omega} (\delta U + \delta W) d\Omega = 0. \quad (1)$$

Hence, quasistatic equilibria are calculated as stationary values of the total potential energy V^Σ , which features an integration of the elastic strain energy density U and the potential of conservative external forces W ; the domain of integration Ω corresponds to the active material volume currently residing in the open control domain.

In the framework of the geometrically linear theory, the total bending strains κ are directly obtained from the displacement field w :

$$\kappa = \nabla \nabla w, \quad (2)$$

where ∇ serves as a placeholder for the appropriate differential operator, i.e.: the above expression simply reduces to $\kappa = w''$ for the beam, but yields a 2D-tensor κ in case of the plate. In a general elastic-plastic state, only the elastic part of the curvature strains κ_e contributes to the strain energy density $U = U(\kappa_e)$ and thus determines the bending moment:

$$M = \frac{\partial U}{\partial \kappa_e}. \quad (3)$$

Naturally, this expression returns M as a scalar for the beam and as a planar tensor for the plate. The notion of elastic strains is necessitated by the possible occurrence of plastic deformations and further motivates an additive decomposition of the total curvature strain increments:

$$\dot{\kappa} = \dot{\kappa}_e + \dot{\kappa}_p, \quad (4)$$

where the incremental dot denotes the total change at a given material particle. This kind of additive decomposition of material strain increments is a common feature of continuum-based small strain plasticity models [5]. The fact that (4) is written in terms of the structural strain measures exemplifies the transition to the aforementioned structural plasticity model.

The structural plasticity model is taken from [7], where elastic-plastic bending solutions of plates are considered as reference load cases; see [8] for an augmentation to incorporate membrane effects. The yield criterion for pure bending features a scalar comparison of the actual bending moment and the yield moment

$$F = f(M) - k_M(A_p)^2 \leq 0, \quad (5)$$

where the current yield moment k_M depends on the plastic work A_p previously done to the material particle. The function $k_M(A_p)$ practically accomplishes the transition to the purely structural model by emulating the through-the-thickness plastification as a kind of isotropic work hardening behaviour. The distinction of elastic and elastic-plastic states is conveniently summarised with complementary Kuhn-Tucker conditions:

$$F \leq 0, \quad \dot{\lambda} \geq 0, \quad \dot{\lambda} F = 0, \quad (6)$$

which introduces the non-negative increments of the consistency parameter $\dot{\lambda}$ that determines the magnitude of plastic flow in the evolution laws for the plastic variables:

$$\dot{\kappa}_p = \dot{\lambda} \frac{\partial f}{\partial M}, \quad \dot{A}_p = M \dot{\kappa}_p. \quad (7)$$

Both the associated flow rule for $\dot{\kappa}_p$ as well as the evolution law for the plastic dissipation $\dot{A}_p$ closely resemble classic metal plasticity theory [9].

So far, no attention has been paid to the axial transport of the structure through the open control domain, as depicted in Figure 1. In particular, the evolution laws of structural plasticity have been stated for material increments $\{\dot{\lambda}, \dot{\kappa}_p, \dot{A}_p\}$, i.e.: changes of plastic variables that the

axially moving particles sustain. Since boundary conditions and external loadings are applied at spatially fixed positions, a mixed Eulerian–Lagrangian kinematic approach is advisable to avoid common deficiencies of purely Lagrangian models in problems of moving structures. This is easily achieved by utilizing the spatial x -coordinate for parametrisation; it is immediately connected to the material arc coordinate s through the prescribed material transport rate v :

$$s = x - vt \quad (8)$$

In the current geometrically linear setting, derivatives with respect to s and x are interchangeable and from the total time derivative $\dot{s} = 0$ we further conclude that $\dot{x} = v$. The plastic variables are treated as functions of x as well, and, correspondingly, their total changes in time feature a convective and a local time derivative at fixed x :

$$\dot{\kappa}_p = v\kappa' - \partial_t \kappa, \quad \dot{A}_p = vA'_p - \partial_t A_p, \quad (9)$$

The respective derivatives on the right-hand side are denoted by a prime and the operator ∂_t , which expands to $\partial_t = \partial/\partial t|_{x=\text{const.}}$. Evidently, the resolution of plastic variables at fixed Eulerian coordinates x leads to an advection problem for the downstream transport of plastic variables, whose material increments are governed by the plastic evolution laws (7).

