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INFLUENCE OF UNCERTAINTIES ON THE STABILITY OF A SELF-EXCITED ELASTIC MULTIBODY SYSTEM

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Abstract. *In this paper, uncertainty analyses based on fuzzy arithmetic are performed and illustrated for a disc brake affected by friction-induced vibrations and modeled as an elastic multibody system. By the use of fuzzy-valued model parameters, the nominal results of conventional, crisp-valued models are extended to the inclusion of uncertainties, and valuable conclusions on the overall effect as well as on the individual influence of uncertain parameters on the output of the model can be drawn. For the definition, the simulation and the evaluation of the fuzzy-parameterized model, the software package FAMOUS is used and coupled to the multibody simulation program Neweul-*M^2.

1 INTRODUCTION

In automotive engineering, the analysis of the undesirable effect of brake squeal still remains a challenging topic [1, 2]. In order to analyze friction-induced vibrations, a special case of self-excited vibrations, the well-established frequency-domain methods of complex eigenvalue analysis [3] has been used for the development of complex, industrial structures. This method, however, is based on linearization of the system's equations of motion and tends to overestimate the number of critical frequencies that show locally unstable vibration behavior, which serve as an indicator for the occurrence of self-induced vibrations.

For time-domain investigations, elastic multibody systems based on the floating frame of reference approach are well suited since they encourage an efficient combination of large, nonlinearly described motions and small, linearly described deformations. Using time integration of the original nonlinear system, the generalized coordinates can be combined and displayed in a phase diagram, where an occurring self-induced vibration is expressed by a limit cycle. However, these results are extremely dependent on the model configuration and very sensitive to the actual parameter values, and so there is a distinct need of accounting for uncertainties.

In Section 2 of this paper, the floating frame of reference approach will be introduced to discretize kinematics and to model contact in elastic multibody systems. In Section 3, model parameters will be considered as uncertain and modeled by so-called fuzzy numbers. Finally, in Section 4, the approach of elastic multibody systems and fuzzy arithmetic will be applied to the time-domain simulation of a simplified brake system with uncertain parameters.

2 CONTACT SIMULATION BASED ON ELASTIC MULTIBODY SYSTEMS

Multibody systems (MBS) are well suited to simulate the kinematics and dynamics of mechanical systems composed by a number of rigid bodies that undergo large, nonlinearly described motions [4]. In many engineering tasks, e.g. in vibration analyses, elastic deformations have to be considered, and thus, the basic theory of MBS is extended to include small deformations in elastic bodies. This results in elastic multibody systems (EMBS), which are particularly efficient for dynamic time simulations.

2.1 Floating frame of reference approach and kinematics

Regarding the modeling of elastic multibody systems, the floating frame of reference approach is used [5]. This approach is well suited for the study of oscillations in general and friction-induced vibrations in particular, since it encourages an efficient decoupling of large, nonlinearly described motions and small, linearly described deformations.

In this approach, a so-called floating frame of reference is attached to the elastic body in order to decouple rigid body motion and elastic deformation. On the one hand, the translation vector $\mathbf{r}_{\mathrm{IR}}$ and the rotation matrix $\mathbf{S}_{\mathrm{IR}}$ of the reference system $K_{\mathrm{R}} \equiv \{\mathbf{r}_{\mathrm{IR}}, \mathbf{S}_{\mathrm{IR}}\}$ represent the large, nonlinearly described rigid motion of the body with respect to the inertial system K_{I}. On the other hand, the small, linearly described elastic deformation of an arbitrary point P can now be described in the reference system by considering two configurations, namely, the reference configuration and the current or deformed configuration.