The typical strategy to simulate this process is known as “operator-split method” [4], which features an alternation of Lagrangian and Eulerian steps: First, a quasistatic statically and plastically admissible equilibrium is computed in the Lagrangian step, which yields new deformed variables and material plastic increments $\{\dot{\kappa}_p, \dot{A}_p\}$ that are further transported downstream according to (9) during the second, Eulerian phase. As simulation time progresses, the transient process gradually approaches a stationary state that is identified by vanishing spatial changes of the primary fields, which is particularly significant for the plastic variables:

$$\partial_t \kappa|_{\text{stationary}} = (\dot{\kappa}_p - v\kappa')|_{\text{stationary}} = 0, \quad \partial_t A_p|_{\text{stationary}} = (\dot{A}_p - vA'_p)|_{\text{stationary}} = 0. \quad (10)$$

Thus, the material particle increments equal the convected increments in case of steady state operation. The primary goal of the following section is to use this knowledge in the development of an improved algorithm that allows for a more efficient computation of stationary states as compared to the established operator-split method.

3 Elastic-plastic return mapping schemes for non-material finite elements

Non-material finite element models with shape functions based on cubic Hermitian polynomials are used for the spatial discretisation of the transverse deflection w in all considered problems. Since the specifics of these implementations are not essential here, we refer to previously published works on this subject [1, 10]. The finite element variables for the approximation of the deflection w and the plastic fields $\{\lambda, \kappa_p, A_p\}$ are collected in the matrices $\mathbf{q}$ and $\mathbf{p}$, respectively. Gaussian quadrature rules are employed to compute the total potential as a sum of

finite element contributions, which is why $\mathbf{q}$ holds the finite element nodal variables, whereas $\mathbf{p}$ contains integration point samples of the plastic variables $\{\lambda, \kappa_p, A_p\}$.

Currently, our primary concern is the discussion of the plastic sub-routines that are required to accomplish the following two tasks:

- Take a predicted solution for $\mathbf{q}$ satisfying equilibrium (1) and compute increments of plastic variables $\dot{\mathbf{p}}$ according to the plastic evolution laws (iterative Lagrangian step).
- Take the material plastic increments $\dot{\mathbf{p}}$ and solve the advection problem (9) in accordance with the spatial resolution of plastic variables $\mathbf{p}$ (Eulerian step).

As visualised in Figure 1, the finite element models feature regular meshes such that the element edges are in perfect alignment with the spatial Cartesian coordinates. Consequently the integration points are also arranged along straight lines parallel to the x -axis. As these lines coincide with trajectories of material particles, we shall refer to them as streamlines. The corresponding numerical schemes of integrating the plastic advection problem are known as streamline integration methods [6]. Here, we distinguish the *Streamline Integration in Time* (SIT) that is based on the established operator-split and the *Streamline Integration in Space* (SIS) that incorporates the Eulerian transport step in an augmented corrector routine in an attempt to compute stationary solutions more effectively.

3.1 Streamline Integration in Time (SIT)

Figure 2 (a) provides a flowchart of the SIT algorithm. Its primary elements reflect the two-stage solution process of the operator-split method, i.e.: fixed-point equilibrium iterations with an elastic predictor and a plastic corrector followed by the transport of plastic variables through the spatially fixed mesh. The equilibrium iteration is called return mapping as the material plastic increments $\dot{\mathbf{p}}$ are obtained from a closest point projection back onto the yield surface. This way, predicted states that would violate the yield criterion (5) due to $F(\mathbf{q}, \mathbf{p}) > 0$ are forced back onto the yield surface, thus fulfilling the consistency condition $F = 0$. After successful completion of the return mapping loop, the Eulerian transport phase is initiated with a new deformed state $\mathbf{q}$ and material plastic increments $\dot{\mathbf{p}}$. These values enter an explicit forward in time backwards in space finite difference scheme to solve the plastic advection problems for each streamline. The finite difference discretization of the plastic variables along a streamline in accordance with the original equation (9) reads:

$${}^t\Delta \mathbf{p}(x_i) = \left(\dot{\mathbf{p}}(x_i) - v \frac{\mathbf{p}(x_i) - \mathbf{p}(x_{i-1})}{x_i - x_{i-1}} \right) \Delta t, \quad (11)$$

where the symbol ${}^t\Delta$ is introduced to emphasize that ${}^t\Delta \mathbf{p}$ is a Eulerian increment. The points x_{i-1} and x_i denote the axial coordinates of two subsequent integration points along the streamline. Note that integration point distances $\Delta x_i = x_i - x_{i-1}$ are not constant even for equidistant finite element meshes due to the specific spacing of the Gauss points in the normalised coordinate domain. Numerical stability of the explicit scheme effectively limits the ratio $v\Delta t/\Delta x_i$ and thus the time step Δt .

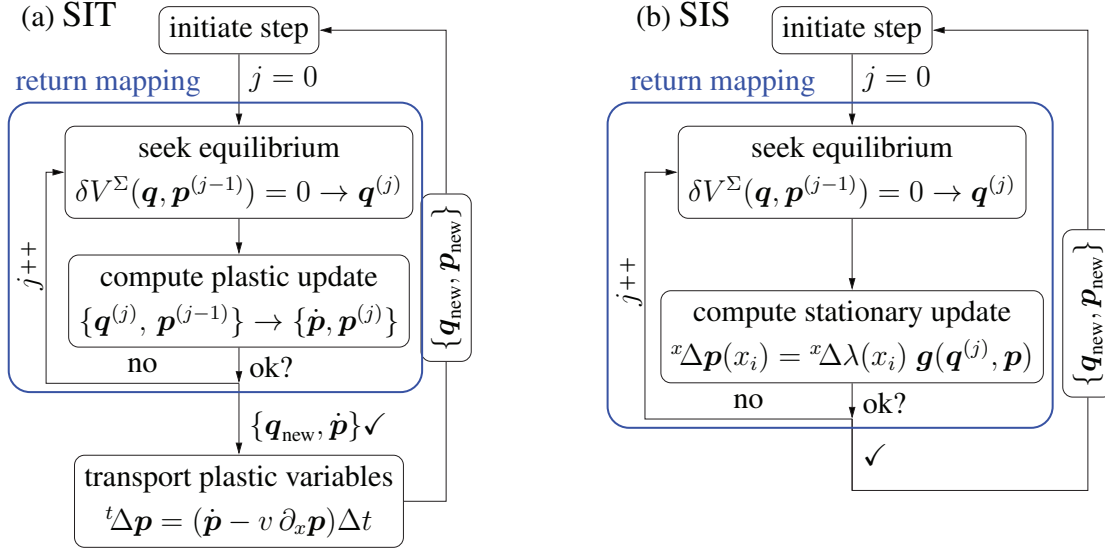


Figure 2: (a) Flowchart of the SIT: the finite difference in (11) for the spatial derivative of $\mathbf{p}$ with respect to x is abbreviated with $\partial_x \mathbf{p}$; (b) Flowchart of the SIS: a finite difference integration of the steady-state initial value problems for the plastic variables is performed at the corrector stage of a modified return mapping algorithm.

3.2 Streamline Integration in Space (SIS)

The primary distinctive feature of the SIS, as visualised in the flowchart of Figure 2 (b), is the incorporation of the advective transport in a modified corrector step of the return mapping algorithm. Consequently, the step-wise solutions of a given load history for the elastic-plastic bending problems is a sequence of stationary states, which is a particularly convenient feature when conducting parameter studies.

The basic idea behind the SIS algorithm is to replace material plastic increments with spatial derivatives owing to the condition of stationary operation, as stated in (10) and rewritten here in terms of $\mathbf{p}$:

$$\dot{\mathbf{p}} = v \mathbf{p}'. \quad (12)$$

This result allows for a convenient reformulation of the plastic evolution laws (7) as ordinary differential equations in the axial coordinate x :

$$\kappa'_p = \lambda' \frac{\partial f}{\partial M}, \quad A'_p = M \kappa'_p, \quad (13)$$

together with corresponding Kuhn-Tucker conditions to govern yielding along a streamline:

$$F \leq 0, \quad \lambda' \geq 0, \quad \lambda' F = 0. \quad (14)$$

Since no plastic deformations occur unless $\lambda' > 0$, the scalar consistency factor λ can be considered to be the driving force for yielding, which is exemplified by the proportionality $\mathbf{p}' \propto \lambda'$