In the following, the reference system is used for describing the kinematic magnitudes of an EMBS. As shown in Figure 1, the relative position of an arbitrary point P in the reference configuration is described by the vector $\mathbf{R}_{\mathrm{RP}}$. In an equivalent way, after being subjected to an elastic deformation, $\mathbf{r}_{\mathrm{RP}}$ represents the relative position of P in the current configuration. As a result, the deformation in between these two configurations is described by the vector $\mathbf{u}_{\mathrm{P}}$, which

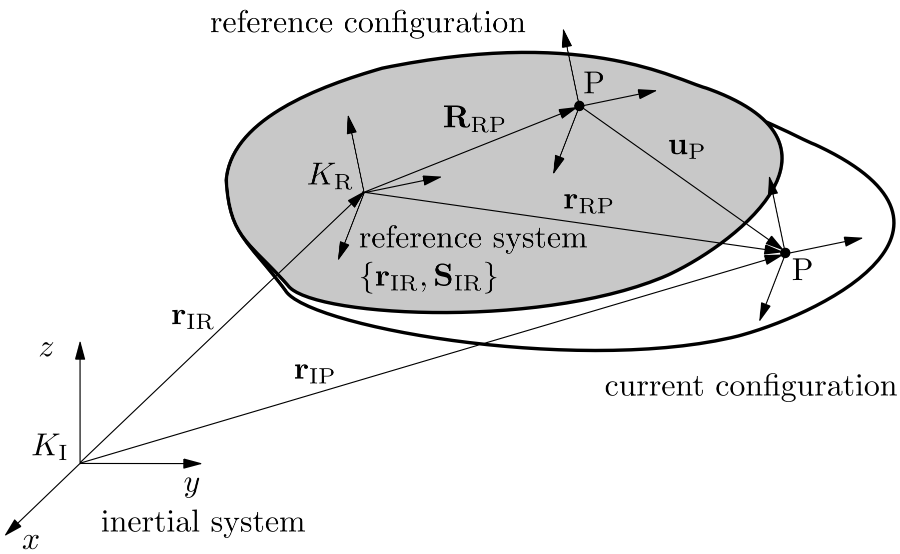

Figure 1: Floating frame of reference approach.

leads to the basic relation

$$\mathbf{r}_{\mathrm{RP}} = \mathbf{R}_{\mathrm{RP}} + \mathbf{u}_{\mathrm{P}}(\mathbf{R}_{\mathrm{RP}}, t). \tag{1}$$

By the use of global Ritz functions, an advantageous separation of the dependent variables is achieved [5]. The deformation of the elastic body is then given as

$$\mathbf{r}_{\mathrm{RP}} = \mathbf{R}_{\mathrm{RP}} + \mathbf{\Phi}_{\mathrm{P}}(\mathbf{R}_{\mathrm{RP}}) \cdot \mathbf{q}(t), \tag{2}$$

where $\mathbf{\Phi}_{\mathrm{P}}(\mathbf{R}_{\mathrm{RP}})$ are the discretization-dependent elastic modes and $\mathbf{q}(t)$ are the time-dependent elastic coordinates of the EMBS simulation. Now, the absolute position of the point P is characterized in the current configuration by

$$\mathbf{r}_{\mathrm{IP}} = \mathbf{r}_{\mathrm{IR}} + \mathbf{r}_{\mathrm{RP}} = \mathbf{r}_{\mathrm{IR}} + (\mathbf{R}_{\mathrm{RP}} + \mathbf{\Phi}_{\mathrm{P}}(\mathbf{R}_{\mathrm{RP}}) \cdot \mathbf{q}(t)), \tag{3}$$

and after differentiating Equation (3) with respect to time, its absolute velocity reads as

$$\mathbf{v}_{\mathrm{IP}} = \mathbf{v}_{\mathrm{IR}} + \tilde{\boldsymbol{\omega}}_{\mathrm{IR}} \cdot \mathbf{r}_{\mathrm{RP}} + \dot{\mathbf{r}}_{\mathrm{RP}} = \mathbf{v}_{\mathrm{IR}} + \tilde{\boldsymbol{\omega}}_{\mathrm{IR}} \cdot \mathbf{r}_{\mathrm{RP}} + \mathbf{\Phi}_{\mathrm{P}}(\mathbf{R}_{\mathrm{RP}}) \cdot \dot{\mathbf{q}}(t). \tag{4}$$

As a result, a complete kinematic description of an elastic body is available. Amongst others, this description is applied when quantifying the vibration amplitudes at specific points or when calculating the relative velocities at the contact interface, as performed in the next subsection for a friction-affected system.

2.2 Contact modeling and dynamics

The modeling of the disc-pad contact interfaces plays a decisive role in the dynamic simulation of friction-induced vibrations. In this investigation, in order to calculate the contact forces that act in the dynamics of an EMBS, a master-slave approach with a linear penalty-factor formulation [6] is used.

Based on the kinematic discretization described in the previous subsection, absolute positions and velocities for the nodes at the contact interfaces are calculated in the inertial frame. For the corresponding slave-master contact pairs, this allows the computation of penetration gaps and local relative velocities in the normal and tangential directions, respectively. In this way, normal and tangential contact forces for each contact node are calculated, which are then transformed into the modal space using Ritz functions, as done for the deformations of the body in Equation (2).

3 FUZZY ARITHMETIC FOR UNCERTAINTY QUANTIFICATION

Uncertainty quantification has acquired the interest of the engineering community in the last decades. The often unknown nature of mechanical systems requires accounting for uncertainties in the modeling process. Fuzzy arithmetical methods are especially well suited for this application and are used to represent epistemic uncertainties, such as vagueness and lack of information [7]. So-called fuzzy numbers represent the lack of knowledge in the form of a number of nested intervals that range from intervals that represent most uncertain scenarios, to nominal values that aim at quantifying situations of complete certainty.

Fuzzy arithmetic has already been successfully applied to multibody systems [8]. In practice, model parameters are defined by means of preferably triangular fuzzy numbers, defined by a nominal value and lower and upper bounds. These are then discretized for different levels α of certainty using so-called α-cuts. At this point, fuzzy arithmetic offers a number of methods, such as the general or reduced transformation methods or sparse-grid approaches to extract the most efficient parameter combinations. The uncertainties can then be propagated through the system by evaluating the time-consuming models with just a reduced number of appropriate samples. If viewed as an input-output system, uncertain model parameters are seen as fuzzy inputs that are first sampled and then evaluated in a system. The obtained results are gathered, and with appropriate retransformation methods, fuzzy outputs are calculated.

In the case of EMBS simulations, uncertainties in model parameters are propagated through the time integration, and as a result, their influence on the time-dependent output signals describing the dynamics of the system is observed. Additionally, measures of influence can be computed, providing quantitative information about the individual effect of the uncertain model parameters.

4 EXAMPLE OF A SELF-EXCITED DISC BRAKE

In this section, fuzzy arithmetic is applied to the elastic multibody system of a disc brake, as shown in Figure 2. The system consists of the brake disc (green), two brake pads (red, blue) and the caliper (olive), modeled as finite-element structures and reduced using modal reduction. The generation of the elastic bodies is performed with the EMBS-specific preprocessor MatMorembs [9], where more advanced methods are available in addition to modal reduction [10, 11].

In the presented friction-affected system, the disc rotates around the y-axis with constant rotational velocity $\omega = 1$ rad/s by means of a rotational joint which is aligned to the y-axis of the inertial system. Lower and upper pads can exhibit displacements in y-direction, and their motion is described by the coordinates y_{lpad} and y_{upad}. The pads are pressed against the disc by means of 25 separate nodal forces $F_{\mathrm{n}} = 16$ kN with a total normal force of 400 kN at each pressure surface. Furthermore, each pad is connected to the caliper via 25 springs with stiffness $k = 200$ kN/m and act in the y-direction in order to couple pad and caliper dynamics.

Regarding the caliper, it is connected to the inertial system on its lower-pad side by four springs $k_\mathrm{c} = 1$ kN/m acting in the y-direction. Its out-of-plane motion is represented by the generalized coordinate y_cali.