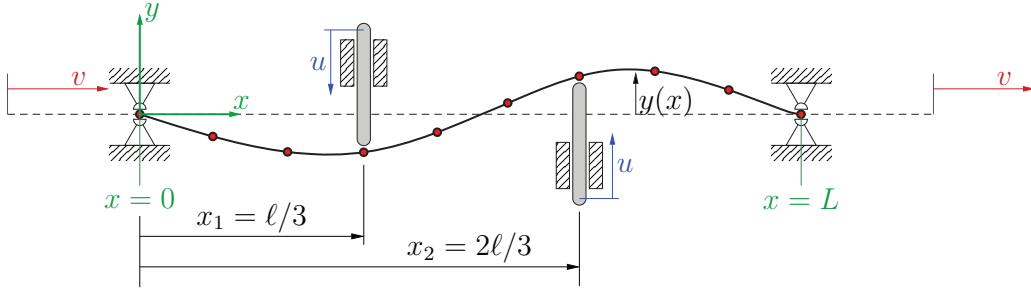


Figure 3: Elastic-plastic bending of an axially moving Euler-Bernoulli beam.

in the ODE system:

$$\mathbf{p}' = \lambda' \mathbf{g}(\mathbf{q}, \mathbf{p}). \quad (15)$$

Within the loop of predictor corrector iterations, the inhomogeneity $\mathbf{g}(\mathbf{q}, \mathbf{p})$ depends on both the predicted deformed state $\mathbf{q}$ as well as the current estimate for the plastic variables. The corresponding finite difference discretization, which may be explicit or implicit depending on how $\mathbf{g}$ is evaluated, reads:

$${}^x\Delta \mathbf{p}(x_i) = {}^x\Delta \lambda(x_i) \mathbf{g}(\mathbf{q}, \mathbf{p}), \quad (16)$$

where ${}^x\Delta \mathbf{p}$, in contrast to ${}^t\Delta \mathbf{p}$, denotes stationary (convective) plastic increments that the material point sustains as it travels the distance of two adjacent integration points Δx_i . Observe that the SIS does not require a time step Δt and is also mesh independent in the sense that the integration point spacings Δx_i do not enter this formula. To close the system of equations, the increments ${}^x\Delta \lambda$ are computed from the consistency condition $F = 0$ to achieve the projection of plastic states onto the current yield surface.

4 Stationary elastic-plastic bending of an axially moving beam

Consider the elastic-plastic bending problem of an axially moving Euler-Bernoulli beam depicted in Figure 3. As the structure passes through the open domain $0 \leq x \leq L$ at constant rate v , it experiences bending and counter-bending by two vertically deflected rigid stamps at spatially fixed positions. The contact is assumed to be frictionless and the stamps only induce transverse forces. Simply supported boundary conditions at the entry and exit of the control domain are chosen to ensure moment-free and thus yield-free boundaries.

This problem has been studied previously in [6] with a slightly less matured version of the SIS algorithm. The basic parameter set is unchanged, but a higher maximum stamp deflection $u_{\max}$ is considered to obtain plastic regions of significant extent in the final stationary state. We refer to [6] for the explicit expression of the structural plasticity law $k_M(A_p)$ and here merely repeat the basic parameter specifications (SI-units are implied): The beam has total length $\ell = 1$, a rectangular cross-section with thickness $h = 0.01$ and width $H = 0.05$. The stamps are placed at $x_1 = \ell/3$ and $x_2 = 2\ell/3$ and displaced simultaneously up to $u_{\max} = 0.025$. The linear elastic, ideal plastic material has Young's modulus $E = 210 \times 10^9$ and yield strength $\sigma_f = 260 \times 10^6$.

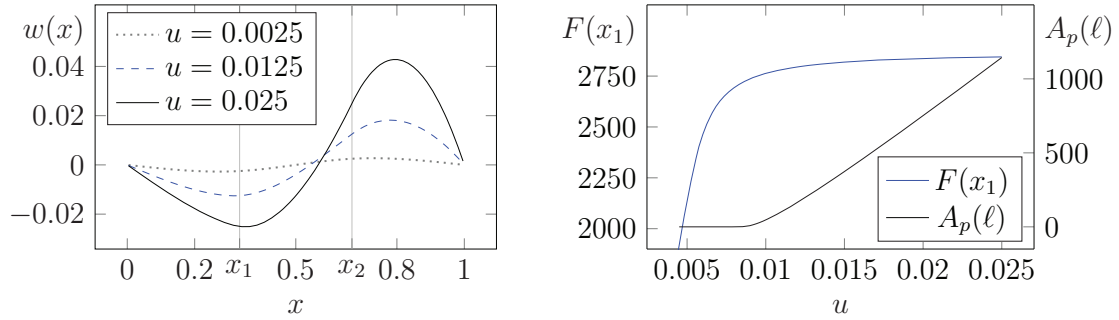


Figure 4: Visualisation of a SIS parameter study featuring: w at different stamp deflection levels (left) as well as the evolution of the forming force at the left stamp $F(x_1)$ and the accumulated plastic work $A_p(\ell)$ at the exit (right).