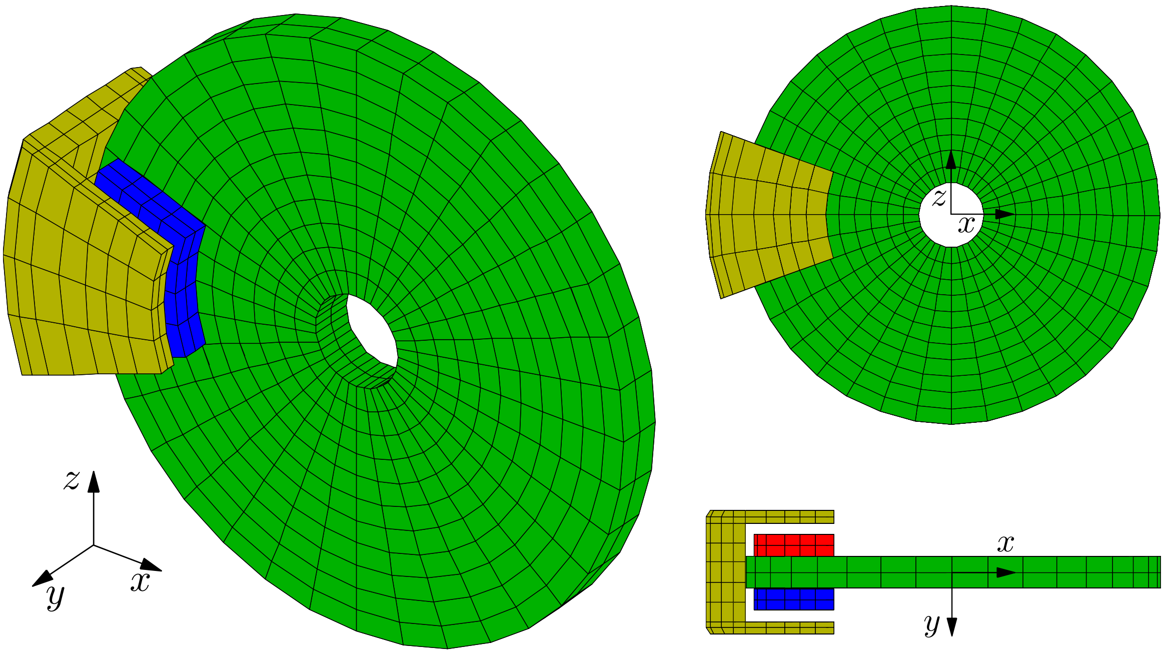

Figure 2: Example of a disc brake with disc, pads and caliper.

As far as contact modeling is concerned, interfaces between each pad and the disc are defined. For the elastic discretization, 25 and 48 nodes are selected, respectively, where the disc nodes form an annular sector for efficient contact computation. As the normal direction force law, a linear penalty formulation is used with a penalty factor of $c_p = 10^8$ N/m, while for the tangential friction law, a constant Coulomb coefficient of $\mu = 0.7$ is selected.

4.1 Nominal results

The model described in the previous section is modeled in the Matlab-based symbolic EMBS-program Neweul-M^2 [12]. Based on the floating frame of reference approach, nonlinear equations of motion are calculated symbolically. With the nominal characteristics described before, a time integration is performed from $t_0 = 0$ ms to $t_\mathrm{end} = 10$ ms. As an ODE integrator, an explicit, fourth-order Runge-Kutta method with a fixed time step of $\Delta t = 10^{-6}$ s is used. Initial conditions for positions and velocities for both rigid and elastic coordinates are set to zero.

In the simulation results shown in Figure 3, the three generalized coordinates y_lpad, y_upad and y_cali are plotted. The caliper displacement y_cali develops a smooth motion in comparison to the pads. If longer simulations are performed, it can be observed that the caliper motion becomes instable for the friction coefficient. Regarding the pad signals, more complex vibrations that oscillate around mean values of 45 μm and -50 μm are observed. Despite both displacements being identical at the beginning of the simulation for both pads, notable differences can be observed from $t = 2$ ms on. This behavior arises due to the influence of the lower-pad side fixation, which inserts a source of asymmetry into the system.