Δt	$A_p(\ell)$	$\epsilon[A_p]$	$F(x_1)$	$\epsilon[F(x_1)]$	$F(x_2)$	$\epsilon[F(x_2)]$
5.00×10^{-2}	1243	8.59 %	-2892	1.41 %	2918	2.57 %
2.50×10^{-2}	1221	6.68 %	-2896	1.56 %	2919	2.64 %
1.25×10^{-2}	1199	4.81 %	-2901	1.73 %	2922	2.74 %
6.25×10^{-3}	1162	1.54 %	-2875	0.82 %	2866	0.78 %
SIS	1144		-2852		2844	

Table 1: Correspondence of SIT and SIS results as the time step of the SIT simulations is decreased.

The SIS yields a sequence of stationary solutions as the deflection of the stamps is gradually increased up to the final level $u_{\max}$, which is a particularly convenient feature when studying parameter dependencies. To demonstrate these capabilities, Figure 4 depicts evolutions of the bending deflections w , of the forming force at the first stamp $F(x_1)$ and of the accumulated plastic work at the exit $A_p(\ell)$. These results were obtained with an equidistant mesh of $n = 18$ finite elements. As loading progresses the local extrema of w exhibit a significant shift in downstream direction. The degressive nature of the forming force $F(x_1)$ is clearly related to the “plastic softening” of the cross-section as the yield moment slowly approaches its limit value; the accumulated plastic work grows accordingly.

Finally, in addition to the results already published in [6], the results of a comparative study between the two algorithms are collected in Table 1. It presents final values of different SIT simulations with decreasing time step size in comparison to a reference SIS simulation. The same mesh of $n = 18$ elements was used and the material transport rate for the SIT was specified as $v = 0.2$. Total simulation time for the SIT was $t_{\max} = 20$, which means that four times the material length ℓ is transported through the control domain; at this point, practically no transient effects remain. We present values for the accumulated plastic work per unit reference length A_p evaluated at $x = \ell$ as well as the forming forces induced by the stamps at positions x_1 and x_2 . Furthermore, the persistent relative deviations, computed as $\epsilon[\alpha] = (\alpha_{\text{SIT}}/\alpha_{\text{SIS}} - 1)$

for each variable $\alpha = \{A_p(\ell), F(x_1), F(x_2)\}$, are given in percent as well. The solutions tend towards each other as the time step Δt is decreased, which comes at the price of increased simulation times for the SIT. Regarding computational efficiency, the final simulation with $\Delta t = 6.25 \times 10^{-3}$ took about an hour on a regular PC, whereas the SIS reference solution was obtained in less than two minutes on the same hardware. The substantial duration of the time-driven simulation is a consequence of the high load magnitude that induces plastic states close to limit yield, as evidenced by the almost horizontal tangent of the forming force depicted in Figure 4.

5 Stationary elastic-plastic bending of an axially moving plate

The plate bending problem is visualised in Figure 1. Kinematic constraints at the boundaries of the control domain resemble a cantilever with material flowing through in axial direction: the plate is clamped at $x = 0$, enters the domain in a fully undeformed planar state and leaves the domain at a free end. An external, self-equilibrated line load acts at the center of the domain $x = \ell/2$. We refer to [7] for the specific set of governing equations for the primary fields, which are now treated as functions of x , y and t .

With the intent to compare the SIT and SIS algorithms, exemplary computations are carried out with the following parameter set (again implying SI-units): The plate has a width of $H = 0.5$, a thickness of $h = 0.01$ and the control volume length is $\ell = 2$. The elastic ideal-plastic material is characterized by a Youngs Modulus of $E = 210 \times 10^9$, a Poisson ratio of $\nu = 0.3$ and a yield strength of $\sigma_f = 250 \times 10^6$. The lateral line load is imposed at $x = \ell/2$ and chosen to be triangular, with peak values of $\pm P$ in the center and at the edges, respectively. The load factor P is incrementally increased up to $P_{max} = 280 \times 10^3$. The set of numerical parameters for the SIT simulations comprises: the material transport rate $v = 0.2$ and the time step $\Delta t = 0.1$. The load increases linearly and reaches its peak value P_{max} at time $t = 2$. The simulation, however, proceeds until $t = 20$ to ensure that transient effects have vanished almost entirely.

As depicted in Figure 5, the plate mesh is refined in axial direction to achieve a better resolution close to the line load at $x = \ell/2$. The simulation frames on the left depict the deformed finite element mesh with an overlay of the plastic work distribution $A_p(x, y, t)$ at different time instances. The sequence clearly demonstrates the transient behaviour simulated by the SIT algorithm: After initiating the computation, the external loading soon induces plastic deformations that are further transported downstream in x -direction. Since the peak load is already reached at $t = 2$, about 90 % of the simulation time are just spent to wait for transient effects to wear off. In total, the computation on a conventional office PC takes 20 minutes with the SIT and only 8 seconds with the SIS. Additionally, the poor ratio of $v\Delta t/\Delta x_{min} \approx 1$ means that the advection problem is simulated with low accuracy. This causes a persistent deviation of approximately $\epsilon[A_p(\ell, y = 0)] = 4\%$, which refers to the final value of the plastic work distribution on the center line of the plate $A_p(x, y = 0, t_{max})$ as depicted on the right of Figure 5. These plots also visualise the transient evolution of this distribution as simulation time progresses.

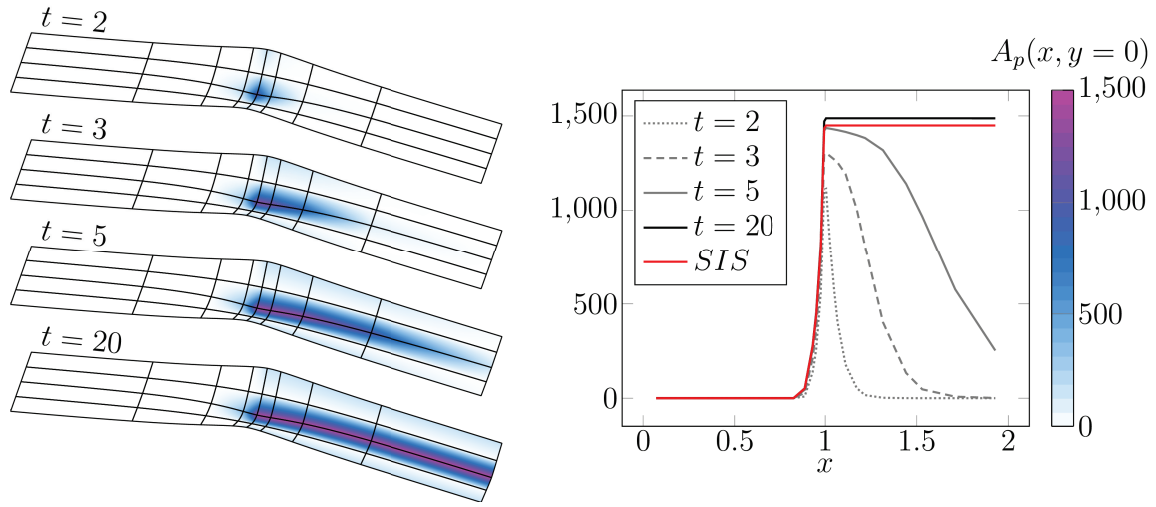


Figure 5: SIT and SIS algorithm applied to benchmark plate problem: The left column depicts the deformed configurations obtained by SIT for time instances $t = 2, 3, 5$ and 20 . The right column shows the respective plastic dissipation A_p for each integration point on the central streamline.

6 Conclusion

Different algorithms have been considered to solve the plastic advection problem in mixed Eulerian–Lagrangian finite element models for continuous bending operations of axially moving structures. The benchmark cases comprise geometrically linear bending for an axially moving Euler–Bernoulli beam and an axially moving plate. The algorithm called *Streamline Integration in Space* proves to be very efficient for the computation of steady-state solutions. It utilizes a transformation of plastic evolution laws that holds in case of stationary operation to modify the corrector step of an elastic-plastic return mapping scheme. Simulation results obtained with a standard method referred to as *Streamline Integration in Time* yield corresponding results at significantly higher computational cost.

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