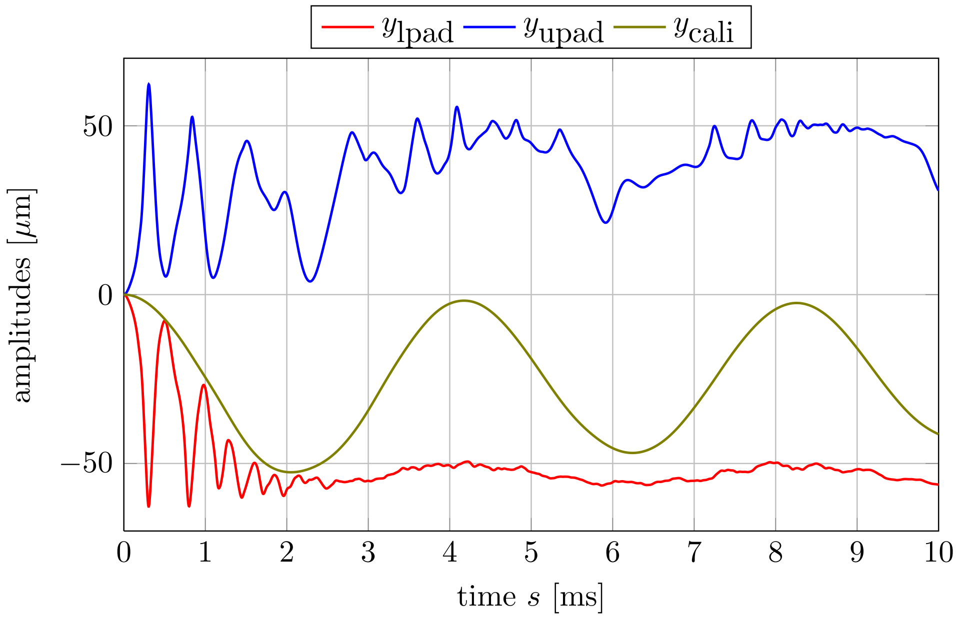

Figure 3: Nominal results of the time integration.

4.2 Consideration of uncertain parameters

Now, uncertainty analyses based on fuzzy arithmetic are performed. By means of triangular fuzzy numbers, uncertainties are considered in the model parameters, which are then propagated through the system. In order to model, evaluate and postprocess system uncertainties, the Matlab-based software package FAMOUS [13] (Fuzzy Arithmetical Modeling of Uncertain Systems) is used.

In a first analysis, the spring stiffnesses k and k_c and the nodal forces F_n are defined as uncertain parameters. For the stiffnesses, lower and upper bounds of $\pm 10\%$ are assumed, while for the nodal forces, a smaller uncertainty of $\pm 1\%$ is selected. The triangular fuzzy numbers are discretized using four α-cuts, i.e. four intervals at $\mu_\alpha = \{0, 0.25, 0.50, 0.75\}$ and a nominal value at $\mu_\alpha = 1$. In order to calculate the fuzzy outputs of the system, the reduced transformation method is applied, which results in 33 evaluations of the model.

Based on this uncertainty analysis, the nominal results in Figure 3 can be expanded towards contour plots, which represent the different certainty levels of the signal. The blue contour lines range from light lines for complete uncertainty ($\mu = 0$) to dark lines for complete certainty ($\mu = 1$). Furthermore, measures of absolute and relative influence of the uncertain model parameters on the overall uncertainty of the results can be calculated. The corresponding results in Figure 4 show a notable influence of the uncertain model parameters on the transversal motion of the upper pad y_{upad}, where the maximal absolute influence reaches 10^{-3} μm. For the lower pad and caliper motions y_{lpad} and y_{cali}, instead, these values barely reach 20% and 40% thereof. These uncertainties are mainly caused by the uncertainty assumed for the nodal force F_n. Despite its assumed uncertainty being of just 1%, its relative influence is about 70% and 60% for the generalized coordinates. As expected, the disc-pad interface stiffness k the influences pad motions more than k_c, and vice versa for the caliper displacement y_{cali}. Finally, the influence of k_c is also substantial up to $t = 2$ ms, falling beyond F_n and k thereafter.

In a second analysis, the two stiffness parameters are considered as uncertain again. Instead of the normal force F_n, which can be considered as an operational or service parameter, the rotational velocity ω is now assumed as uncertain with a worst-case uncertainty of $\pm 10\%$. In

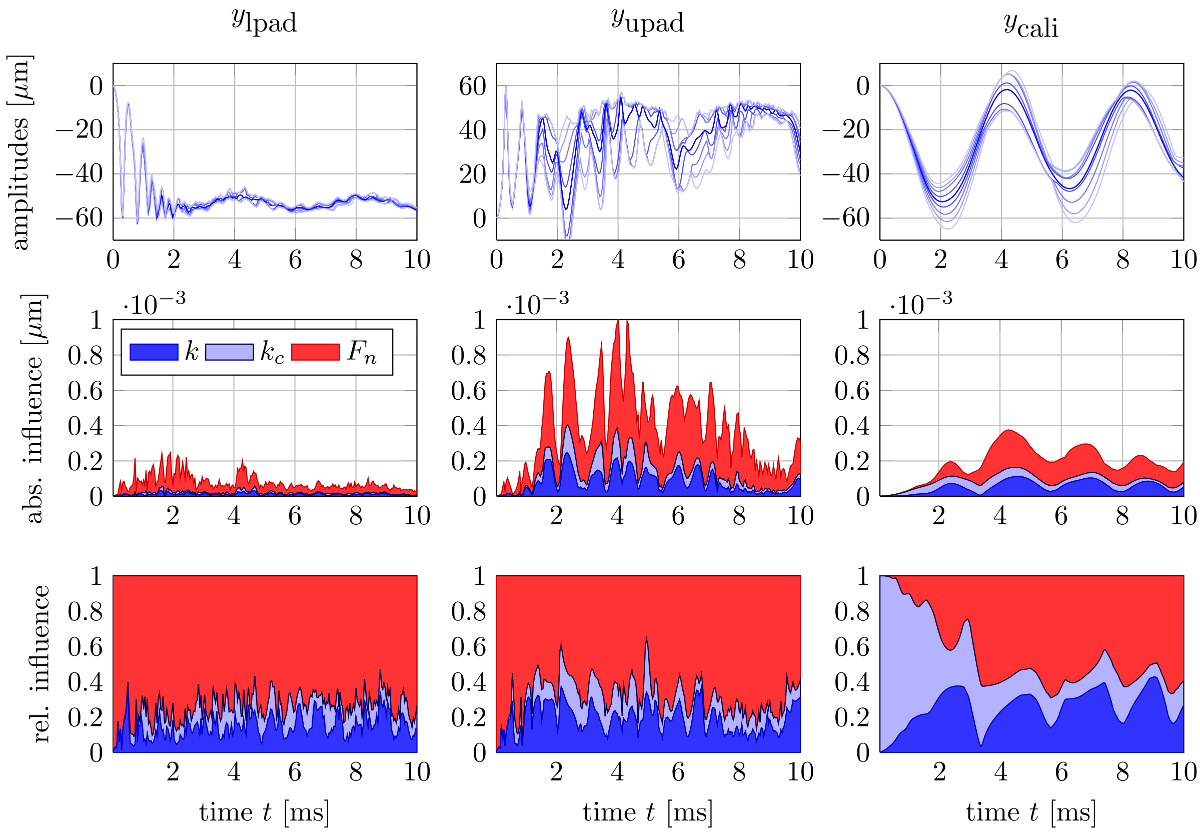

Figure 4: Stiffnesses k and k_{c} with $\pm 10\%$ and normal force F_{n} with $\pm 1\%$ uncertainty.

comparison to the first uncertainty analysis, the maximal absolute influence of the system is reduced to $0.4 \cdot 10^{-3}$, see Figure 5. Indeed, the influence of the stiffness parameters k and k_{c} excels the operational parameter ω, whose relative influence barely exceeds 20% and 10% for pad and caliper motions, respectively. As expected and already observed in the previous analysis, the uncertainty in the stiffness parameter k is relevant for the brake pads especially at the beginning of the simulation. For the relative influence of k_{c} in the caliper displacement y_{cali}, the same behavior as in the first uncertainty analysis is observed.

5 CONCLUSIONS

In this paper, elastic multibody systems are proposed to investigate the dynamic behavior of a brake system. The advantageous decoupling of large rigid body motion and small elastic deformations enables an efficient computation of the contact and the vibration amplitude in the time domain. Since these models may be sensitive to the current model parameter configuration, there is a distinct need for the systematic consideration of uncertainties. For this reason, uncertainty analyses based on fuzzy arithmetical methods are proposed, providing valuable insight and quantitative information. In the elastic multibody system of a simplified brake system, the influence of modeling stiffnesses together with operational parameters, such as applied nodal forces and rotational speed, is investigated.

The results obtained from the uncertainty analyses emphasize the importance of the applied force in the dynamical behavior of the system. Furthermore, in future time-domain investigations, particular attention should be paid on the definition of interfaces between the elastic bodies.

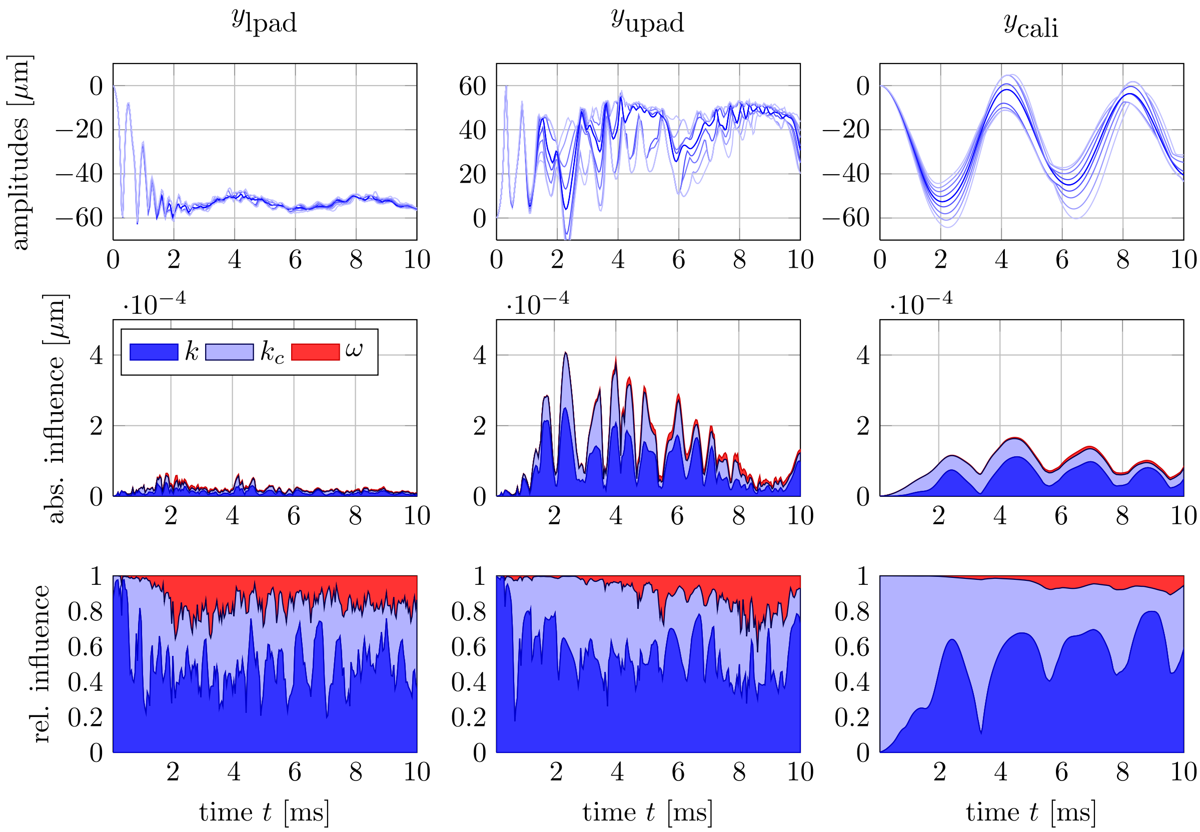

Figure 5: Stiffnesses k and k_c with $\pm 10\%$ and rotational velocity ω with $\pm 1\%$ uncertainty.

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